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SOME FIXED POINT RESULTS ON SYMMETRIC SOFT *G*-COMPLETE METRIC SPACES

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Abstract

In the present paper, we have studied some important fixed point results on symmetric soft G-complete metric spaces using soft mappings. Afterwards the ideas of soft G-totally bounded, soft G-compact metric spaces are given and a fixed point result on soft G-compact metric spaces using soft mapping have conferred. We have also discussed about the converse of the above result with suitable examples.

1. Introduction

In 1999, the idea of soft set was first initiated by D. Molodtsov [10]. After that Maji et al. [8] studied this theory in detail. Currently, in different branches of mathematics and its applications, research on soft set theory are progressing at a very fast pace. The concept of soft mappings are given by several researcher such as Kharal et al. [7], Majumder et al. [9] and Nazmul et al. [13] in their own form, soft real set, soft real number and soft metric spaces was introduced and studied some of their important properties by Das and Samanta [1, 2].

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On the other hand, many researchers have worked on the generalization of metric spaces. After Frechet and Hausdorff, Gahler [6] in 1963 introduced 2-metrics and subsequently a more general form n-metric space and claimed that it is an extension of metric space (1-metric space). Also, Mustafa et al. [11, 12] introduced another approach called G-metric space and discussed some important fixed point results on this spaces.

Recently, Guler et al. [4] have studied the behavior of G-metric spaces in soft set setting and gave the notion of soft G-metric spaces. They have mentioned an existence and uniqueness theorem of fixed point in this spaces. Afterwards, Guler and Yildirim [5] have also introduced soft G-complete metric spaces and some fixed point results are investigated.

As a continuation, our prime aim to extend this study of fixed point results in soft G-complete metric space. In this paper, firstly we have discussed some different fixed point results on symmetric soft G-complete metric spaces using soft mappings. After that we have introduced soft G-totally bounded, soft G-compact spaces and the celebrated existence and uniqueness theorem of fixed point on soft G-compact spaces is established. We have also studied the behavior of the converse of the above result with suitable examples.

2. Preliminaries

In this section, some preliminary definitions and results are stated which are used in the main section. Unless otherwise mentioned, X, E, $P(X)$ are respectively denotes an initial universal set, the set of parameters and the power set of X.

Definition 2.1 [10]. Let $F : A \to P(X)$ be a mapping. Then the pair (F, A) is said to be a soft set over X; where $A \subseteq E$.

Definition 2.2 [1, 2]. A soft element of $\widetilde{X}$ is a function $\varepsilon : E \to X$. A soft element ε of $\widetilde{X}$ is said to belongs to a soft set (F, E) over X, that is $\varepsilon \mathrel{\widetilde{\in}} (F, E)$, if $\varepsilon(e) \in F(e), \forall e \in E$. Thus $F(e) = \{\varepsilon(e) : \varepsilon \mathrel{\widetilde{\in}} (F, E)\}$.

Remark 2.3 [2]. The collection of all soft elements of $\widetilde{X}$ is denoted by $SE(\widetilde{X})$.

Definition 2.4 [1]. Let $\mathscr{B}(\mathbb{R})$ be the collection of all non-empty bounded subsets of $\mathbb{R}$, the set of real numbers. Then the pair (F, A) is said to be

(i) a soft real set if $F : A \to \mathscr{B}(\mathbb{R})$.

(ii) a soft real number if $F : A \to \mathbb{R}$. The soft real number and the collection of all soft real numbers are denoted by the symbols $\tilde{r}$ and $\mathbb{R}(A)$ respectively. Also we have denoted a particular type soft real number by $\bar{s}$ where $\bar{s}(e) = s, \forall e \in A$.

Definition 2.5 [1]. Let $(F, A), (G, A) \tilde{\in} \mathbb{R}(A)$. Then,

(1) $(F, A), (G, A)$ if $F(e) = G(e), \forall\, e \in A$.

(2) $(F + G)(e) = F(e) + G(e), \forall\, e \in A$.

(3) $(F - G)(e) = F(e) - G(e), \forall\, e \in A$.

(4) $(F \cdot G)(e) = F(e) \cdot G(e), \forall\, e \in A$.

(5) $\frac{F}{G}(e) = \frac{F(e)}{G(e)}, \forall\, e \in A$ provided $G(e) \neq 0$.

Definition 2.6 [2]. Let $\tilde{r}, \tilde{s}$ be two soft real numbers. Then,

(a) $\tilde{r} \,\tilde{\leq}\, (\tilde{<})\tilde{s}$ if $\tilde{r}(e) \leq (<)\tilde{s}(e), \forall e \in A$.

(b) $\tilde{r} \,\tilde{\geq}\, (\tilde{>})\tilde{s}$ if $\tilde{r}(e) \geq (>)\tilde{s}(e), \forall e \in A$.

Definition 2.7 [4]. Let X be a nonempty set and E be the nonempty set of parameters. A mapping $\tilde{G} : SE(\tilde{X}) \times SE(\tilde{X}) \times SE(\tilde{X}) \to \mathbb{R}(E)^*$, where $\mathbb{R}(E)^*$ be the set of all non-negative soft real numbers, is said to be a soft generalized metricor soft G-metric on $\tilde{X}$ if $\tilde{G}$ satisfies the following conditions:

$(\tilde{G}_1)$ $\tilde{G}(\tilde{x}, \tilde{y}, \tilde{z}) = \bar{0}$ if $\tilde{x} = \tilde{y} = \tilde{z}$,

$(\tilde{G}_2)$ $\bar{0} \,\tilde{<}\, \tilde{G}(\tilde{x}, \tilde{y}, \tilde{z})$ for all $\tilde{x}, \tilde{y} \,\tilde{\in}\, SE(\tilde{X})$ with $\tilde{x} \neq \tilde{y}$,

$(\widetilde{G}_3)$ $\widetilde{G}(\widetilde{x}, \widetilde{y}, \widetilde{z}) \widetilde{\leq} \widetilde{G}(\widetilde{x}, \widetilde{y}, \widetilde{z})$ for all $\widetilde{x}, \widetilde{y}, \widetilde{z} \widetilde{\in} SE(\widetilde{X})$, with $\widetilde{y} \neq \widetilde{z}$,

$(\widetilde{G}_4)$ $\widetilde{G}(\widetilde{x}, \widetilde{y}, \widetilde{z}) = \widetilde{G}(\widetilde{x}, \widetilde{z}, \widetilde{y}) = \widetilde{G}(\widetilde{y}, \widetilde{z}, \widetilde{x}) = \dots$,

$(\widetilde{G}_5)$ $\widetilde{G}(\widetilde{x}, \widetilde{y}, \widetilde{z}) \widetilde{\leq} \widetilde{G}(\widetilde{x}, \widetilde{a}, \widetilde{a}) + \widetilde{G}(\widetilde{a}, \widetilde{y}, \widetilde{z})$ for all $\widetilde{x}, \widetilde{y}, \widetilde{z}, \widetilde{a} \widetilde{\in} SE(\widetilde{X})$.

