Advances and Applications in Mathematical Sciences
Volume 22, Issue 12, October 2023, Pages 2297-2308
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RICCI SOLITONS IN *f*-KENMOTSU MANIFOLDS WITH THE QUARTER-SYMMETRIC NON-METRIC CONNECTION

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Abstract

In this paper, we have some curvature conditions in 3-dimensional *f*-Kenmotsu manifolds with the quarter-symmetric non-metric connection. We also have that this manifold is not always ξ-projective flat. And we have shown that 3-dimensional *f*-Kenmotsu manifold with the quarter-symmetric non-metric connection is also an η-Einstein manifold and the Ricci soliton defined on this manifold is said to be expanding or shrinking with respect to values of *f* and λ constant.

1. Introduction

In 1972, Kenmotsu [6] studied a class of contact Riemannian manifold satisfying some special conditions and named this manifold as Kenmotsu manifold.

The manifold M, with the structure (ϕ, ξ, η, g) is called normal if $[\phi, \phi] + 2d\eta \otimes \xi = 0$ and it is almost cosymplectic if $d\eta = 0$ and $d\phi = 0$. A normal and almost cosymplectic manifold is called cosympectic. Olszak and Rosca [10] studied geometrical aspect of *f*-Kenmotsu manifolds and gave some curvature conditions. Also the other mathematicians proved that a Ricci-symmetric *f*-Kenmotsu Manifold is an Einstein Manifold. Later on, in 2010, authors also proved that Ricci semi-symmetric α-Kenmotsu manifolds are Einstein manifolds.

2020 Mathematics Subject Classification: 53C15, 53C25.
Keywords: Kenmotsu manifold, Ricci solitons, quarter-symmetric non-metric connection.
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Received September 17, 2022; Accepted June 22, 2023

In 1983, Sharma and Sinha [13] started to study of the Ricci Solitons. Later on Ricci Solitons in contact manifolds were extensively studied by Cornelia Livia Bejan and Mircea Crasmareanu [2].

In 2012, the theory of Ricci solitons on Kenmotsu manifolds were studied by Nagaraja and Premalatha [2] and a deep study was done by S. C. Rastogi [11], [12] on quarter-symmetric non-metric connection.

Starting with the introduction, we have some fundamental notions used in this study, in section 2. In section 3, we have the introduction of f-Kenmotsu Manifold. In the next section 4 we study f-Kenmotsu manifold with quarter-symmetric non-metric connection and proved that this manifold is not always ξ-projective flat. In the last section we prove that f-Kenmotsu manifold with the quarter-symmetric non-metric connection is η-Einstein manifold and the Ricci soliton defined on this manifold is classified with respect to the values of f and λ constant.

2. Preliminaries

Consider a 3-dimensional differentiable manifold M with an almost contact structure (ϕ, ξ, η, g) satisfying

$$\phi^2 X = -X + \eta(X)\xi,$$

$$\eta \circ \phi = 0,$$

$$\phi\xi = 0,$$

$$\eta(\xi) = 1,$$

$$g(X, \xi) = \eta(X),$$

$$g(X, \phi Y) = -g(\phi X, Y),$$

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y) \tag{2.1}$$

for any vector fields $X, Y \in \chi(M)$, where ϕ is a (1,1) tensor field, ξ is a vecter field, η is a 1-form and g is Riemannian metric. Then M is called an almost contact manifold. For an almost contact manifold M, we have [16]

$$(\nabla_X \phi)Y = \nabla_X \phi Y - \phi(\nabla_X Y), \tag{2.2}$$

$$(\nabla_X \eta)Y = \nabla_X \eta Y - \eta(\nabla_X Y). \tag{2.3}$$

Let $\{e_1, e_2, e_3, \ldots, e_n\}$ be orthonormal basis of $T_p(M)$. R be Riemannian curvature tensor, S be Ricci curvature tensor, Q be Ricci operator, then $\forall X, Y \in \chi(M)$ it follows that [5]

