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m-ZUMKELLER LABELING ALGORITHM FOR JEWEL AND JELLY FISH GRAPH

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Abstract

In this paper, we provide algorithms to label jewel graphs and jelly fish graphs by m-Zumkeller numbers.

1. Introduction

A positive integer n is Zumkeller if we can partition the set of all the positive divisors of an integer n into two disjoint subsets such that sum of each partition subset is $\frac{\sigma(n)}{2}$, where $\sigma(n)$ gives the sum of all the positive divisors of n. Various properties of Zumkeller numbers are discuued in [4]. In [3] S. Sriram, R. Govindarajan and K. Thirusangu proved that jewel graph and jelly fish graph are Zumkeller graph.

Generalizing the concept of Zumkeller number H. Patodia and H. K. Saikia defined a new type of number as m-Zumkeller number in [2]. A positive integer n is an m-Zumkeller number if we can partition the set of all the positive divisors of n into two disjoint subsets of equal product.

Let $G = (V, E)$ be a graph. A one-one function $f : V \to N$ is said to be an m-Zumkeller labeling of the graph G, if the induced function $f^* : E \to N$

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defined as $f^*(xy) = f(x)f(y)$ is an m-Zumkeller number for all $xy \in E$ and $x, y \in V$. A graph is said to be an m-Zumkeller graph if it admits an m-Zumkeller labeling. In [1] the m-Zumkeller labeling of complete bipartite graphs and wheel graphs are discussed. In this paper we prove that jewel graph and jelly fish graph are also m-Zumkeller graph.

2. Properties of *m*-Zumkeller numbers

Various properties of m-Zumkeller numbers discussed in [2] are given below-

1. If n is an m-Zumkeller number, then $\tau(n) \geq 4$, where $\tau(n)$ gives the number of positive divisors of n.

2. The integer $n = \prod_{i=1}^{r} p_i^{\alpha_i}$ (where $p_i^{'s}$ are distinct primes) is an m-Zumkeller number if and only if $4 \mid \alpha_i \tau(n) \forall i = 1, 2, \ldots, r$.

3. The product of distinct prime numbers i.e. $\prod_{i=1}^{r} p_i$ (where $p_i^{'s}$ are distinct primes, $r \geq 2$) are m-Zumkeller numbers.

4. The integers of the form $2^k \prod_{i=1}^{r} p_i$ where k is any positive integer and $p_i^{'s}$ are distinct odd primes are m-Zumkeller numbers.

Example 2.1. The integers 6, 8, 10, 14, 15, 16, 21, 22, 24 are the first few m-Zumkeller numbers.

For 24, the positive divisors of 24 are 1, 2, 3, 4, 6, 8, 12, 24. This divisors of 24 can be partitioned into two subsets, $P = \{1, 2, 12, 24\}$ and $Q = \{3, 4, 6, 8\}$ such that the product of all the elements in each subset is 576. Hence, 24 is an m-Zumkeller number.

3. Jewel graph $J(n)$

Definition 3.1. The jewel graph J_n is the graph with vertex set $V(J_n) = \{u, v, x, y, u_i : 1 \leq i \leq n\}$ and edge set $E(J_n) = \{ux, uy, xy, xv, yv, uu_i, vu_i : 1 \leq i \leq n\}$.

3.1 *m*-Zumkeller labeling algorithm for jewel graph $J(n)$

Input. A Jewel graph J_n having $n+4$ vertices and $2n+5$ edges.

Output. m-Zumkeller jewel graph

Procedure. *mZum_lab_jewel* graph.

$V(J_n) = \{u_i \mid 1 \leqslant i \leqslant n+4\}$ be the vertex set of J_n

$E(J_n) = \{u_1u_2, u_1u_3, u_1u_4, u_2u_3, u_3u_4, u_2u_i, u_4u_i : 5 \leqslant i \leqslant n+4\}$ be the edge set of J_n

$p_1 := a$ prime number $\neq 2 \leqslant 13$

$p_2 := a$ prime number $\neq 2 \leqslant 13$

$p_3 := a$ prime number $\neq 2 \leqslant 13$

$p_4 := a$ prime number $\neq 2 \leqslant 13$

$p_1 \neq p_2 \neq p_3 \neq p_4$

do

begin

$f(u_{2i-1}) = 2p_1$

$f(u_{2i+1}) = 2p_2$

While $i = 1$

end

for $i : 1$ to 2 do

begin

$f(u_{2i}) = 2^i p_3$

end

for $i : 1$ to n do

begin

$f(u_{i+4}) = 2^{i+2} p_4$

end

end *mZum_lob_jewel* graph.

Proposition 3.1. *The jewel graph J_n is an m-Zumkeller graph.*

Proof. Let J_n be a jewel graph with vertex set

$V(J_n) = \{u_1, u_2, u_3, u_4, u_{i+4} \mid 1 \leqslant i \leqslant n\}$ and edge set

$E(J_n) = \{u_1u_2, u_1u_3, u_1u_4, u_2u_3, u_3u_4, u_2u_{i+4}, u_4u_{i+4} : 1 \leqslant i \leqslant n\}$.