The soft set $\widetilde{X}$ with a soft G-metric $\widetilde{G}$ on $\widetilde{X}$ is said to be a soft G-metric space and is denoted by $(\widetilde{X}, \widetilde{G}, E)$.

Definition 2.8 [4]**.** A soft G-metric space $(\widetilde{X}, \widetilde{G}, E)$ is symmetric if

$$(G_6)\ \tilde{G}(\tilde{x}, \tilde{y}, \tilde{y}) = \tilde{G}(\tilde{x}, \tilde{x}, \tilde{y}) \text{ for all } \widetilde{x}, \widetilde{y} \widetilde{\in} SE(\widetilde{X}).$$

Proposition 2.9 [4]**.** *For any soft G-metric* $\widetilde{G}$ *on* $\widetilde{X}$, *we can construct a soft metric* $d_{\widetilde{G}}$ *on* $\widetilde{X}$ *defined by,*

$$d_{\widetilde{G}}(\widetilde{x}, \widetilde{y}) = \widetilde{G}(\widetilde{x}, \widetilde{y}, \widetilde{y}) + \widetilde{G}(\widetilde{x}, \widetilde{x}, \widetilde{y}).$$

Definition 2.10 [4]**.** Let $(\widetilde{X}, \widetilde{G}, E)$ be a soft G-metric space. For $\widetilde{a} \widetilde{\in} SE(\widetilde{X})$ and $\widetilde{r} \widetilde{>} \overline{0}$, the G-ball with a center ea and radius $\widetilde{r}$ is

$$B_{\widetilde{G}}(\widetilde{a}, \widetilde{r}) = \{\widetilde{x} \widetilde{\in} SE(\widetilde{X}) : \widetilde{G}(\widetilde{a}, \widetilde{x}, \widetilde{x}) \widetilde{<} \widetilde{r}\} \widetilde{\subseteq} SE(\widetilde{X}).$$

Definition 2.11 [4]**.** Let $(\widetilde{X}, \widetilde{G}, E)$ be a soft G-metric space and $\{x_n\}$ be a sequence of soft elements in $\widetilde{X}$. The sequence $\{x_n\}$ is said to be soft G-convergent at $\widetilde{x}$ in $\widetilde{X}$ if for every $\widetilde{\epsilon} \widetilde{\geq} \overline{0}$, chosen arbitrarily, $\exists$ natural number $N = N(\widetilde{\epsilon})$ such that $\overline{0} \widetilde{\leq} \tilde{G}(x_n, x_n, \tilde{x}) \widetilde{<} \tilde{\epsilon}$ whenever $n \widetilde{\geq} N$.

Definition 2.12 [5]**.** Let $(\widetilde{X}, \widetilde{G}, E)$ be a soft G-metric space and $\{x_n\}$ be a sequence of soft elements in $\widetilde{X}$. The sequence $\{x_n\}$ is said to be soft G-Cauchy if for every $\widetilde{\epsilon} \widetilde{\geq} \overline{0}$, chosen arbitrarily, $\exists$ natural number k such that $\tilde{G}(x_n, x_n, x_l) \widetilde{<} \tilde{\epsilon}$ whenever $n, m, l \widetilde{\geq} k$. i.e.; $\tilde{G}(x_n, x_n, x_l) \to \tilde{0}$ as $i, j \to \infty$.

Definition 2.13 [5]**.** A soft G-metric space $(\widetilde{X}, \widetilde{G}, E)$ is said to be soft G-

complete if every soft G-Cauchy sequence in $(\widetilde{X}, \widetilde{G}, E)$ is soft G-convergent in $(\widetilde{X}, \widetilde{G}, E)$.

Definition 2.14 [5]**.** A soft G-metric space $(\widetilde{X}, \widetilde{G}, E)$ is soft G-complete if and only if $(\widetilde{X}, d_{\widetilde{G}}, E)$ is complete soft metric space.

Definition 2.15 [4]**.** Let $(\widetilde{X}, \widetilde{G}, E)$ be a soft G-metric space. Let $T : (\widetilde{X}, \widetilde{G}, E) \to (\widetilde{X}, \widetilde{G}, E)$ be a mapping and $x_0 \tilde{\in} SE(\tilde{X})$ be a soft element such that $T(x_0) = x_0$, then x_0 is called a fixed point of T.

Theorem 2.16 [5, 6]**.** *Let $(\widetilde{X}, \widetilde{G}, E)$ be a soft G-complete space and $T : (\widetilde{X}, \widetilde{G}, E) \to (\widetilde{X}, \widetilde{G}, E)$ be a mapping that satisfies the following condition for all $\tilde{x}, \tilde{y}, \tilde{z} \tilde{\in} SE(\widetilde{X})$,*

$$\widetilde{G}(T\tilde{x}, T\tilde{y}, T\tilde{z}) \tilde{\leq} \bar{a}\, \widetilde{G}(\tilde{x}, T\tilde{x}, T\tilde{x}) + \bar{b}\, \widetilde{G}(\tilde{y}, T\tilde{y}, T\tilde{y}) + \bar{c}\, \widetilde{G}(\tilde{z}, T\tilde{z}, T\tilde{z})$$

$$+ \bar{d}\, \widetilde{G}(\tilde{x}, \tilde{y}, \tilde{z}) \tag{2.1}$$

where $\bar{0} \tilde{\leq} +\bar{a} + \bar{b} + \bar{c} + \bar{d} \tilde{<} \bar{1}$. Then T has a unique fixed point.

3. Main results

In this section, we have studied some important fixed point results in symmetric soft G-complete metric spaces. The definitions of soft G-totally bounded space, soft G-compact space are given and celebrated fixed point theorem on soft G-compact space is established. We have also studied the behavior of converse of the above theorem with suitable examples.