$$S(X, Y) = \sum_{i=1}^{n} g(R(e_i, X)Y, e_i), \tag{2.4}$$

$$QX = -\sum_{i=1}^{n} R(e_i, X)e_i \tag{2.5}$$

$$S(X, Y) = g(QX, Y) \tag{2.6}$$

In *f*-Kenmotsu manifold, if the Ricci tensor S satisfy the condition

$$S(X, Y) = \alpha g(X, Y) + \beta\eta(X)\eta(Y) \tag{2.7}$$

α, β be certain scalars, then the manifold M is said to be η-Einstein manifold. If $\beta = 0,$ the manifold is Einstein manifold.

In a three dimensional Riemannian manifold, the curvature tensor R is defined as

$$R(X, Y)Z = S(Y, Z)X - S(X, Z)Y + g(Y, Z)QX - g(X, Z)QY$$
$$-\frac{\tau}{2}[g(Y, Z)X - g(X, Z)Y] \tag{2.8}$$

where S is the Ricci tensor, Q is Ricci operator and τ is the scalar curvature.

Now, let M be an n-dimensional Riemannian manifold with the Riemannian connection ∇. A linear connection $\tilde{\nabla}$ is said to be a quarter-symmetric connection on M if its tortion tensor $\tilde{T}$ satisfies

$$\tilde{T}(X, Y) = \eta(Y)\phi X - \eta(X)\phi Y \tag{2.9}$$

where $\tilde{T} \neq 0$ and η is a 1-form. If moreover $\tilde{\nabla} g = 0$ then the connection is called quarter-symmetric metric connection.

If $\tilde{\nabla} g \neq 0$ then the connection is called quarter-symmetric non-metric connection [17].

For $n \geq 1$, the manifold M is locally projectively flat iff the projective curvature tensor P vanishes. We define the projective curvature tensor P as

$$P(X, Y)Z = R(X, Y)Z - \frac{1}{2n}[S(Y, Z)X - S(X, Z)Y] \tag{2.10}$$

for any $X, Y, Z \in \chi(M)$ where S is the Ricci tensor and R is the curvature tensor of M. If $P(X, Y)\xi = 0$ for any $X, Y \in \chi(M)$, the manifold M is called ξ-projective flat [16].

A Ricci Soliton is defined on a Riemannian manifold (M, g) as a natural generalization of an Einstein metric. We define Ricci Soliton as a triple (g, V, λ) with g a Riemannian metric, V a vector field and λ be a real scalar such that

$$L_V g + 2S + 2\lambda g = 0 \tag{2.11}$$

where L_V denotes the Lie derivative operator along the vector field V and S is a Ricci tensor of M. The Ricci soliton is said to be shrinking, steady and expanding accordingly λ is -ve, 0, +ve respectively.

3. *f*-Kenmotsu manifolds

A three dimensional almost contact manifold M with the structure (ϕ, ξ, η, g) is an f-Kenmotsu manifold if the covariant derivative of ϕ satisfies [16],

$$(\nabla_X \phi)Y = f[g(\phi X, Y)\xi - \eta(Y)\phi X] \tag{3.1}$$

where $f \in C^{\infty}(M, R)$ such that $df \wedge \eta = 0$. If $f^2 + f' \neq 0$ where $f' = \xi f$, then M is called Regular [3]. If $f = \alpha =$ constant $\neq 0$, M is called α-Kenmotsu manifold. If $f = 1$ then manifold is called 1-Kenmotsu manifold also called Kenmotsu Manifold.

