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MULTISET TOPOLOGY AND ITS APPLICATIONS: A SURVEY

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Abstract

In real life, a number of situations occur where we have to deal with collections of elements in which duplicates are significant. There are enormous repetitions in nature like many hydrogen atoms, many strands of DNA, repeated roots of a polynomial equation representing a physical phenomenon etc. A useful mathematical model for representing multi attribute objects is a multiset (mset) or a bag. Multiset topology (M-topology) is the generalization of general topology in multiset setting. In this article, a detailed investigation is done on evolution of multiset topology till date. Some of the applications of multiset and its topology in DNA and RNA mutations are also discussed.

1. Introduction

Theory of sets plays a very important role in mathematics. Every branch (field) of mathematics use set theoretic concepts in some way or the other. Set is very helpful in formulating various mathematical structures with limitation that no elements in the set is repeated. But in real life, we face a number of situations or problems where we have to deal with collection of elements or objects where duplicates are significant. The advances in science and technology have also given rise to many problems (or situations) where the objects under analysis are characterized by many qualitative and/or

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quantitative features (attributes). Enormous repetitions have been observed in the physical world [1, 2, 3, 4, 5, 6, 7, 8]. For example, there are many strands of DNA, many hydrogen atoms in a molecule, repeated roots of a polynomial representing a physical situation, repeated prime factors of a positive integer, repeated sequence of (bases) A, G, C and T (or U) in strands of DNA and RNA etc. To explain such situations the classical theory of sets proves inadequate because of the limitations that no element in the set is repeated.

This led to the introduction of Multiset Theory, which was first studied by Blizard [1]. A multiset (mset) or bag is an unordered collection of objects (elements) in which repeated occurrences of elements are allowed. An example of multiset is the collection of prime factors of 720 i.e., {2, 2, 2, 2, 3, 3, 5}.

Multiset has various applications in logic, philosophy, linguistics, physics, mathematics and computer science. An extensive survey on multiset and its applications was done by Singh et al. [9].

Along with analysis and algebra, topology is considered as one of the very important branch of abstract mathematics. The development of topology was primarily motivated by investigations of real-world problems. After the formal foundation of the subject, topology has grown up as an abstract area of mathematics. Within the past few decades topology has also grown up as an important tool or component of applied mathematics to deal with or to model various problems of physics, biology, economics, engineering, chemistry etc. Recently in 2012, Girish and John [10] generalized the concept of point set topology in multiset context and introduced multiset topology (M-topology). The introduction of M-topology has opened the door for theoretical development and practical applications of the subject. In this article, we make an extensive survey on the development of multiset topology and its applications in various fields.

The rest of the article proceeds as follows. We describe the basic ideas of multiset in section 2. Section 3 discusses various theories (concepts) which have been developed in the field of multiset topology over the years. In section 4, we survey some applications of M-topological ideas in DNA and RNA mutation. Finally, section 5 concludes.

2. Preliminaries on Multisets

This section is a collection of basic definitions and notations of multiset theory as introduced by Cerf et al. [11], Peterson [12], Yager [8], Jena et al. [5]. Over the years various concepts have been introduced in multiset theory but we discuss some of them which are necessary for our study.

Definition 1 [5]**.** An mset is a collection of elements which contains duplicates. Formally, if X be a set of elements, then an mset M drawn from X is represented by a mapping, Count M or $C_M(x)$ and is defined as $C_M(x) : X \to N$ where N represents set of non-negative integers.

For each $x \in X, C_M(x)$ represents the number of occurrences of the element x in M. $C_M(x)$ is also called multiplicity of x in M.

Definition 2 [8]**.** If M be an mset drawn from a set X then the support set of M is denoted by M^* and is defined by $M^* = \{x \in X : C_M(x) > 0\}$.