Theorem 3.1. *Let $(\widetilde{X}, \widetilde{G}, E)$ be a symmetric soft G-complete metric space and U, V be two self-maps on $(\widetilde{X}, \widetilde{G}, E)$ satisfying the following conditions,*

$$\widetilde{G}(U(\tilde{x}), U(\tilde{x}), V(\tilde{y})) \tilde{\leq} \bar{a}\, \widetilde{G}(\tilde{x}, \tilde{x}, \tilde{y}) + b\, \widetilde{G}(\tilde{x}, \tilde{x}, U(\tilde{x})) + \bar{c}\, \widetilde{G}(\tilde{y}, \tilde{y}, V(\tilde{y}));$$

$$\forall\, \tilde{x}, \tilde{y} \tilde{\in} SE(\widetilde{X}),$$

where $\overline{a}, \overline{b}$ *and* $\overline{c}$ *are non negative soft reals such that* $\overline{0} \,\tilde{\leq}\, +\overline{a} + \overline{b} + \overline{c} + \overline{d} \,\tilde{<}\, \overline{1}$. *Then U and V have a unique common fixed point.*

Proof. Let $x_0 \tilde{\in} SE(\tilde{X})$ be a soft element.

Let us define a sequence $\{x_n\}$ by,

$$x_{2k+1} = U(x_{2k}), x_{2k+2} = V(x_{2k+1}); k = 0, 1, 2, \ldots$$

Then,

$$\tilde{G}(x_{2k+1}, x_{2k+1}, x_{2k+2}) = \tilde{G}(U(x_{2k}), U(x_{2k}), V(x_{2k+1}))$$

$$\tilde{\leq} \overline{a}\tilde{G}(x_{2k}, x_{2k}, x_{2k+1}) + \overline{b}\tilde{G}(x_{2k}, x_{2k}, x_{2k+1})$$

$$\overline{c}\tilde{G}(x_{2k+1}, x_{2k+1}, x_{2k+2})$$

$$\Rightarrow (\overline{1} - \overline{c})\tilde{G}(x_{2k+1}, x_{2k+1}, x_{2k+2}) \tilde{\leq} (\overline{a} - \overline{b})\tilde{G}(x_{2k}, x_{2k}, x_{2k+1})$$

$$\Rightarrow \tilde{G}(x_{2k+1}, x_{2k+1}, x_{2k+2}) \tilde{\leq} \frac{\overline{a} + \overline{b}}{\overline{1} - \overline{c}}\tilde{G}(x_{2k}, x_{2k}, x_{2k+1})$$

Let $\overline{h_1} = \dfrac{\overline{a} + \overline{b}}{\overline{1} - \overline{c}}$. Then $\overline{0} \,\tilde{\leq}\, \overline{h_1} \,\tilde{<}\, \overline{1}$ as $\overline{a}, \overline{b}, \overline{c}$ are non negative soft reals with $\overline{0} \,\tilde{\leq}\, +\overline{a} + \overline{b} + \overline{c} + \overline{d} \,\tilde{<}\, \overline{1}$.

Therefore, $\tilde{G}(x_{2k+1}, x_{2k+1}, x_{2k+2}) \tilde{\leq} \overline{h_1}\, \tilde{G}(x_{2k}, x_{2k}, x_{2k+1})$.

Again,

$$\tilde{G}(x_{2k+2}, x_{2k+2}, x_{2k+3}) = \tilde{G}(V(x_{2k+1}, Vx_{2k+1}, Ux_{2k+2}))$$

$$= \tilde{G}(V(x_{2k+1}, Ux_{2k+2}, Ux_{2k+2})), \text{ since } \tilde{G} \text{ is symmetric.}$$

$$= \tilde{G}(U(x_{2k+2}, Ux_{2k+2}, Vx_{2k+1})),$$

$$\tilde{\leq} \overline{a}\, \tilde{G}(x_{2k+2}, x_{2k+2}, x_{2k+1}) + \overline{b}\, \tilde{G}(x_{2k+2}, x_{2k+2}, x_{2k+3})$$

$$+ \overline{c}\, \tilde{G}(x_{2k+1}, x_{2k+1}, x_{2k+2})$$

$$= \overline{a}\, \tilde{G}(x_{2k+2}, x_{2k+1}, x_{2k+1}) + \overline{b}\, \tilde{G}(x_{2k+2}, x_{2k+2}, x_{2k+3})$$

$$\bar{c}\ \tilde{G}(x_{2k+1}, x_{2k+1}, x_{2k+2}),\ \text{since } \tilde{G} \text{ is symmetric.}$$

$$\Rightarrow (\bar{1} - \bar{c})\tilde{G}(x_{2k+2}, x_{2k+2}, x_{2k+3}) \tilde{\leq} (\bar{a} + \bar{c})\ \tilde{G}(x_{2k+1}, x_{2k+1}, x_{2k+2})$$

$$\Rightarrow \tilde{G}(x_{2k+2}, x_{2k+2}, x_{2k+3}) \tilde{\leq} \frac{\bar{a} + \bar{c}}{\bar{1} - \bar{b}} \tilde{G}(x_{2k+1}, x_{2k+1}, x_{2k+2})$$

Let $\overline{h_2} = \frac{\bar{a} + \bar{c}}{\bar{1} - \bar{b}}$. Then $\bar{0} \tilde{\leq} \overline{h_2} \tilde{<} \bar{1}$, as $\bar{a}, \bar{b}, \bar{c}$ are non negative soft reals with $\bar{0} \tilde{\leq} +\bar{a} + \bar{b} + \bar{c} + \bar{d} \tilde{<} \bar{1}$. Therefore,

$$\tilde{G}(x_{2k+2}, x_{2k+2}, x_{2k+3}) \tilde{\leq} \overline{h_2}\ \tilde{G}(x_{2k+1}, x_{2k+1}, x_{2k+2}).$$

Taking $\bar{h} = \max\ \{\overline{h_1}, \overline{h_2}\}$. Then,

$$\tilde{G}(x_{2k+1}, x_{2k+1}, x_{2k+2}) \tilde{\leq} \bar{h}\ \tilde{G}(x_{2k}, x_{2k}, x_{2k+1})$$

i.e., $\tilde{G}(x_{2k+1}, x_{2k+2}, x_{2k+2}) \tilde{\leq} \bar{h}\ \tilde{G}(x_{2k}, x_{2k+1}, x_{2k+1})$, since $\tilde{G}$ is symmetric and,

$$\tilde{G}(x_{2k+2}, x_{2k+2}, x_{2k+3}) \tilde{\leq} \bar{h}\ \tilde{G}(x_{2k+1}, x_{2k+1}, x_{2k+2})$$

i.e., $\tilde{G}(x_{2k+2}, x_{2k+3}, x_{2k+3}) \tilde{\leq} \bar{h}\ \tilde{G}(x_{2k+1}, x_{2k+2}, x_{2k+2})$, since $\tilde{G}$ is symmetric