By (2.1) and (2.3), we have

$$(\nabla_X \eta)Y = fg(\phi X, \phi Y), \tag{3.2}$$

from (3.1),we have [14]

$$\nabla_X \xi = f[X - \eta(X)\xi]. \tag{3.3}$$

Also from (2.7), in a 3-dimensional *f*-Kenmotsu manifold

$$R(X, Y)Z = \left(\frac{\tau}{2} + 2f^2 + 2f'\right)(X \wedge Y)Z$$

$$- \left(\frac{\tau}{2} + 3f^2 + 3f'\right)[\eta(X)(\xi \wedge Y)Z + \eta(Y)(X \wedge \xi)Z] \tag{3.4}$$

and

$$S(X, Y) = \left(\frac{\tau}{2} + f^2 + f'\right)g(X, Y) - \left(\frac{\tau}{2} + 3f^2 + 3f'\right)\eta(X)\eta(Y). \tag{3.5}$$

Thus from (3.5), we get

$$S(X, \xi) = -2(f^2 - f')\eta(X). \tag{3.6}$$

By (3.4) and (3.5), we get

$$R(X, Y)\xi = -(f^2 + f')[\eta(Y)X - \eta(X)Y] \tag{3.7}$$

$$R(X, Y)\xi = -(f^2 + f')(\eta(X)\xi - X), \tag{3.8}$$

$$QX = \left(\frac{\tau}{2} + f^2 + f'\right)X - \left(\frac{\tau}{2} + 3f^2 + 3f'\right)\eta(X)\xi. \tag{3.9}$$

From (2.10) and using (3.7) and (3.6), we have that

Theorem 1. *A 3-dimensional f-Kenmotsu manifold is always ξ-projectively flat.*

4. *f*-Kenmotsu Manifolds with the Quarter-symmetric Non-metric Connection

Let ∇ be a Riemannian connection of *f*-Kenmotsu manifold and $\tilde{\nabla}$ be a linear connection then this linear connection $\tilde{\nabla}$ defined as

$$\tilde{\nabla}_X Y = \nabla_X Y - \eta(X)\phi Y - g(X, Y)\xi \tag{4.1}$$

where $X, Y \in \chi(M)$ be any vector field and η be 1-form, is called the quarter-symmetric non-metric connection [15]. Now, using (2.2), (3.1) and (4.1) we have

$$(\tilde{\nabla}_X \phi)Y = f[g(\phi X, Y)\xi - \eta(Y)\phi X] + g(\phi X, Y)\xi \tag{4.2}$$

for any vector field $X, Y \in \chi(M)$ where ϕ be (1,1) tensor field , is ξ is a vector field, η is 1-form and $f \in C^{\infty}(M, R)$ so that $df \wedge \eta = 0$. As a result of $df \wedge \eta = 0$, we have

$$df = f', X(f) = f'\eta(X) \tag{4.3}$$

where $f' = \xi f$ [10]. If $f = 0$, the manifold is cosymplectic. If $f = \alpha \neq 0$, then the manifold is α-Kenmotsu. An f-Kenmotsu manifold with quarter-symmetric non-metric connection is called regular, if $f^2 + f' + f - 2f\phi \neq 0$.

From (2.2), (4.2) we have

$$\tilde{\nabla}_X \xi = f[X - \eta(X)\xi] - \eta(X)\xi. \tag{4.4}$$

Using (2.2), (4.1) and (3.2), we get

$$(\tilde{\nabla}_X \eta) = fg(\phi X, \phi Y). \tag{4.5}$$

We define the curvature tensor $\tilde{R}$ of any f-Kenmotsu manifold M with respect to quarter-symmetric non-metric connection $\tilde{\nabla}$ as

$$\tilde{R}(X, Y)\xi = \tilde{\nabla}_X \tilde{\nabla}_Y \xi - \tilde{\nabla}_Y \tilde{\nabla}_X \xi - \tilde{\nabla}_{[X, Y]}\xi \tag{4.6}$$

