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A DISCRETE SEIR MODEL WITH LATENT PERIOD AND APPLICATION TO THE TRANSMISSION OF COVID-19

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Abstract

A discrete time SEIR model with incubation period is established that can be applied to the transmission of COVID-19. The basic reproduction number 0 R of the model is defined, and the threshold properties of R_0 are verified via numerical simulation. The effects of some nonpharmaceutical intervention (NPI) measures, such as wearing mask, acid testing, on the control for the disease transmission, are obtained via both sensitivity analysis and numerical studies.

1. Introduction

The COVID-19 pandemic in recent years has caused deep influences on the people's daily life, disrupted the global economic, social systems and posed comprehensive threats to the people's health [1]. To fight against COVID-19, intervention strategies have been implemented in various degrees to alleviate the infection intensity. These interventions mainly refers to non-pharmaceutical interventions (NPIs), including mask wearing, hand washing, adaptation or closure of school (business), travel restrictions, limits and restrictions on public and private gatherings [2]. Effects of some NPIs on the transmission of COVID-19 have been analyzed in the literature [3, 4, 5]. For example, it is shown that maintaining social distance can reduce the risk of interpersonal communication associated with COVID-19 [5]; surgical masks can prevent the spread of droplets from infectious individuals [4]. However, those results on the effectiveness of NPIs are not obtained quantitatively.

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Various mathematical models are established to study quantitatively the transmission character and the effects of control measures on ceasing the transmission of infectious diseases [6, 7]. Those models are mainly continuous models described by ordinary differential equation systems. In this paper, a discrete mathematical model described by difference equation systems [8] will be constructed, and the effects of some NPIs on the transmission of the infectious diseases will be analyzed quantitatively.

The rest of paper is organized as follows: in section 2, a discrete model is constructed and the equilibrium of the model is given, also the basic reproduction number of the model is defined and sensitivity analysis of the basic reproduction number with respect to the model parameters is carried out; In section 3, numerical simulations are given to indicate the effect of some NPIs on the transmission of COVID-19; Concluding remarks are drawn in the last section.

2. Construction and Analysis of the Discrete Model

2.1. Model Construction

The population in considered region is divided into 4 groups: the susceptible (S), the exposed (E), the infected (I) and the recovered (R). The number of these 4 groups at the nth time step is denoted as $S(n)$, $E(n)$, $I(n)$, $R(n)$, respectively. Thus, $S(n+1)-S(n)$ is the variation of the number in the susceptible group from nth time step to the $(n+1)$th time step. Supposed that the recruitment rate of the population is b, the contact infection rate is β, and the natural death rate of the population is d, we have

$$S(n+1)-S(n) = b-\beta S(n)I(n)-dS(n) \tag{1}$$

The individuals who infect COVID-19 virus are moved to the exposed group. Some of them in this group become infected after an average incubation period τ for the virus and move to the infected group with a rate α; some of them who show no symptoms and self-recovered move to the recovered group with a rate δ, therefore,

$$E(n+1)-E(n) = \beta S(n)I(n)-\alpha\beta S(n-\tau)I(n-\tau)\exp(-d\tau)-dE(n)-\delta E(n) \tag{2}$$

As for the infected group, we suppose that

$$I(n+1) - I(n) = \alpha\beta S(n-\tau)I(n-\tau)\exp(-d\tau) - dI(n) - \mu I(n) - \gamma I(n), \quad (3)$$

where μ is the disease caused death rate, γ is the recovery rate. Lastly,

$$R(n+1) - R(n) = \gamma I(n) + \delta E(n) - dR(n) \quad (4)$$

reflects the variation of number in the recovered group.

Combining equations (1)-(4), we obtain the discrete model that described the transmission of COVID-19 among the population. The total population in the region is

$$= S + E + I + R.$$

The initial values of difference equation system (1)-(4) are given as follows:

$$S(k) = s_k,\ k = -\tau,\ \tau + 1,\ B,\ 1,\ 0;\ I(k) = i_k,\ k = -\tau,\ -\tau + 1,\ B,\ 1,\ 0;$$

$$E(0) = e_0;\ R(0) = r_0 \quad (5)$$

where $s_k,\ i_k(k = -\tau,\ -\tau + 1,\ B,\ 1,\ 0),\ e_0,\ r_0$ are all non-negative.

2.2. Equilibrium and Basic Reproduction Number

The equilibrium of (1)-(4) is the solution(s) of the following algebraic equation:

$$\begin{cases} b - \beta SI - dS = 0 \\ \beta SI - \alpha\beta SI\exp(-d\tau) - dE - \delta E = 0 \\ \alpha\beta SI\exp(-d\tau) - dI - \mu I - \gamma I = 0 \\ \gamma I + \delta E - dR = 0 \end{cases} \quad (6)$$

It is straightforward by solving (6) to obtain the following result about the of equilibrium of model (1)-(4).