Definition 3 [9]**.** If the number of distinct elements and their occurrences in an mset is finite then the mset is called finite mset, otherwise it is called an infinite mset. Thus, an infinite mset means either the number of elements in it is infinite or occurrence of one or more elements is infinite i.e., $C_M(x) \geq \aleph_0$.

For mset with infinite multiplicities, see [13, 14, 15, 16, 17]. Various efforts have also been done to study msets with negative multiplicities, see [14, 17, 18, 19, 20, 21].

A multiset M drawn from the set $X = \{x_1, x_2, \ldots, x_n\}$ can be represented as $M = \{m_1/x_1, m_2/x_2, \ldots, m_n/x_n\}$, where m_1 is occurrences of the element x_i in the set M, see [8].

Definition 4 (Dressed Epsilon, $\in^k$) [9]**.** If an element x in an mset M occurres k times i.e., $C_M(x) = k$ then we write $x \in^k M$.

Definition 5 [5]**.** If M and N are two msets drawn from the set X, then we have the following definitions

1. $M = N$ if $C_M(x) = C_N(x), \forall x \in X$.

2. $M \subseteq N$ (M is a submset of N) if $C_M(x) \leq C_N(x), \forall x \in X$.

3. $P = M \cup N$ if $C_P(x) = \max\{C_M(x), C_N(x)\}, \forall x \in X$

4. $P = M \cap N$ if $C_P(x) = \min\{C_M(x), C_N(x)\}, \forall x \in X$.

5. $P = M \oplus N$ if $C_P(x) = C_M(x) + C_N(x), \forall x \in X$.

6. $P = M \ominus N$ if $C_P(x) = C_M(x) - C_N(x), \forall x \in X$.

Where $\oplus$ and $\ominus$ denotes mset addition and mset subtraction respectively.

For any arbitrary collection of msets the above definition can be generalized as:

Definition 6 [10]**.** If $\{M_1, M_2, M_3, \ldots\}$ be a collection of msets drawn from $[X]^m$ then

(1) $\bigcup_{i \in I} M_i = \{C_{\cup M_i}(x)/x : C_{\cup M_i}(x) = Max\{C_{M_i}(x) : x \in X\}\}$

(2) $\bigcap_{i \in I} M_i = \{C_{\cap M_i}(x)/x : C_{\cap M_i}(x) = Min\{C_{M_i}(x) : x \in X\}\}$

(3) $\oplus_{i \in I} M_i = \{C_{\oplus M_i}(x)/x : C_{\oplus M_i}(x) = \sum_{i \in I}\{C_{M_i}(x) : x \in X\}\}$.

Definition 7 [5]**.** Let X be a set of elements from which msets are drawn. Then $[X]^m$ represents the collection of all msets whose elements are in M such that all the elements have count less than or equal to m. And the set $[X]^\infty$ is the collection of all the msets drawn from the set X such that there is no limit of occurrences of an element in an mset.

Definition 8 [10]**.** (1) Whole submset: A whole submset of an mset M is a submset every element of whose has the same multiplicity as that of in M i.e., a submset N of M is called whole submset if $C_M(x) = C_N(x)$, for every x in N.

(2) Partial whole submset: A submset N of an mset M is called partial whole submset if at least one element in N has the same multiplicity as in M i.e., $C_M(x) = C_N(x)$, for some x in N.

(3) Full submset: A submset N of an mset M is called full submset if N has the same support set as M with the condition that $C_N(x) \leq C_M(x)$, for all x in N.

Example 2.1. Let us consider the mset $M = \{5/x, 3/y, 7/z, 4/q\}$ then

(1) The submset $N = \{5/x, 7/z, 4/q\}$ is a whole submset and partial whole submset of M.

(2) The submset $N = \{2/x, 3/y, 2/z, 3/q\}$ is a partial whole submset and full submset of M.

(3) The submset $N = \{2/x, 3/z, 4/q\}$ is a partial whole submset of M.

(4) The submset $N = \{2/x, 2/y, 5/z, 2/q\}$ is a full submset of M.