using (4.1), (4.4) and (3.3) we obtain

$$\begin{aligned} \tilde{\nabla}_X \tilde{\nabla}_Y \xi = {} & X(f)Y - X(f)\eta(Y)\xi + f\nabla_X Y - fX\eta(Y)\xi - \eta(X)f\phi Y \\ & - fg(X, Y)\xi - \eta(Y)f^2 X - \eta(Y)fX + 2\eta(X)\eta(Y)f\xi \\ & + \eta(X)\eta(Y)\xi + f^2\eta(X)\eta(Y)\xi - X\eta(Y)\xi, \end{aligned} \tag{4.7}$$

and

$$\begin{aligned} - \tilde{\nabla}_{[X, Y]}\xi = {} & -f\nabla_X Y + f\nabla_Y X + f\eta(Y)\phi X - f\eta(X)\phi Y \\ & + fX\eta(Y)\xi - fY\eta(X)\xi + X\eta(Y)\xi - Y\eta(X)\xi. \end{aligned} \tag{4.8}$$

Using (4.7) and (4.8) in (4.6), we have

$$\widetilde{R}(X, Y)\xi = X(f)Y - Y(f)X - X(f)\eta(Y)\xi + Y(f)\eta(X)\xi$$
$$- f^2\eta(Y)X + f^2\eta(X)Y - f\eta(Y)X + f\eta(X)Y$$
$$+ 2\eta(Y)f\phi X - 2\eta(X)f\phi Y. \tag{4.9}$$

By using (4.3) in (4.9), we have

$$\widetilde{R}(X, Y)\xi = -(f^2 + f' + f - 2f\phi)(\eta(Y)X - \eta(X)Y). \tag{4.10}$$

From (4.10), we get

$$\widetilde{R}(\xi, Y)\xi = -(f^2 + f' + f - 2f\phi)(\eta(Y)\xi - Y). \tag{4.11}$$

and

$$\widetilde{R}(X, \xi)\xi = -(f^2 + f' + f - 2f\phi)(X - \eta(X)\xi). \tag{4.12}$$

In (4.10), taking inner product with Z, we get

$$g(\widetilde{R}(X, Y)\xi, Z) = -(f^2 + f' + f - 2f\phi)(\eta(Y)g(X, Z) - \eta(X)g(Y, Z)). \tag{4.13}$$

With the help of these result we have the following lemma.

Lemma 1. *Let M be* 3*-dimensional f-Kenmotsu manifold with the quarter-symmetric non-metric connection.* $\widetilde{S}$ *be Ricci curvature and* $\widetilde{Q}$ *be Ricci operator*, *then*

$$\widetilde{S}(X, \xi) = -(f^2 + f' + f - 2f\phi)\,\eta(X), \tag{4.14}$$

and

$$\widetilde{Q}\xi = -(f^2 + f' + f - 2f\phi)\xi. \tag{4.15}$$

Proof. Contracting (4.13) with Y and Z and taking summation over $i = 1, 2, 3, \ldots, n$, using (2.4) we have (4.14). And also by using (2.6) and (2.1) in (4.14), we get (4.15).

Lemma 2. *Let M be* 3*-dimensional f-Kenmotsu manifold with quarter symmetric non-metric connection.* $\widetilde{S}$ *be Ricci tensor*, τ *be scaler curvature tensor and* $\widetilde{Q}$ *be Ricci operator. Then it follows that*

$$\widetilde{S}(X, Y) = \left(\frac{\tau}{2} + f^2 + f' + f - 2f\phi\right)g(X, Y)$$

$$- \left(\frac{\tau}{2} + 3f^2 + 3f' + 3f - 6f\phi\right)\eta(X)\eta(Y), \tag{4.16}$$

and

$$\widetilde{Q}X = \left(\frac{\tau}{2} + f^2 + f' + f - 2f\phi\right)X - \left(\frac{\tau}{2} + 3f^2 + 3f' + 3f - 6f\phi\right)\eta(X)\xi. \tag{4.17}$$

Proof. Taking inner product of (4.12) with Y, we get

$$g(\widetilde{R}(X, \xi), Y) = -(f^2 + f' + f - 2f\phi)(g(X, Y) - \eta(X)\eta(Y)). \tag{4.18}$$