Now

$$f^*(u_1u_2) = f(u_1)f(u_2) = (2p_1)(2p_3) = 2^2 p_1p_3$$

$$f^*(u_1u_3) = f(u_1)f(u_3) = (2p_1)(2p_2) = 2^2 p_1p_2$$

$$f^*(u_1u_4) = f(u_1)f(u_4) = (2p_1)(2^2 p_3) = 2^3 p_1p_3$$

$$f^*(u_2u_3) = f(u_2)f(u_3) = (2p_3)(2p_2) = 2^2 p_2p_3$$

$$f^*(u_3u_4) = f(u_3)f(u_4) = (2p_2)(2^2 p_3) = 2^3 p_2p_3$$

$$f^*(u_2u_{i+4}) = f(u_2)f(u_{i+4}) = (2p_3)(2^{i+2} p_4) = 2^{i+3} p_3p_4$$

$$f^*(u_4u_{i+4}) = f(u_4)f(u_{i+4}) = (2^2 p_3)(2^{i+2} p_4) = 2^{i+4} p_3p_4.$$

Hence we have seen that each edge of the jewel graph has an *m*-Zumkeller label on it. Thus the jewel graph J_n is an *m*-Zumkeller graph. □

Example 3.1. The Jewel graph J_2 is an *m*-Zumkeller graph for $p_1 = 3, p_2 = 5, p_3 = 7$ and $p_4 = 11$ which is shown in figure 1.

4. Jelly fish graph $J(m, n)$

Definition 4.1. The Jelly Fish graph $J(m, n)$ is obtained from a 4-cycle u_1, u_2, u_3, u_4 by joining u_1 and u_3 with an edge and appending *m* pendent edges to u_2 and *n* pendent edges to u_4.

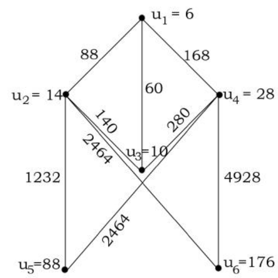

Figure 1. m-Zumkeller labeling of J_2.

4.1 *m*-Zumkeller labeling algorithm for jelly fish graph $J(m, n)$

Input. A Jelly Fish graph $J(m, n)$ having $m+n+4$ vertices and $m+n+5$ edges.

Output. m-Zumkeller jelly fish graph.

Procedure. *mZum_lab_jelly* fish graph.

$V(J(m, n)) = \{u_i : 1 \leqslant i \leqslant 4, v_i : 1 \leqslant i \leqslant m, v_i' : 1 \leqslant i \leqslant n\}$ be the vertex set of $J(m, n)$

$E(J(m, n)) = \{u_1u_2, u_1u_3, u_1u_4, u_2u_3, u_3u_4, u_2v_i : 1 \leqslant i \leqslant m, u_4v_i' : 1 \leqslant i \leqslant n\}$.
be the edge set of $J(m, n)$

$p_1 := a$ prime number $\neq 2 \leqslant 13$

$p_2 := a$ prime number $\neq 2 \leqslant 13$

$p_3 := a$ prime number $\neq 2 \leqslant 13$

$p_4 := a$ prime number $\neq 2 \leqslant 13$

$p_1 \neq p_2 \neq p_3 \neq p_4$

do

begin

$f(u_{2i-1}) = 2p_1$

$f(u_{2i+1}) = 2p_2$

While $i = 1$

end

for i : 1 to 2 do

begin

$f(u_{2i}) = 2^i p_3$

end

for i : 1 to m do

begin

$f(v_i) = 2^i p_2$

end

for i : 1 to n do

begin

$f(v_i') = 2^i p_4$

end *mZum_lab_jelly* fish graph.

Proposition 4.1. *The jelly fish graph* $J(m, n)$ *is an m-Zumkeller graph.*

Proof. Let $J(m, n)$ be a jelly fish graph with vertex set

$V(J(m, n)) = \{u_i : 1 \leqslant i \leqslant 4, v_i : 1 \leqslant i \leqslant m, v_i' : 1 \leqslant i \leqslant n\}$ and edge set

$E(J(m, n)) = \{u_1u_2, u_1u_3, u_1u_4, u_2u_3, u_3u_4, u_2v_i : 1 \leqslant i \leqslant m, u_4v_i' : 1 \leqslant i \leqslant n\}.$

Now

$$f^*(u_1u_2) = f(u_1)f(u_2) = (2p_1)(2p_3) = 2^2 p_1p_3$$

$$f^*(u_1u_3) = f(u_1)f(u_3) = (2p_1)(2p_2) = 2^2 p_1p_2$$

$$f^*(u_1u_4) = f(u_1)f(u_4) = (2p_1)(2^2 p_3) = 2^3 p_1p_3$$

$$f^*(u_2u_3) = f(u_2)f(u_3) = (2p_3)(2p_2) = 2^2 p_2p_3$$

$$f^*(u_3u_4) = f(u_3)f(u_4) = (2p_2)(2^2p_3) = 2^3p_2p_3$$

$$f^*(u_2v_i) = f(u_2)f(v_i) = (2p_3)(2^ip_2) = 2^{i+1}p_2p_3$$

$$f^*(u_4v_i') = f(u_4)f(v_i') = (2^2p_3)(2^ip_4) = 2^{i+2}p_3p_4.$$

Hence we have seen that each edge of the jewel graph has an m-Zumkeller label on it. Thus the jewel graph J_n is an m-Zumkeller graph. □

Example 4.1. The Jelly Fish graph $J(4, 5)$ is an m-Zumkeller graph, its m-Zumkeller labeling with $p_1 = 3, p_2 = 5, p_3 = 7, p_4 = 13$ is shown in figure 2.

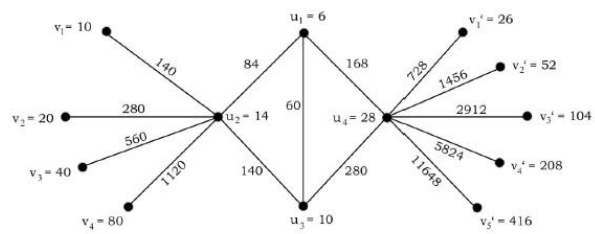

Figure 2. m-Zumkeller labeling of $J(4, 5)$.

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