Definition 9 [10]**.** (1) Power whole mset: If $M \in [X]^m$ be an mset, then the power whole mset of M, denoted by $PW(M)$ is the collection of all whole submsets of M.

(2) Power full mset: The collection of all full submsets of an mset is called its power full mset.

(3) Power mset: If $M \in [X]^m$ be an mset, then the collection of all submsets of M is called its power mset and is denoted by $P(M)$. If N is a submset of M such that $N = \phi$, then multiplicity of N in $P(M)$ is 1. If N is non empty, then $N \in^k P(M)$, where $k = \prod_z \binom{m}{n}$, the product is taken over distinct elements z of N, m and n are the occurrences of z in M and N respectively. If $M = \{6/x, 4/y\}$ be an mset then $N = \{2/x, 3/y\}$ has the multiplicity $\binom{6}{2}\binom{4}{3}$ in $P(M)$.

(4) Power set of an mset: If $M \in [X]^m$ be an mset and $P(M)$ be the power mset, then the power set of M denoted by $P^*(M)$, is the support set of $P(M)$.

Example 2.2. If $M = \{3/x, 2/y\}$ be an mset then

1. Power whole mset of M is $PW(M) = \{\phi, M, \{3/x\}, \{2/y\}\}$

2. Power full mset is $PF(M) = \{\{3/x, 1/y\}, \{1/x, 1/y\}, \{2/x, 1/y\}, \{2/x, 2/y\}, M\}$.

3. Power mset

$$P(M) = \{\phi, M, 2/\{3/x, 1/y\}, 3/\{1/x, 2/y\}, 6/\{1/x, 1/y\}, 6/\{2/x, 1/y\},$$
$$3/\{2/x, 2/y\}, 1/\{3/x\}, 1/\{2/y\}, 3/\{2/x\}, 3/\{1/x\}, 2/\{2/y\}\}$$

4. Power set of mset M is

$$P^*(M) = \{\phi, M, \{3/x, 1/y\}, \{1/x, 2/y\}, \{1/x, 1/y\}, \{2/x, 1/y\},$$
$$\{2/x, 2/y\}, \{3/x\}, \{2/y\}, \{2/x\}, \{1/x\}, \{2/y\}\}$$

Definition 10 [23]**.** If M_1 and M_2 are two msets drawn from a set X, then the cartesian product of M_1 and M_2 is defined as $M_1 \times M_2 = \{(m/x, n/y)/mn : x \in^m M_1, y \in^n M_2\}$.

Definition 11 [23]**.** An mset relation on an mset M is a submset R of $M \times M$ such that every $(m/x, n/y) \in R$ has the count equal to $(m/x, n/y)$ $C_1(x, y) \times C_2(x, y)$. Where $C_1(x, y)$ and $C_2(x, y)$ are counts (multiplicities) of x and y in M respectively i.e., $C_1(x, y) = C_M(x)$ and $C_2(x, y) = C_M(y)$. If $(m/x, n/y) \in R$ then we say that m/x is related to n/y and we write $m/xRn/y$.

Example 2.3. If $M = \{3/x, 5/y, 6/z\}$ be an mset, then

$$R = \{(2/x, 3/y)/6, (3/x, 3/z)/9, (2/y, 3/z)/6, (3/x, 3/x)/9\}$$

is an mset relation on M.

$$S = \{(3/x, 2/y)/4, (3/x, 3/x)/9, (3/y, 4/z)/10, (3/z, 2/z)/6\}$$

But $S \subseteq M \times M$ is not an mset relation on M as $C_S(x, y) = 4 \neq 3 \times 2$ and $C_S(y, z) = 10 \neq 3 \times 4$. For more about multiset relations and multiset functions, see [23, 24].

3. Multiset Topology

In this section we make a survey on various theories developed in the field of multiset topology. Topology in multiset context was introduced in 2012 and various developments are yet to be done in this area. Since its introduction various theories have been developed in this field and some applications of multiset topology in mathematical biology are also established.