By putting $X = \xi, Y = X, Z = Y$ in (2.8), using (4.14) and taking contraction with ξ, we obtain

$$g(\widetilde{R}(\xi, X)Y, \xi) = \widetilde{S}(X, Y) + 4(f^2 + f' + f - 2f\phi)\eta(X)\eta(Y)$$

$$- 2(f^2 + f' + f - 2f\phi)g(X, Y) - \frac{\tau}{2}[g(X, Y) - \eta(X)\eta(Y)]. \tag{4.19}$$

With the help of (4.18) and (4.19), we have (4.16). Now using (4.16) and (2.6), we get

$$g\left(\widetilde{Q}X - \left[\left(\frac{\tau}{2} + f^2 + f' + f - 2f\phi\right)X - \left(\frac{\tau}{2} + 3f^2 + 3f' + 3f - 6f\phi\right)\eta(X)\xi\right], Y\right) = 0. \tag{4.20}$$

Since $Y \neq 0$ in (4.20), this leads the proof of (4.17).

Example (a 3-dimensional *f*-Kenmotsu manifold with quarter-symmetric non-metric connection). Let us consider the 3-dimensional manifold $M = (x, y, z) \in R^3, z \neq 0$ where (x, y, z) are the standard coordinates in R^3. The vector fields

$$e_1 = z^2 \frac{\partial}{\partial x}, \; e_2 = z^2 \frac{\partial}{\partial y}, \; e_3 = \frac{\partial}{\partial z}$$

are linearly independent at each point of M. Let g be the Riemannian metric defined as

$$g(e_1, e_1) = g(e_2, e_2) = g(e_3, e_3) = 1, \; g(e_1, e_2) = g(e_2, e_3) = g(e_3, e_1) = 0.$$

Now consider a $(1, 1)$ tensor field ϕ defined by $\phi(e_1) = -e_2$, $\phi(e_2) = e_1$, $\phi(e_3) = 0$ then using linearity of g and ϕ, for any $Z, W \in \chi(M)$ we have

$$\eta(e_3) = 1,$$

$$\phi^2(Z) = -Z + \eta(Z)e_3,$$

$$g(\phi Z, \theta W) = g(Z, W) - \eta(Z)\eta(W).$$

Now by computation directly, we get

$$[e_1, e_2] = 0, [e_2, e_3] = -\frac{2}{z} e_2, [e_1, e_3] = -\frac{2}{z} e_1.$$

By the use of these above equations, we have

$$\nabla_{e_1} e_1 = \frac{2}{z} e_3, \nabla_{e_2} e_2 = \frac{2}{z} e_3, \nabla_{e_3} e_3 = 0,$$

$$\nabla_{e_1} e_1 = \nabla_{e_2} e_2 = \nabla_{e_3} e_1 = \nabla_{e_3} e_3 = 0. \tag{4.21}$$

Now in this example we consider for quarter-symmetric non-metric connection, using (4.1) and (4.21) we have

$$\tilde{\nabla}_{e_i} e_i = \left(\frac{2}{z} - 1\right) e_3, \tilde{\nabla}_{e_3} e_3 = -e_3, \tilde{\nabla}_{e_i} e_3 = -\frac{2}{z} e_i, \tilde{\nabla}_{e_i} e_j = 0 = \tilde{\nabla}_{e_3} e_i \tag{4.22}$$

where $i \neq j = 1, 2$.

We know that

$$\tilde{R}(X, Y)Z = \tilde{\nabla}_X \tilde{\nabla}_Y Z - \tilde{\nabla}_Y \tilde{\nabla}_X Z - \tilde{\nabla}_{[X, Y]} Z. \tag{4.23}$$

Using (4.22) and (4.23) we get

$$\tilde{R}(e_i, e_3)e_3 = \left(\frac{2}{z} - \frac{6}{z^2}\right) e_i, \tilde{R}(e_i, e_j)e_3 = 0$$

$$\tilde{R}(e_i, e_j)e_j = \left(\frac{2}{z} - \frac{6}{z^2}\right) e_i, \tilde{R}(e_i, e_3)e_j = 0, \tag{4.24}$$

$$\tilde{R}(e_3, e_i)e_i = \left(\frac{2}{z} - \frac{6}{z^2}\right) e_3$$

where $i \neq j = 1, 2$.