Therefore,

$$\tilde{G}(x_n, x_{n+1}, x_{n+1}) \tilde{\leq} \bar{h}\ \tilde{G}(x_{n-1}, x_n, x_n)$$

$$\tilde{\leq} \bar{h}^2\ \tilde{G}(x_{n-2}, x_{n+1}, x_{n-1})$$

$$\vdots$$

$$\tilde{\leq} \bar{h}^n\ \tilde{G}(x_0, x_1, x_1)$$

Thus, for all $n, m \in \mathbb{N}$, $n < m$, we have,

$$\tilde{G}(x_n, x_m, x_m) \tilde{\leq} \tilde{G}(x_n, x_{n+1}, x_{n+1}) + \tilde{G}(x_{n+1}, x_{n+2}, x_{n+2})$$

$$+ \tilde{G}(x_{n+2}, x_{n+3}, x_{n+3}) + \ldots + \tilde{G}(x_{m-1}, x_m, x_m)$$

$$\tilde{\leq} ((\bar{h})^n (\bar{h})^{n+1} + \ldots + (\bar{h})^{m-1})\tilde{G}(x_0, x_1, x_1)$$

$$\tilde{\leq} \frac{(\bar{h})^n}{\bar{1} - \bar{h}} \tilde{G}(x_0, x_1, x_1).$$

i.e.; $\tilde{G}(x_n, x_m, x_m) \to \bar{0}$ as $n, m \to \infty$, since $\bar{0} \tilde{\leq} \bar{h} \tilde{<} \bar{1}$.

Now for $n, m, l \in \mathbb{N}$, (G_5) of definition 2.7, implies that,

$$\tilde{G}(x_n, x_m, x_l) \tilde{\leq} \tilde{G}(x_n, x_m, x_m) + \tilde{G}(x_l, x_m, x_m)$$

Taking limit as $n, m, l \to \infty$, we get,

$$\tilde{G}(x_n, x_m, x_l) \to \bar{0}.$$

So $\{x_n\}$ is a soft G-Cauchy sequence and by completeness of $(\tilde{X}, \tilde{G}, E)$, there exists $\tilde{t} \tilde{\in} SE(\tilde{X})$ such that $\{x_n\}$ is soft G-converges to $\tilde{t}$, i.e.; $x_n \to \tilde{t}$ as $n \to \infty$.

Now,

$$\tilde{G}(U(\tilde{t}), x_n, x_n) \tilde{\leq} \bar{a} \tilde{G}(\tilde{t}, \tilde{t}, x_{n-1}) + \bar{b} \tilde{G}(\tilde{t}, \tilde{t}, U(\tilde{t}))$$

$$+ \bar{c} \tilde{G}(x_{n-1}, x_{n-1}, x_n)$$

$$\tilde{\leq} \bar{a} \tilde{G}(\tilde{t}, \tilde{t}, \tilde{t}) + \bar{b} \tilde{G}(U(\tilde{t}), \tilde{t}, \tilde{t})$$

$$\tilde{\leq} \bar{c} \tilde{G}(\tilde{t}, \tilde{t}, \tilde{t}), [\text{by taking limit } n \to \infty]$$

$$\Rightarrow \tilde{G}(U(\tilde{t}), \tilde{t}, \tilde{t}) + \bar{b} \tilde{G}(U(\tilde{t}), \tilde{t}, \tilde{t})$$

$$\Rightarrow U(\tilde{t}) = \tilde{t}, \ [\text{since } \bar{0} \tilde{\leq} \bar{a} + \bar{b} + \bar{c} \tilde{<} \bar{1}]$$

Again,

$$\tilde{G}(\tilde{t}, \tilde{t}, V(\tilde{t})) = \tilde{G}(U(\tilde{t}), U(\tilde{t}), V(\tilde{t}))$$

$$\tilde{\leq} \bar{a} \tilde{G}(\tilde{t}, \tilde{t}, \tilde{t}) + \bar{b} \tilde{G}(\tilde{t}, \tilde{t}, U(\tilde{t}))$$

$$+ \bar{c} \tilde{G}(\tilde{t}, \tilde{t}, V(\tilde{t}))$$

$$\tilde{\leq} \bar{a} \tilde{G}(\tilde{t}, \tilde{t}, \tilde{t}) + \bar{b} \tilde{G}(\tilde{t}, \tilde{t}, \tilde{t})$$

$$+ \bar{c}\, \tilde{G}(\tilde{t}, \tilde{t}, V(\tilde{t}))$$

$$\Rightarrow (\bar{1} - \bar{c})\tilde{G}(\tilde{t}, \tilde{t}, V(\tilde{t})) \tilde{\leq} \bar{0}$$

$$\Rightarrow V(\tilde{t})) = \tilde{t}, \ \text{[since } \bar{0} \tilde{\leq} \bar{a} + \bar{b} + \bar{c} \tilde{<} \bar{1}]$$

Therefore, $U(\tilde{t}) = \tilde{t} = V(\tilde{t})$.

i.e.; U and V have a common fixed point.

To show uniqueness, let $\tilde{t}^* \tilde{\in} SE(\tilde{X})$ be another fixed point of U and V with $\tilde{t} \neq \tilde{t}^*$.

Now,

$$\tilde{G}(\tilde{t}, \tilde{t}, \tilde{t}^*) = \tilde{G}(U(\tilde{t}), U(\tilde{t}), V(\tilde{t}^*))$$

$$\tilde{\leq} \bar{a}\, \tilde{G}(\tilde{t}, \tilde{t}, \tilde{t}^*) + \bar{b}\, \tilde{G}(\tilde{t}, \tilde{t}, U(\tilde{t}))$$

$$+ \bar{c}\, \tilde{G}(\tilde{t}, \tilde{t}, V(\tilde{t}^*))$$

$$= \bar{a}\, \tilde{G}(\tilde{t}, \tilde{t}, \tilde{t}^*) + \bar{b}\, \tilde{G}(\tilde{t}, \tilde{t}, \tilde{t})$$

$$+ \bar{c}\, \tilde{G}(\tilde{t}, \tilde{t}, \tilde{t}^*)$$

$$\Rightarrow \tilde{G}(\tilde{t}, \tilde{t}, \tilde{t}^*) = (\bar{a} + \bar{c})\tilde{G}(\tilde{t}, \tilde{t}, \tilde{t}^*)$$

$$\Rightarrow \tilde{t} = \tilde{t}^* \ \text{[since } \bar{0} \tilde{\leq} \bar{a} + \bar{b} + \bar{c} \tilde{<} \bar{1}]$$