Definition 12 [10, 22]**.** If $M \in [X]^m$ be an mset and $\tau \subseteq P^*(M)$, then τ is called a multiset topology (or M-topology) if it satisfies the following

(1) ϕ and M are in τ.

(2) Union of arbitrary collections of elements of τ are in τ.

(3) Intersections of finitely many elements of τ are in τ.

The multiset M along with the topology τ is called a multiset topological space (or M-topological space) and its denoted by (M, τ). Multiset topology on a multiset M is basically a collection of submsets of M, called open msets such that τ and M both are open. Also arbitrary unions and finite intersections of open msets (submsets) are open.

Example 3.1 [10]**.** If $M \in [X]^m$ be an mset then some trivial example of M-topologies are

(i) $P^*(M)$ is an M-topology on M and called discrete M-topology.

(ii) The collection $\{\phi, M\}$ is called indiscrete M-topology.

(iii) The power whole mset $PW(M)$ is an M-topology on mset M.

(iv) The power full mset with the empty mset i.e., $PF(M) \cup \{\phi\}$ forms an M-topology.

3.1 M-basis and sub M-basis

In this section, we define the concepts of basis and sub basis in M-topological space and also we discuss M-topologies generated by M-basis and sub M-basis.

Definition 13 (M-basis) [10, 22]**.** If M be an mset, then an M-basis for an M-topology on M is a collection $\boldsymbol{B}$ of submsets of M (called M-basis element) such that

(1) For each $x \in^m M$, for some $m > 0$, $\exists$ at least one $B \in \boldsymbol{B}$ containing m/x.

(2) If for each $m/x \in M_1 \cap M_2$, where $M_1, M_2 \in \boldsymbol{B}$, $\exists$ an $M_3 \in \boldsymbol{B}$ such that $m/x \in M_3 \subseteq M_1 \cap M_2$.

Definition 14 (Sub M-basis) [10, 22]**.** If τ be an M-topology on an mset M, then a sub collection S of τ is called a sub M-basis if the collection of finite intersections of the members of S is an M-basis for M.

From the definitions it is being observed that sub M-bases have been defined for a given topology, but the concept of sub M-basis for some topology have remained untouched.

Topology generated by M-basis: A submset N of an mset M is said to be open in M (i.e., an element of the topology) if for each $m/x \in N$, $\exists$ an M-basis element $B \in \boldsymbol{B}$ such that $m/x \in B$ and $B \subseteq N$. It is also very clear that every basis element is also an element of the topology.

Topology generated by sub M-basis: If S be a sub M-basis, then the M-topology generated by S is the collection of unions of all finite intersections of the elements of S.

3.2 M-topology from mset Relation

Girish and John [10] showed that a binary multiset relation on an mset M can induce topologies. In this section, we discuss (as in [10]) about the topologies induced by binary mset relation.

Definition 15. If $M \in [X]^m$ be an mset and R be an mset relation on M, then

(a) Post mset of $x \in^k M$ is defined as the set $\{n/y : m/xRn/y$, for some $m\}$ and the post mset of $x \in^k M$ is denoted as k/xR.

(b) Pre mset of $x \in^p M$ is defined as the set $\{r/y : r/yRq/x, \text{ some } q\}$ and the pre mset of $x \in^p M$ is denoted as Rp/x.

(c) Pre class and Post class are defined as $P_- = \{Rm/x : x \in^m M\}$ and $P_+ = \{m/xR \in^m M\}$ respectively.

If R be an mset relation on an mset M, then the post class and pre class defined above form sub M-basis and hence they induce topologies on M.

Example 3.1. Consider the mset $M = \{3/x, 4/y, 7/z\}$ and the relation

$$R = \{(3/x, 3/x)/9, (2/x, 4/y)/8, (2/y, 5/z)/10, (3/z, 2/y)/6,$$
$$(3/y, 2/x)/6, (2/z, 3/x)/6\}$$

on M.