Using (2.4) and (4.24), we verify that

$$\widetilde{S}(e_i, e_i) = -\frac{10}{z^2} + \frac{2}{z} + 1, \; i = 1, 2, \; \widetilde{S}(e_3, e_3) = -\frac{12}{z^2} + \frac{4}{z}. \tag{4.25}$$

Now using (2.10), (4.24) and (4.25), we have that

$$\widetilde{P}(e_1, e_2)e_3 = 0, \; \widetilde{P}(e_i, e_3)e_3 = \left(\frac{4}{3z} - \frac{8}{z^2}\right)e_i$$

This leads to the following Lemma:

Lemma 3. *A* 3-d*imensional f-Kenmotsu manifold with the quarter-symmetric non-metric connection is not necessarily* ξ*-projectively flat*.

5. Ricci Solitons in *f*-Kenmotsu Manifold with the quarter-symmetric non-metric connection

Consider a 3-dimensional *f*-Kenmotsu manifold with the quarter-symmetric non-metric connection. Let V be pointwise collinear with ξ (i.e. $V = b\xi$, where b is a function). Then $(L_V g + 2S + 2\lambda g)(X, Y) = 0$ implies

$$(Xb)\eta(Y) + bg(\widetilde{\nabla}_X \xi, Y) + (Yb)\eta(X) + bg(X, \widetilde{\nabla}_Y \xi) + 2\widetilde{S}(X, Y) + 2\lambda g(X, Y) = 0. \tag{5.1}$$

Using (4.4) in (5.1), we get

$$0 = (Xb)\eta(Y) + (Yb)\eta(X) + 2bfg(X, Y) - 2bf\eta(X)\eta(Y) - b\eta(X)\eta(Y) + 2\widetilde{S}(X, Y) + 2\lambda g(X, Y) \tag{5.2}$$

substitute Y with ξ in (5.2),we obtain

$$Xb - 2b\eta(X) + \xi b\eta(X) - 4(f^2 + f' + f - 2f\phi)\eta(X) + 2\lambda\eta(X) = 0 \tag{5.3}$$

again substituting X with ξ in (5.3)

$$\xi b = 2(f^2 + f' + f - 2f\phi) + b - \lambda \tag{5.4}$$

putting (5.3) in (5.4), we have

$$b = [2(f^2 + f' + f - 2f\phi) + b - \lambda]\eta \tag{5.5}$$

applying d on (5.5)

$$0 = db = [2(f^2 + f' + f - 2f\phi) + b - \lambda]d\eta \tag{5.6}$$

since $d\eta \neq 0$, we have

$$[2(f^2 + f' + f - 2f\phi) + b - \lambda] = 0. \tag{5.7}$$

Now using (5.5) and (5.7), it is obtain that b is constant. Hence from (5.2), we can verify

$$\widetilde{S}(X, Y) = -(bf + \lambda)g(X, Y) + b(f - 1)\eta(X)\eta(Y) \tag{5.8}$$

which results that M is η-Einstein manifold. This gives a following theorem:

Theorem 2. *If in a* 3- *dimensional f-Kenmotsu manifold with quarter-symmetric non-metric connection*, *the metric g is a Ricci solitons and V is a pointwise collinear with* ξ, *then V is a constant multiple of* ξ *and M is* η-*Einstein manifold of the form* (5.8) *and Ricci Solitons is expanding or shrinking according as* $\lambda = 2(f^2 + f' + f - 2f\phi) + b$ *is positive or negative*.

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