Theorem 1. (i) *There always exists the disease free equilibrium* $E_0(b/d,\ 0,\ 0,\ 0)$; (ii) *Provided*

$$\frac{b\alpha\beta\exp(-d\tau)}{d(\mu + \gamma + d)} > 1 \quad (7)$$

then there exists a positive equilibrium $E_1(\overline{S},\ \bar{I},\ \overline{E},\ \overline{R})$ *besides* E_0, *where*

$$\overline{S} = \frac{\mu + \gamma + d}{\alpha\beta \exp(-d\tau)} \quad (8)$$

$$\overline{E} = \frac{1}{d + \delta}\left[(1 - \alpha \exp(-d\tau))\frac{b\alpha\beta \exp(-d\tau) - d(\mu + \gamma + d)}{\alpha\beta \exp(-d\tau)}\right] \quad (9)$$

$$\bar{I} = \frac{b\alpha\beta \exp(-d\tau) - d(\mu + \gamma + d)}{\beta(\mu + \gamma + d)} \quad (10)$$

$$\overline{R} = \frac{\gamma}{d} \cdot \frac{b\alpha\beta \exp(-d\tau) - d(\mu + \gamma + d)}{\beta(\mu + \gamma + d)}$$

$$+ \frac{\delta}{d(d + \delta)}\left[(1 - \alpha \exp(-d\tau))\frac{b\alpha\beta \exp(-d\tau) - d(\mu + \gamma + d)}{\alpha\beta \exp(-d\tau)}\right] \quad (11)$$

From Theorem 1, we define the basic reproduction number R_0 of model (1)-(4) as

$$R_0 = \frac{b\alpha\beta \exp(-d\tau)}{d(\mu + \gamma + d)} \quad (12)$$

That is, model (1)-(4) only has the disease free equilibrium E_0 if $R_0 < 1$; if $R_0 > 1$, model (1)-(4) has the positive equilibrium E_1 besides E_0.

2.3. Sensitivity Analysis

The value of basic reproduction number R_0 is closely related the severity of the transmission of the infectious diseases. The disease will vanish in the population gradually if $R_0 < 1$ and it will be endemic in the population if $R_0 > 1$. The larger of R_0 is, the more severity of the infectious disease. Hence, control measures are adopted to reduce the value of R_0 in practice. Some control measures seem to be more effective than others. How to find the control measures that are more effective and easy to apply in the real world is key important to cease or to slow down the transmission of the infectious diseases.

In this part, we use partial elasticity of the basic reproduction number R_0 with respect to each parameter to explain the sensitivity of the variation of R_0. For a given function $z = f(x, y)$, we denote $Ez/Ex = (\partial z/\partial x) \cdot (x/z) = a$ as the elasticity of z with respect to the variable x. If $a > 0$, then the value of z increases $a\%$ when the value of x increases 1%; and if $a < 0$, then the value of z decreases $|a|\%$ when the value of x increases 1%.

Direct computation gives:

$$\frac{ER_0}{Eb} = \frac{ER_0}{E\alpha} = \frac{ER_0}{E\beta} = 1 \tag{13}$$

$$\frac{ER_0}{E\tau} = -d\tau \tag{14}$$

$$\frac{ER_0}{Ed} = -\tau R_0 - \frac{R_0}{d} - \frac{R_0}{\mu + \gamma + d} \tag{15}$$

$$\frac{ER_0}{E\mu} = \frac{ER_0}{E\gamma} = -\frac{R_0}{\mu + \gamma + d} \tag{16}$$

From (13)-(16), we observe that one can decrease the value of b, α, β or increase the value of τ, d, μ, γ to reduce the R_0 value. But with the practical meaning of these parameters considered, it is difficult or impossible at present to influence the value of b, τ, d, μ. That is, decreasing the value of the transmitting rate from the exposed group to the infected group α, decreasing the contact infection rate β or increasing the recovery rate γ is more appropriate in practice to control the transmission of COVID-19. For example, acid-testing for COVID-19 in population can find timely the individual who is exposed to the virus. Moreover, if home-staying for clinic observation or other measures are adopted to these exposed individuals, the value of α will be decreased; NPIs, such as wearing masks, keeping social distance or reducing social gathering can decrease the value of β; positive clinic treatment for the infected can increase the value of γ. In the following, we will indicate the effectiveness of some NPIs quantitatively via numerical studies for model (1)-(4).