Now the post msets are

$(3/x)R = \{3/x, 4/y\} = (2/x)R, (2/y)R = \{2/x, 5/z\}$

$(2/z)R = \{2/y, 3/x\} = (3/z)R$

Pre msets are

$R(3/x) = \{3/x, 2/z, 3/y\} = R(2/x), R(2/y) = \{3/z, 2/x\} = R(4/y)$

$R(5/z) = \{2/y\}$

Post class and Pre class are

$P_+ = \{\{3/x, 4/y\}, \{2/x, 5/z\}, \{3/x, 2/y\}\}$ and

$P_- = \{\{3/x, 3/y, 2/z\}, \{2/x, 3/z\}, \{2/y\}\}$

Now, consider β_+ and β_- to be the collection of intersections of elements from P_+ and P_- respectively, then

$\beta_+ = \{\phi, M, \{3/x, 4/y\}, \{2/x, 5/z\}, \{3/x, 2/y\}, \{2/x\}\}$

$\beta_- = \{\phi, M, \{3/x, 3/y, 2/z\}, \{2/x, 3/z\}, \{2/y\}, \{2/x, 2/z\}\}$

M-topologies generated by M-bases β_+ and β_- are

$\tau_1 = \{\phi, M, \{3/x, 4/y\}, \{2/x, 5/z\}, \{3/x, 2/y\}, \{3/x, 4/y, 5/z\}, \{2/x\}\}$ and

$\tau_2 = \{\phi, M, \{3/x, 3/y, 2/z\}, \{2/x, 3/z\}, \{2/y\}, \{2/x, 2/z\}, \{2/x, 2/y, 2/z\},$

$\{3/x, 3/y, 3/z\}, \{2/x, 2/y, 3/z\}\}$

For more about topologies induced by mset relation, see [10].

3.3 M-connectedness

In 2016, Mahalakshmi and Thangavelu [25] introduced the concept of connectedness and related theorems in M-topological spaces. Again in 2020, Rajish Kumar and Sunil J. John [26] discussed M-connectedness in subspace M-topologies.

Definition 16 [25]**.** If (M, τ) be an M-topological space, then

(a) M-separation: An M-separation of an mset M is a pair of disjoint nonempty open submsets M_1 and M_2 such that $M = M_1 \cup M_2$.

(b) Connected space: An M-topological space (M, τ) is called M-connected if there does not exist an M-separation of the mset M.

A submset N of an M-space (M, τ) is connected if it is M-connected as sub space of M i.e., the M-connectedness of a submset N of an mset is determined by subspace M-topology on N. As we have two subspace M-topologies viz. open subspace M-topology and closed subspace M-topology (see [26]), so we have following two types of M-connectedness on a submset N.

Definition 17 [26]**.** If N is a submset of a mset M, then

(a) MO-connectedness: The submset N is called MO-connected if it is connected in open subspace M-topology.

(b) MC-connectedness: The submset N is called MC-connected if it is connected in closed subspace M-topology.

Example 3.2. Consider the mset $M = \{2/x, 3/y, 4/z\}$ with the M-topology

$$\tau = \{\phi, M, \{2/x, 2/y\}, \{2/x, 2/y, 2/z\}, \{2/x, 3/y\}, \{2/x, 3/y, 4/z\}\}$$

then (M, τ) is M-connected.

Example 3.3. Consider the mset $M = \{3/a, 5/b, 7/c, 9/d\}$ with the M-topology $\tau_1 = \{\phi, M, \{3/a, 9/d\}, \{5/b, 7/c\}\}$ then (M, τ_1) is M-disconnected and the pair $M_1 = \{3/a, 9/d\}$ and $M_2 = \{5/b, 7/c\}$ of open submsets of M is an M-separation. To know more about M-connectedness, see [25, 26].

3.4 M-compactness

In 2017, Mahanta and Samanta [27] have introduced the concept of compactness in multiset topological space and also discussed various analogous results of compact topological space in the context of M-topological space.