Hence, U and V have a unique common fixed point. □

Theorem 3.2. *Let $(\tilde{X}, \tilde{G}, E)$ be a symmetric soft G-complete metric space and U, V be two self-maps on $(\tilde{X}, \tilde{G}, E)$ satisfying the following conditions,*

$$\tilde{G}(U(\tilde{x}), U(\tilde{x}), V(\tilde{y})) \tilde{\leq} \bar{a}\, \tilde{G}(\tilde{x}, \tilde{x}, \tilde{y}) + b\, \tilde{G}(U(\tilde{x}), U(\tilde{x}), \tilde{x}) + \bar{c}\, \tilde{G}(V(\tilde{y}), V(\tilde{y}), \tilde{y});$$

$$\forall\, \tilde{x}, \tilde{y} \tilde{\in} SE(\tilde{X}),$$

where $\bar{a}, \bar{b}$ and $\bar{c}$ are non negative soft reals such that

$\overline{0} \,\widetilde{\leq}\, +\overline{a} + \overline{b} + \overline{c} + \overline{d} \,\widetilde{<}\, \overline{1}$. *Then U and V have a unique common fixed point.*

Proof. Since $(\widetilde{X}, \widetilde{G}, E)$ is a symmetric soft G-matric space, so $\widetilde{G}(\widetilde{x}, \widetilde{y}, \widetilde{y}) = \widetilde{G}(\widetilde{x}, \widetilde{x}, \widetilde{y})$. Thus the result follows from Theorem 3.1. □

Theorem 3.3. *Let* $(\widetilde{X}, \widetilde{G}, E)$ *be a symmetric soft G-complete metric space and* $T : (\widetilde{X}, \widetilde{G}, E) \to (\widetilde{X}, \widetilde{G}, E)$ *be a mapping which satisfies the following conditions,*

$$\widetilde{G}(T(\widetilde{x}), T(\widetilde{x}), T(\widetilde{y})) \,\widetilde{\leq}\, \overline{a}\, \widetilde{G}(\widetilde{x}, \widetilde{x}, \widetilde{y}) + b\, \widetilde{G}(\widetilde{x}, \widetilde{x}, T(\widetilde{x})) + \overline{c}\, \widetilde{G}(\widetilde{y}, \widetilde{y}, T(\widetilde{y}));$$

$$\forall\, \widetilde{x}, \widetilde{y} \,\widetilde{\in}\, SE(\widetilde{X}),$$

where $\overline{a}, \overline{b}$ *and* $\overline{c}$ *are non negative soft reals such that* $\overline{0} \,\widetilde{\leq}\, +\overline{a} + \overline{b} + \overline{c} + \overline{d} \,\widetilde{<}\, \overline{1}$. *Then T has a unique fixed point.*

Proof. The result follows from Theorem 3.1 by setting $U = V = T$. □

Theorem 3.4. *Let* $(\widetilde{X}, \widetilde{G}, E)$ *be a symmetric soft G-complete metric space and* U, V *be two self-maps on* $(\widetilde{X}, \widetilde{G}, E)$ *satisfying the following conditions,*

$$\widetilde{G}(U^n(\widetilde{x}), U^n(\widetilde{x}), V^n(\widetilde{y})) \,\widetilde{\leq}\, \overline{a}\, \widetilde{G}(\widetilde{x}, \widetilde{x}, \widetilde{y}) + b\, \widetilde{G}(\widetilde{x}, \widetilde{x}, U^n(\widetilde{x})) + \overline{c}\, \widetilde{G}(\widetilde{y}, \widetilde{y}, V^n(\widetilde{y}));$$

$$\forall\, \widetilde{x}, \widetilde{y} \,\widetilde{\in}\, SE(\widetilde{X}),$$

where $\overline{a}, \overline{b}$ *and* $\overline{c}$ *are non negative soft reals such that* $\overline{0} \,\widetilde{\leq}\, \overline{a} + \overline{b} + \overline{c} + \overline{d} \,\widetilde{<}\, \overline{1}$ *and* $n \in \mathbb{N}$.

Then U and V have a unique common fixed point.

Proof. Let $H = U^n$ and $T = V^n$, $n \in \mathbb{N}$. Then H and T satisfying the conditions given in Theorem 3.1. So from Theorem 3.1, we get a unique fixed point $\widetilde{t}\,(\widetilde{\in}\, SE(\widetilde{X}))$ of H and T. i.e.; $H(\widetilde{t}) = \widetilde{t}$ and $T(\widetilde{t}) = \widetilde{t}$.

Now,

$$G(U(\widetilde{t}), U(\widetilde{t}), \widetilde{t}) = G(H(U(\widetilde{t})), H(U(\widetilde{t})), T(\widetilde{t}))$$

$$\tilde{\leqq}\ \bar{a}\, G(U(\tilde{t}), U(\tilde{t}), \tilde{t}) + \bar{b}\, G(U(\tilde{t}), U(\tilde{t}), H(U(\tilde{t})))$$

$$+ \bar{c}\, G(\tilde{t}, \tilde{t}, T(\tilde{t})), \text{ from Theorem 3.1}$$

$$= \bar{a}\, G(U(\tilde{t}), U(\tilde{t}), \tilde{t}) + \bar{b}\, G(U(\tilde{t}), U(\tilde{t}), U(\tilde{t})) + \bar{c}\, G(\tilde{t}, \tilde{t}, \tilde{t})$$

$$\text{Therefore } G(U(\tilde{t}), U(\tilde{t}), \tilde{t})\ \tilde{\leqq}\ \bar{a}\, G(U(\tilde{t}), U(\tilde{t}), \tilde{t})$$

$$\Rightarrow G(U(\tilde{t}), U(\tilde{t}), \tilde{t}) = \bar{0}, \text{ since } \bar{0}\ \tilde{\leqq}\ \bar{a} + \bar{b} + \bar{c}\ \tilde{<}\ \bar{1}$$

$$\text{i.e., } U(\tilde{t}) = \tilde{t}.$$

Again,

$$G(\tilde{t}, \tilde{t}, V(\tilde{t})) = G(H(\tilde{t}), H(\tilde{t}), T(V(\tilde{t})))$$