3. Numerical Studies

In this section, we show some transmission properties of model (1)-(4) via numerical simulations. First, we simulate the threshold properties for the basic reproduction number R_0. That is, the disease free equilibrium E_0 is stable when $R_0 < 1$ and the endemic equilibrium (the positive equilibrium) exists which is stable when $R_0 > 1$.

3.1. Threshold Properties of R_0

We set the parameters in model (1)-(4) as the following:

$$b = 50; \beta = 4.5 \times 10^{-7}; d = 3.9139 \times 10^{-5}; \alpha = 0.35; \tau = 15; \delta = 0.06;$$

$$\mu = 0.04; \gamma = 0.2;$$

From (12), we have

$$R_0 = \frac{b\alpha\beta \exp(-d\tau)}{d(\mu + \gamma + d)} = 0.8337 < 1 \tag{17}$$

The initial values are given as followed:

$$S(k) = 1000; I(k) = 100, \ k = -15, -14, \text{B}, 1, 0; E(0) = 10; R(0) = 10;$$

The variation of the number of the infected group is shown in Figure 1. We observe that the disease free equilibrium is stable. This shows that the disease will gradually cease its transmission among the population.

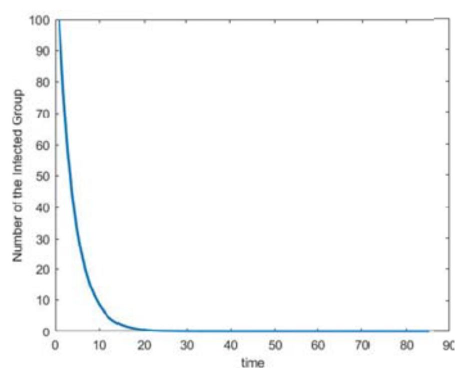

Figure 1. Variation of the number of the infected group when $R_0 < 1$. The disease free equilibrium is stable.

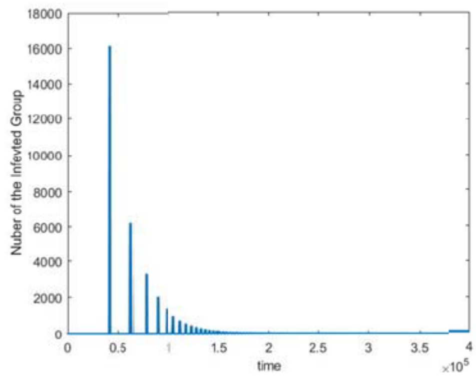

Figure 2. Variation of the number of the infected group when $R_0 > 1$.

Moreover, we set the parameters in model (1)-(4) as the following:

$$b = 100; \beta = 1.3 \times 10^{-6}; d = 3.9139 \times 10^{-5}; \alpha = 0.25; \tau = 10; \delta = 0.05;$$

$$\mu = 0.05; \gamma = 0.3;$$

Form (12), we have

$$R_0 = \frac{b\alpha\beta \exp(-d\tau)}{d(\mu + \gamma + d)} = 2.2713 > 1 \tag{18}$$

The initial values are given as followed:

$$S(k) = 10000; I(k) = 500, \ k = -10, -9, \text{B}, 1, 0; E(0) = 100; R(0) = 20;$$

The variation of the number of the infected group is shown in figure 2. From Figure 2, the existence of the positive equilibrium when $R_0 > 1$ and it is stable are difficult to observe due to the range of the number of the infected group is large but the value of $\bar{I}$ defined as (10) is 41.2858 is relatively small. We further set the time range in the interval [190000, 400000], and the variation of the number of the infected group in this interval is shown in Figure 3. From Figure 3, one can easily observe that the positive equilibrium exists and it is stable.

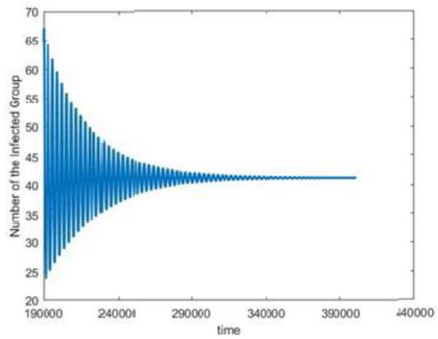

Figure 3. Variation of the number of the infected group when $R_0 > 1$. The endemic equilibrium, whose value is 41.2858, is stable.

3.2. Effects of NPI Measures

As mentioned in Section 2.3, wearing masks, Keeping social distance or reducing social gathering can reduce the value of the contact infectious rate β; Acid-testing for COVID-19 and home-staying for clinic observation to these exposed individuals can decrease the value α. In this part, we further analyze the effectiveness of some NPI measures via numerical studies.