Definition 18 [27]**.** If $M \in [X]^m$ be an mset and (M, τ) be an M-topological space then

(a) Cover: A collection C of submsets of M is called a cover of M if $M \subseteq \bigcup_{N \in C} N$.

(b) Sub Cover: If C is a cover of M then a sub collection $C^* \subseteq C$ is called sub cover of M if C^* is also a cover of M.

(c) Open cover: A cover C of M is called an open cover if $C \subseteq \tau$.

Compact M-topology: If $M \in [X]^m$ be an mset and (M, τ) be a M-topological space, then M is said to be compact if for every open cover C of M, there is a finite sub cover of C which is also a cover of M.

A sub multiset N is compact in a M-topological space (M, τ_M) if it is compact in the sub M-topology induced by τ_M.

Now, we discuss the year wise development of various concepts and theories in the field of multiset topology.

In 2014, Mahanta and Das [28] have introduced the concept of semi open mset (SOM) and semi closed mset (SCM) and studied their various properties. They generalized the concept of compactness in M-topology as semi compactness. They also studied semi whole compactness, semi full compactness, semi partial compactness along with certain characterizations.

In 2015, Sheikh et al. [29] have extended the notions of γ-operations, preopen msets, α-open msets, semi open msets, *b*-open msets and β-open msets in M-topological space. They also discussed various relationships among these submsets in M-topological space. Same authors in [30] introduced the notion of separation axioms $T_i(i = 0, 1, 2, 2\frac{1}{2}, 3, 4, 5)$ on multiset topological spaces and studied some of their properties. They also showed the preservation of separation axioms under hereditary properties. Again, Sheikh et al. [31] introduced the notions of supra M-topological spaces. They defined supra pre open msets, supra α-open msets, supra semi open msets, supra α-open msets and also discussed various properties and relationships among these submsets. This is the generalization of the work done in [29]. The importance is that supra M-topological spaces are wider and more general than M-topological spaces.

In 2016, Mahanta and Das [32] introduced the concept of exterior and boundary in multiset topological space and they have established some relationships among the concepts of boundary, closure, interior and exterior of an mset in M-topological space. They also characterized exterior and boundary in terms of open set, closed sets, clopen sets and limit points and finally obtained the condition of having empty exterior for an mset in M-topological space. Kandil et al. [33] discussed a multiset topology induced by multiset proximity relation, they introduced the concept of proximal neighbourhood in multiset context. They also discussed mset δ-neighbourhood in the multiset proximity space and defined *p*-inclusion topology, *p*-exclusion topology, co-finite topology in multiset context. Amudhambigai et al. [34] introduced the concepts of p_s-open (closed) msets, p_s-interior, p_s-continuous mset functions, p_s-connected mset spaces and discussed some of the properties of p_s-connected spaces.

In 2017, Mahanta and Samanta [27] have introduced the concept of compactness in multiset topology and have discussed some of the properties of compact multiset topological space. They have redefined the concept of functions in multiset context. Amudhambigai et al. [35] introduced the concepts of λ-open msets, λ-continuous mset function, quasi λ-open mset function and λ-irresolute mset functions in an mset topological space. They also discussed some of their properties. Amudhambigai and Revathi [36] have established various relationships among α-interior, α-closure, α-exterior and α-boundary in multiset topological spaces.

In 2018, Tripathy and Shravan [37] have introduced the notion of generalized closed (open) msets and have investigated some of their properties. They have also introduced the $T_{\frac{1}{2}}$ separation axiom and discussed some of its properties. Again, in the same year Tripathy and Shravan [38] introduced the notion of local functions on multiset ideal topological space using the concept of q-neighbourhood. Amudhambigai et al. [39] discussed the concepts of regular b-closed msets and regular b-continuous mset functions with examples. They also discussed the concepts of rb-convergence, rb-accumulates, rb-S-closed M-spaces. rb-convergent sequences, rb-Hausdroff spaces.