$$\tilde{\leqq}\ \bar{a}\, G(\tilde{t}, \tilde{t}, V(\tilde{t})) + \bar{b}\, G(\tilde{t}, \tilde{t}, H(\tilde{t})) + \bar{c}\, G(V(\tilde{t}), V(\tilde{t}), T(V(\tilde{t}))),$$

from Theorem 3.1

$$= \bar{a}\, G(\tilde{t}, \tilde{t}, V(\tilde{t})) + \bar{b}\, G(\tilde{t}, \tilde{t}, \tilde{t}) + \bar{c}\, G(V(\tilde{t}), V(\tilde{t}), V(\tilde{t}))$$

$$\text{Therefore } G(\tilde{t}, \tilde{t}, V(\tilde{t}))\ \tilde{\leqq}\ \bar{a}\, G(\tilde{t}, \tilde{t}, V(\tilde{t}))$$

$$\Rightarrow G(\tilde{t}, \tilde{t}, V(\tilde{t})) = \bar{0}, \text{ since } \bar{0}\ \tilde{\leqq}\ \bar{a} + \bar{b} + \bar{c}\ \tilde{<}\ \bar{1}$$

$$\text{i.e., } V(\tilde{t}) = \tilde{t}.$$

Therefore, U and V have same fixed point $\tilde{t}$.

To prove uniqueness, let $\tilde{t}(\neq \tilde{t})\ \tilde{\in}\ SE(\tilde{X})$, be another fixed point of U and V.

Then,

$$G(\tilde{t}, \tilde{t}, \tilde{t}^*) = G(U(\tilde{t}), U(\tilde{t}), V(\tilde{t}))$$

$$\tilde{\leqq}\ \bar{a}\, G(\tilde{t}, \tilde{t}, \tilde{t}^*) + \bar{b}\, G(\tilde{t}, \tilde{t}, U(\tilde{t})) + \bar{c}\, G(\tilde{t}^*, \tilde{t}^*, V(\tilde{t}^*))$$

$$= \bar{a}\, G(\tilde{t}, \tilde{t}, \tilde{t}^*) + \bar{b}\, G(\tilde{t}, \tilde{t}, \tilde{t}) + \bar{c}\, G(\tilde{t}^*, \tilde{t}^*, \tilde{t}^*),$$

as t, t^* are fixed of both U and V.

i.e., $G(\tilde{t}, \tilde{t}, \tilde{t}^*) = \bar{a}\, G(\tilde{t}, \tilde{t}, \tilde{t}^*)$

i.e., $\tilde{t} = \tilde{t}^*$ as $\bar{0} \tilde{\leq} \bar{a} + \bar{b} + \bar{c} \tilde{<} \bar{1}$

Therefore, U and V have unique fixed point in $SE(\tilde{X})$. □

Theorem 3.5. *Let* $(\tilde{X}, \tilde{G}, E)$ *be a soft G-complete space and* $T : (\tilde{X}, \tilde{G}, E) \to (\tilde{X}, \tilde{G}, E)$ *be a mapping that satisfies the following condition for all* $\tilde{x}, \tilde{y}, \tilde{z} \in SE(\tilde{X})$,

$$\tilde{G}(T^n\tilde{x}, T^n\tilde{y}, T^n\tilde{z}) \tilde{\leq} \bar{a}\, \tilde{G}(\tilde{x}, T^n\tilde{x}, T^n\tilde{x}) + \bar{b}\, \tilde{G}(\tilde{y}, T^n\tilde{y}, T^n\tilde{y}) + \bar{c}\, \tilde{G}(\tilde{z}, T^n\tilde{z}, T^n\tilde{z}) + \bar{d}\, \tilde{G}(\tilde{x}, \tilde{y}, \tilde{z}) \tag{3.1}$$

where $\bar{0} \tilde{\leq} \bar{a} + \bar{b} + \bar{c} \tilde{<} \bar{1}$ *and* $n \in \mathbb{N}$. *Then T has a unique fixed point.*

Proof. Let us put $H = T^n, n \in \mathbb{N}$. Then,

$$\tilde{G}(H\tilde{x}, H\tilde{y}, H\tilde{z}) \tilde{\leq} \bar{a}\, \tilde{G}(\tilde{x}, H\tilde{x}, H\tilde{x}) + \bar{b}\, \tilde{G}(\tilde{y}, H\tilde{y}, H\tilde{y}) + \bar{c}\, \tilde{G}(\tilde{z}, H\tilde{z}, H\tilde{z}) + \bar{d}\, \tilde{G}(\tilde{x}, \tilde{y}, \tilde{z}) \tag{3.2}$$

where $\bar{0} \tilde{\leq} \bar{a} + \bar{b} + \bar{c} \tilde{<} \bar{1}$.

Therefore by Theorem 2.16, H has unique fixed point $\tilde{u}$. i.e.; $H(\tilde{u}) = \tilde{u}$.

Now, $TH(\tilde{u}) = T(\tilde{u})$ and $TH = HT (= T^{n+1})$, whence $H(T\tilde{u}) = T\tilde{u}$ and

$$G(\tilde{u}, T\tilde{u}, T\tilde{u}) = G(H\tilde{u}, HT\tilde{u}, HT\tilde{u})$$

$$\tilde{\leq} \bar{a}\, \tilde{G}(\tilde{u}, H\tilde{u}, H\tilde{u}) + \bar{b}\, \tilde{G}(T\tilde{u}, HT\tilde{u}, HT\tilde{u}) + \bar{c}\, \tilde{G}(T\tilde{u}, HT\tilde{u}, HT\tilde{u}) + \bar{d}\, \tilde{G}(\tilde{u}, T\tilde{u}, T\tilde{u})$$

$$= \bar{a}\, \tilde{G}(\tilde{u}, \tilde{u}, \tilde{u}) + \bar{b}\, \tilde{G}(T\tilde{u}, T\tilde{u}, T\tilde{u}) + \bar{c}\, \tilde{G}(T\tilde{u}, T\tilde{u}, T\tilde{u}) + \bar{d}\, \tilde{G}(\tilde{u}, T\tilde{u}, T\tilde{u})$$

$$= \overline{d}\, \widetilde{G}(\widetilde{u},\, T\widetilde{u},\, T\widetilde{u}), \text{ where } \overline{0} \,\widetilde{\leq}\, \overline{a} + \overline{b} + \overline{c} \,\widetilde{<}\, \overline{1}$$

Hence, $G(\widetilde{u},\, T\widetilde{u},\, T\widetilde{u}) = \overline{0}$ i.e.; $T(\widetilde{u}) = \widetilde{u}$.