First, we choose the values of parameters in model (1)-(4) such that positive equilibrium exists and be stable. Set the parameters in model (1)-(4) as the following:

$$b = 100; \beta = 2.5 \times 10^{-6}; d = 3.9139 \times 10^{-5}; \alpha = 0.1; \tau = 10; \delta = 0.05;$$

$$\mu = 0.05; \gamma = 0.05; \tag{19}$$

From (12), we have

$$R_0 = \frac{b\alpha\beta \exp(-d\tau)}{d(\mu + \gamma + d)} = 0.3825 > 1 \tag{20}$$

The initial values are given as followed:

$$S(k) = 1000; I(k) = 800; k = -10, -9, \text{B}, 1, 0; E(0) = 100; R(0) = 20;$$

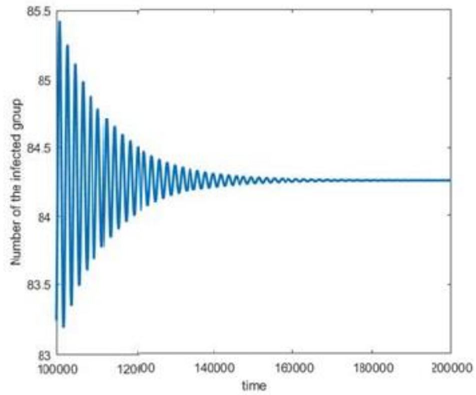

Figure 4. Variation of the number of the infected group. The value of R_0 is given in (20). The endemic equilibrium, whose value is 84.2662, is stable.

The variation of the number of the infected group is shown in figure 4. The time range in figure 4 is set in the interval [100000, 200000]. From Figure 4, one can easily observe that the positive equilibrium, whose value is 54.2662, exists and it is stable.

3.2.1 Simulation for decreasing the value β

Followed, we show that by decreasing the value of β via NPI measures, the value of the basic reproduction number of model (1)-(4) will decrease. And the infectious disease will disappear gradually if the NPI measures that the value of β now is decreased to 3.9×10^{-7}, the other parameters are the same as in (19). From (12), we have

$$R_0 = \frac{b\alpha\beta \exp(-d\tau)}{d(\mu + \gamma + d)} = 0.9957 < 1, \tag{21}$$

The variation of the number of the infected group is shown in figure 5. From Figure 5, one can observe that the disease free equilibrium is stable.

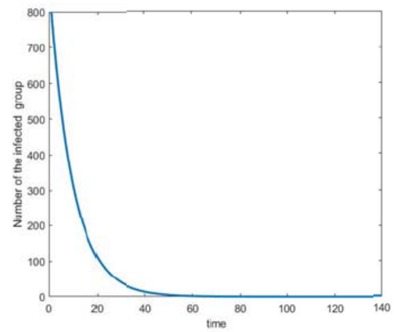

Figure 5. Variation of the number of the infested group when R_0 is given as (21). The disease free equilibrium is stable.

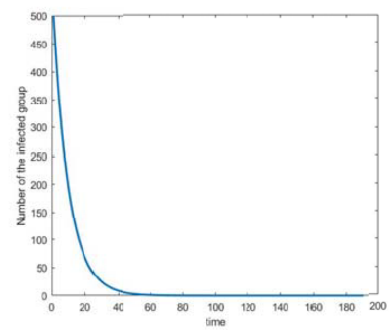

Figure 6. Variation of the number of the infected group when R_0 is given as (22). The disease free equilibrium is stable.

3.2.2 Simulation for decreasing the value α

We further show that by decreasing the value of α via NPI measures, the infectious disease will disappear gradually if the basic reproduction number is decreased to a value less than 1. Supposed that the value of α now is decreased to 0.015, the other parameters are the same as in (19). From (12), we have

$$R_0 = \frac{b\alpha\beta \exp(-d\tau)}{d(\mu + \gamma + d)} = 0.9574 < 1, \tag{22}$$

The variation of the number of the infected group is shown in Figure 6. From Figure 6, one can observe that the disease free equilibrium is stable.

4. Conclusions

A discrete SEIR mathematical model with latent period is constructed and is applied to the transmission of COVID-19 virus. The basic reproduction number is defined. The effects of some non-pharmaceutical intervention (NPI) measures on the control for the disease transmission, such as wearing mask, acid testing, are obtained via both sensitivity analysis and numerical studies. It is shown in section 3.2.1 and 3.2.2 that the infectious disease will disappear gradually if the NPI measures are efficient enough such that the basic reproduction number is less than 1.

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