In 2019, Tripathy and Shravan [40] have introduced the concept of mixed topology $T_1(T_2)$ by using the concept of quasi-coincidence of multiset and multi point and q-neighbourhood. Then some of their properties have been discussed. Zakaria et al. [41] have introduced (generalized) the notion of filters in multiset context, many deviations between multiset filters and ordinary filters have been presented. They have also mentioned the relation between multiset filters and multiset ideals. Various properties of smelters, mset ultrafilters and convergence of multiset filters have been discussed. They have also introduced the concept of basis and sub basis of multiset filters. Mahalakshmi and Thangavelu [42] discussed various combinatorial properties of multisets and the properties that are related to combinatorics and topology are investigated in the domain of msets. Amudhambigai et al. [43] have introduced the concept of π-open mset and π-continuous mset function along with the notion of π-connectedness in M-topological space. They have also introduced the notion of separation axioms $(T_{\frac{1}{2}}, T_1, T_2)$ via π-continuous M-set function.

In 2020, Kumar and John [26] discussed how the mset topology is different from general topology. They have shown that M-topology induces two subspace M-topologies on a submset. They also discussed the condition under which these two subspace M-topologies coincide. They have also introduced two types of M-connectedness and M-separations in M-topology. Shravan and Tripathy [44] have introduced the concept of metric between two multi points in a finite mset and studied various properties of multi metric space. They have also studied notion of metrizability in M-topological space and its important properties. The Uryshon's Lemma in M-topological space is also investigated.

In 2021, Kumar and John [45] discussed how the role of exterior, interior and boundary in multiset topology is different from their roles in general topology. They also found that the results which are true in general topology need not be true in multiset topology. They have also discussed the conditions under which those results are true.

4. Applications of M-topology

In this section, we discuss applications of M-topological concepts in different branches of science. From the available literature we find few applications of M-topological concepts only in the field of Mathematical Biology. We make a brief survey on the applications of M-topological concepts on Mathematical Biology.

In 2018, Sharkasy et al. [46, 47] used the concepts of multiset and multiset topology to detect mutations in DNA and RNA. As DNA and RNA are repeated sequence of (bases) A, C, G, T (or U) so the strands of DNA and RNA can be represented as multisets. We can consider a DNA or RNA strand (or sequence) to be an mset M_1 and its mutated sequence or any other sequence to be an mset M_2. From which we can construct M-topology by defining an mset relation between M_1 and M_2. They have given a very important theorem that if the length of the genes are more than four nucleotides and there is no mutation, then the resulting M-topology is a discrete M-topology. Moreover, they have shown various metrics and semi metrics between set of "types" (i.e., of DNA or RNA sequences) which are found to be very much useful to measure similarities and differences and also to detect the occurrence of mutations and the locations of the occurrence of mutations in DNA or RNA strands (or sequences). The metrics and semi metrics that they have defined are $\mu(A, B)$, $\rho(A, B)$, $\rho^*(A, B)$, $\rho^{**}(A, B)$,

$d^*(M_1, M_2), S(M_1, M_2)$, where A, B, M_1, M_2 are set of "types". They showed that similarities between two gene sequences obtained by these metrics or semi metrics are same as that are in NCBI and NCBI BLAST websites. Based on their findings they have also developed some computer programs to detect mutations.

5. Conclusion

Multiset topology is the generalization of general topology in multiset context. In this article we have done an extensive survey on development of various theories and concepts in the field of multiset topology. From our study we have seen that the subject multiset topology is in its infant stage and many more developments are yet to be done. Also we have given a survey on the applications of various concepts of M-topology in detecting and locating of mutations of DNA and RNA and also finding similarities and differences between two gene sequences. At the end we can say that there are abundant scopes for applications of multiset topological ideas in various branches of sciences. The area of M-topology and its applications will be a fertile field for research in future.

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