If possible, let $\widetilde{v} \,\widetilde{\in}\, SE(\widetilde{X})$ be another fixed point of T.

Then $T^n(\widetilde{v}) = T^{n-1}(T(\widetilde{v})) = T^{n-1}(\widetilde{v}) = \ldots = T(\widetilde{v}) = \widetilde{v}$.

Now, $\widetilde{G}(\widetilde{v},\, H\widetilde{v},\, H\widetilde{v}) = \widetilde{G}(\widetilde{v},\, T^n\widetilde{v},\, T^n\widetilde{v}) = \widetilde{G}(\widetilde{v},\, \widetilde{v},\, \widetilde{v}) = \overline{0}$.

i.e.; $H(\widetilde{v}) = \widetilde{v}$, which is contradiction that H has unique fixed point. Therefore T has a unique fixed point. □

Definition 3.6. Let $(\widetilde{X}, \widetilde{G}, E)$ be a soft G-metric space and $(F, A) \,\widetilde{\subseteq}\, (\widetilde{X}, E)$. Then (F, A) is called soft G-totally bounded if for a given $\widetilde{\epsilon} \,\widetilde{>}\, \overline{0}$, there exist finitely many points $\widetilde{x}_i \,\widetilde{\in}\, SE(\widetilde{X})$ such that $(F, A) \,\widetilde{\subseteq}\, \bigcup_{i=1}^{n} B_{\widetilde{G}}(x_i,\, \widetilde{\epsilon})$.

Definition 3.7. A soft G-metric space $(\widetilde{X}, \widetilde{G}, E)$ is said to be a soft G-compact if it is G-complete and G-totally bounded.

Theorem 3.8. *Let $(\widetilde{X}, \widetilde{G}, E)$ be a soft G-metric space and $T : (\widetilde{X}, \widetilde{G}, E) \to (\widetilde{X}, \widetilde{G}, E)$ be a self-map satisfies,*

$$G(T(x_1), T(x_2), T(x_3)) \,\widetilde{<}\, \tilde{G}(x_1, x_2, x_3) \text{ where } x_1, x_2, x_3 \,\widetilde{\in}\, SE(\tilde{X}) \text{ such that}$$

$$x_1 \neq x_2 \neq x_3. \tag{3.3}$$

If $(\widetilde{X}, \widetilde{G}, E)$ is a soft G-compact space, then T has a unique fixed soft point in $\widetilde{X}$.

Proof. Let us define a function $\phi : \widetilde{X} \to [\overline{0}, \overline{1})$ by,

$$\phi(\widetilde{x}) = \widetilde{G}(\widetilde{x},\, T(\widetilde{x}),\, T^2(\widetilde{x})), \;\forall\, \widetilde{x} \in \widetilde{X}$$

Since, $(\widetilde{X}, \widetilde{G}, E)$ is soft G-compact, the function ϕ assumes its minimum value. So, there is $\tilde{a} \tilde{\in} SE(\tilde{X})$ such that,

$$\tilde{G}(\tilde{a}, T(\tilde{a}), T^2(\tilde{a})) \tilde{\leq} \tilde{G}(\tilde{x}, T(\tilde{x}), T^2(\tilde{x})), \forall \tilde{x} \tilde{\in} SE(\tilde{X}).$$

Now if possible let $T(\tilde{a}) \neq \tilde{a}$, then using 3.3,

$$\tilde{G}(T(\tilde{a}), T(T(\tilde{a})), T(T^2(\tilde{a}))) \tilde{<} \tilde{G}(\tilde{a}, T(\tilde{a}), T^2(\tilde{a}))$$

which contradicts the fact that $\tilde{G}(\tilde{a}, T(\tilde{a}), T^2(\tilde{a}))$ is minimum.

So, $T(\tilde{a}) = \tilde{a}$.

Now we have to show the uniqueness. For this, let $a_1, a_2, a_3 \tilde{\in} SE(\tilde{X})$ such that $a_1 \neq a_2 \neq a_3$ and $T(a_1) = a_1, T(a_2) = a_2, T(a_3) = a_3$.

Thus, $\tilde{G}(a_1, a_2, a_3) = \tilde{G}(T(a_1), T(a_2), T(a_3)) \tilde{<} \tilde{G}(a_1, a_2, a_3)$,

which is impossible.

So, $a_1 = a_2 = a_3$.

Therefore, T has a unique fixed soft point. □

Remark 3.9. The converse of the above theorem is not true which is discussed by the following two examples.

In Example 3.10, we have show that a self-map T on a G-metric space $(\tilde{X}, \tilde{G}, E)$ satisfying the condition $\tilde{G}(T(x_1), T(x_2), T(x_3)) \tilde{<} \tilde{G}(x_1, x_2, x_3)$, $\forall x_1, x_2, x_3 \tilde{\in} SE(\tilde{X})$ has a fixed soft point however $(\tilde{X}, \tilde{G}, E)$ is not soft G-compact.

And in Example 3.11, we have show that a self-map T on a G-metric space has a fixed soft point though $(\tilde{X}, \tilde{G}, E)$ is not soft G-compact and T does not satisfy the condition $\tilde{G}(T(x_1), T(x_2), T(x_3)) \tilde{<} \tilde{G}(x_1, x_2, x_3)$, $\forall x_1, x_2, x_3 \tilde{\in} SE(\tilde{X})$.

Example 3.10. Let $\tilde{X}(\lambda) = [0, 1)$ and $\tilde{G} : SE(\tilde{X}) \times SE(\tilde{X}) \times SE(\tilde{X}) \to \mathbb{R}(E)^*$ be a mapping defined by $\tilde{G}(\tilde{x}, \tilde{y}, \tilde{z})(\lambda) = | \tilde{x}(\lambda) - \tilde{y}(\lambda) |$

$+ \mid \tilde{y}(\lambda) - \tilde{z}(\lambda) \mid + \mid \tilde{z}(\lambda) - \tilde{x}(\lambda) \mid$ for each $\lambda \in E$ and $\tilde{x}, \tilde{y}, \tilde{z} \,\tilde{\in}\, SE(\tilde{X})$. Then $(\tilde{X}, \tilde{G}, E)$ is a soft G-metric space.

Now we consider a sequence $\{x_n\}$ of soft elements in $\tilde{X}$, where $x_n(\lambda) = 1 - \frac{1}{n}$, for each $n \in \mathbb{N}$ and for each $\lambda \in E$.

Then $\{x_n\}$ is a soft G-Cauchy but is not soft G-convergent in $(\tilde{X}, \tilde{G}, E)$.

So, $(\tilde{X}, \tilde{G}, E)$ is not soft G-complete and hence $(\tilde{X}, \tilde{G}, E)$ is not soft G-compact.

Again let $T : (\tilde{X}, \tilde{G}, E) \to (\tilde{X}, \tilde{G}, E)$ be a self-map defined by $T(\tilde{x}) = \frac{\tilde{x}}{\overline{2}}$. Then clearly T has a fixed soft point $\overline{0}$.

Also for any $x_1, x_2, x_3 \,\tilde{\in}\, SE(\tilde{X})$ with $x_1 \neq x_2 \neq x_3$ and for any $\lambda \in E$,

$$\tilde{G}(T(x_1), T(x_2), T(x_3))(\lambda) = G\left(\frac{x_1}{\overline{2}}, \frac{x_2}{\overline{2}}, \frac{x_3}{\overline{2}}\right)(\lambda)$$

$$= \tilde{G}\left(\frac{1}{2}x_1(\lambda), \frac{1}{2}x_2(\lambda), \frac{1}{2}x_3(\lambda)\right)$$

$$= \left|\frac{1}{2}x_1(\lambda) - \frac{1}{2}x_2(\lambda)\right| + \left|\frac{1}{2}x_2(\lambda) - \frac{1}{2}x_3(\lambda)\right|$$

$$+ \left|\frac{1}{2}x_3(\lambda) - \frac{1}{2}x_1(\lambda)\right|$$

$$= \frac{1}{2}[\mid x_1(\lambda) - x_2(\lambda) \mid + \mid x_2(\lambda) - x_3(\lambda) \mid$$

$$+ \mid x_3(\lambda) - x_1(\lambda) \mid]$$

$$\tilde{<} \tilde{G}(x_1, x_2, x_3)(\lambda).$$

So, $\tilde{G}(T(x_1), T(x_2), T(x_3)) \,\tilde{<}\, \tilde{G}(x_1, x_2, x_3), \forall x_1, x_2, x_3 \,\tilde{\in}\, SE(\tilde{X})$.

Thus the self-map $T : (\tilde{X}, \tilde{G}, E) \to (\tilde{X}, \tilde{G}, E)$ satisfies the condition

$\tilde{G}(T(x_1), T(x_2), T(x_3)) \tilde{<} \tilde{G}(x_1, x_2, x_3), \forall x_1, x_2, x_3 \tilde{\in} SE(\tilde{X})$ has a fixed soft point though $(\tilde{X}, \tilde{G}, E)$ is not soft G-compact space.

Example 3.11. Let $\tilde{X}(\lambda) = [0, 1)$ and $\tilde{G} : SE(\tilde{X}) \times SE(\tilde{X}) \times SE(\tilde{X}) \to \mathbb{R}(E)^*$ be a mapping defined by $\tilde{G}(\tilde{x}, \tilde{y}, \tilde{z})(\lambda) = |\tilde{x}(\lambda) - \tilde{y}(\lambda)| + |\tilde{y}(\lambda) - \tilde{z}(\lambda)| + |\tilde{z}(\lambda) - \tilde{x}(\lambda)|$ for each $\lambda \in E$ and $\tilde{x}, \tilde{y}, \tilde{z} \tilde{\in} SE(\tilde{X})$. Then $(\tilde{X}, \tilde{G}, E)$ is a soft G-metric space.

Now we consider a sequence $\{x_n\}$ of soft elements in $\tilde{X}$, where $x_n(\lambda) = 1 - \frac{1}{n}$, for each $n \in \mathbb{N}$ and for each $\lambda \in E$.

Then $\{x_n\}$ is a soft G-Cauchy but is not soft G-convergent in $(\tilde{X}, \tilde{G}, E)$.

So, $(\tilde{X}, \tilde{G}, E)$ is not soft G-complete and hence $(\tilde{X}, \tilde{G}, E)$ is not soft G-compact. Again let $T : (\tilde{X}, \tilde{G}, E) \to (\tilde{X}, \tilde{G}, E)$ be a self-map defined by $T(\tilde{x}) = \overline{2} \cdot \tilde{x}$. Then clearly T has a fixed soft point $\overline{0}$.

Also for any $x_1, x_2, x_3 \tilde{\in} SE(\tilde{X})$ with $x_1 \neq x_2 \neq x_3$ and for any $\lambda \in E$,

$$\tilde{G}(T(x_1), T(x_2), T(x_3))(\lambda) = G(\overline{2} \cdot x_1, \overline{2} \cdot x_2, \overline{2} \cdot x_3)(\lambda)$$

$$= \tilde{G}(2 \cdot x_1(\lambda), 2 \cdot x_2(\lambda), 2 \cdot x_3(\lambda))$$

$$= |2 \cdot x_1(\lambda) - 2 \cdot x_2(\lambda)| + |2 \cdot x_2(\lambda) - 2 \cdot x_3(\lambda)|$$

$$+ |2 \cdot x_3(\lambda) - 2 \cdot x_1(\lambda)|$$

$$= 2 \cdot [|x_1(\lambda) - x_2(\lambda)| + |x_2(\lambda) - x_3(\lambda)|$$

$$+ |x_3(\lambda) - x_1(\lambda)|]$$

$$\not\leq \tilde{G}(x_1, x_2, x_3)(\lambda).$$

So, $\tilde{G}(T(x_1), T(x_2), T(x_3)) \tilde{<} \tilde{G}(x_1, x_2, x_3), \forall x_1, x_2, x_3 \tilde{\in} SE(\tilde{X})$.

Thus we have a self map on a non soft G-compact space and does not satisfy the condition $\tilde{G}(T(x_1), T(x_2), T(x_3)) \tilde{<} \tilde{G}(x_1, x_2, x_3), \forall x_1, x_2, x_3$ $\tilde{\in} SE(\tilde{X})$ has a fixed soft point.

4. Conclusion

In the present paper, our main object to extend the theory of fixed point in soft G-complete metric spaces. So we have tried to established some important fixed point results in symmetric soft G-complete metric spaces. After that we have introduced soft G-totally bounded, soft G-compact metric spaces and a fixed point results on soft G-compact metric space is conferred. Behavior of the converse is also studied with suitable examples. We hope this study have a great importance to researchers to extend fixed point results in the field of soft metric spaces.

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