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A STUDY ON FUZZY ROUGH ALGEBRAIC BORDER AND EXTERIOR

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Abstract

The concepts of fuzzy rough algebraic border and fuzzy rough algebraic exterior sets are introduced in this paper. The properties of fuzzy rough algebraic border and fuzzy rough algebraic exterior are discussed. There are some intriguing characterizations and attributes studied.

1. Introduction

Authors [14], [8], [9], and [3] discussed fuzzy topological spaces, fuzzy sets, and their applications. Soft set theory, intuitionistic fuzzy set theory, soft fuzzy set theory, and other approaches are used to address a variety of additional uncertainties that crop up in real-world situations. In [12] and [13], a few of such theories' characteristics are covered. The idea of rough set was first presented by Pawlak [7]. R. Biswas and S. Nanda investigated the rough group and rough subgroup [1]. The rough topological space was researched by B.P. Mathew and S. J. John [5]. S. Majmudar and S. Nanda [6] examined the idea of a fuzzy rough set. The fuzzy rough topological group and

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fuzzy rough group were examined and discussed by the author [11]. Calda, Jafari, and Noiri [2] researched the ideas of G-exterior and G-border. The concepts of fuzzy rough algebraic border and fuzzy rough algebraic exterior sets are introduced in this paper. The properties of fuzzy rough algebraic border and fuzzy rough algebraic exterior are discussed. There are some intriguing characterizations and attributes studied.

2. Preliminaries

Definition 2.1 [11]**.** Let X be the fuzzy topological space, and let be a fuzzy set. The boundary Bd of λ, is then determined as $Bd(\lambda) = cl(\lambda) \cap cl(\lambda)'$. Obviously, $Bd(\lambda)$ is a fuzzy closed set.

Definition 2.2 [2]**.** The fuzzy rough border of any fuzzy rough set in (X, T) is defined as the intersection of the fuzzy rough set with the closure of its compliment.

Definition 2.3 [2]**.** The fuzzy rough exterior of any fuzzy rough set in (X, T) is defined as the interior of the compliment of the fuzzy rough set.

3. Fuzzy Rough Algebraic Border and Exterior

Definition 3.1. Let (X, TM) be a fuzzy rough algebraic TM system and be any fuzzy rough algebraic. Then the fuzzy rough algebraic boundary of A, is denoted and defined as

$$\mathcal{F}_{\mathcal{RTM}}Bd(A) = \mathcal{F}_{\mathcal{RTM}}cl(A) \cap \mathcal{F}_{\mathcal{RTM}}cl(A)'.$$

Definition 3.2. Let A be a fuzzy rough algebraic in a fuzzy rough algebraic TM system (X, TM). Then the fuzzy rough algebraic border of A is defined and denoted by

$$\mathcal{F}_{\mathcal{RTM}}Br(A) = A \cap \mathcal{F}_{\mathcal{RTM}}cl(A)'.$$

Proposition 3.1. *Let A be any fuzzy rough algebraic in a fuzzy rough algebraic TM system (X, TM). Then*

(i) $\mathcal{F}_{\mathcal{RTM}}Br(A) \subset \mathcal{F}_{\mathcal{RTM}}cl(A)^{'}$.

(ii) $\mathcal{F}_{\mathcal{RTM}}\ \text{int}(\mathcal{F}_{\mathcal{RTM}}Br(A)) \subseteq A$.

(iii) $\mathcal{F}_{\mathcal{RTM}}Br(\mathcal{F}_{\mathcal{RTM}}Br(A)) \subseteq \mathcal{F}_{\mathcal{RTM}}Br(A)$.

Proof.

i) $\mathcal{F}_{\mathcal{RTM}}Br(A) = A \cap \mathcal{F}_{\mathcal{RTM}}cl(A)^{'}$

$$\subseteq \mathcal{F}_{\mathcal{RTM}}cl(A')$$

Hence $\mathcal{F}_{\mathcal{RTM}}Br(A) \subseteq \mathcal{F}_{\mathcal{RTM}}cl(A')$.

ii) $\mathcal{F}_{\mathcal{RTM}}\ \text{int}(\mathcal{F}_{\mathcal{RTM}}Bd(A)) = \mathcal{F}_{\mathcal{RTM}}\ \text{int}(A \cap \mathcal{F}_{\mathcal{RTM}}cl(A'))$

$$\subseteq A \cap \mathcal{F}_{\mathcal{RTM}}cl(A')$$

$$= A$$

Hence $\mathcal{F}_{\mathcal{RTM}}\ \text{int}(\mathcal{F}_{\mathcal{RTM}}Br(A)) \subseteq A$.

iii) $\mathcal{F}_{\mathcal{RTM}}Br(\mathcal{F}_{\mathcal{RTM}}Br(A)) = \mathcal{F}_{\mathcal{RTM}}Br(A \cap \mathcal{F}_{\mathcal{RTM}}cl(A'))$

$$\subseteq (A \cap \mathcal{F}_{\mathcal{RTM}}cl(A')) \cap (\mathcal{F}_{\mathcal{RTM}}cl(A \cap \mathcal{F}_{\mathcal{RTM}}cl(A'))^{'})$$

$$\subseteq A \cap \mathcal{F}_{\mathcal{RTM}}cl(A')$$

Hence $\mathcal{F}_{\mathcal{RTM}}Br(\mathcal{F}_{\mathcal{RTM}}Br(A)) \subseteq \mathcal{F}_{\mathcal{RTM}}Br(A)$.

Proposition 3.2. *If A is any fuzzy rough algebraic open in (X, TM) then* $\mathcal{F}_{\mathcal{RTM}}Br(A) \subseteq A$.

Proof. Since A is a fuzzy rough algebraic open, then A' is a fuzzy rough algebraic closed. Then

$$\mathcal{F}_{\mathcal{RTM}}Br(A) = A \cap \mathcal{F}_{\mathcal{RTM}}cl(A')$$

$$\subseteq A \cap A'$$

$$\subseteq A.$$

Hence $\mathcal{F}_{\mathcal{RTM}}Br(A) \subseteq A$.

Proposition 3.3. *Let C and D be any two fuzzy rough algebraic in (X, TM) then $\mathcal{F}_{\mathcal{RTM}}Br(C) \cup \mathcal{F}_{\mathcal{RTM}}Br(D)$.*

Proof.

$$\mathcal{F}_{\mathcal{RTM}}Br(C \cup D) = (C \cup D) \cap \mathcal{F}_{\mathcal{RTM}}cl(C \cup D)'$$

$$= (C \cup D) \cap (\mathcal{F}_{\mathcal{RTM}}cl(C' \cap D'))$$

$$\subseteq (C \cup D) \cap (\mathcal{F}_{\mathcal{RTM}}cl(C') \cap \mathcal{F}_{\mathcal{RTM}}cl(D'))$$

$$= (\mathcal{F}_{\mathcal{RTM}}Br(C') \cap \mathcal{F}_{\mathcal{RTM}}cl(D'))$$

$$\cup (\mathcal{F}_{\mathcal{RTM}}Br(D) \cap \mathcal{F}_{\mathcal{RTM}}cl(C'))$$

$$= \mathcal{F}_{\mathcal{RTM}}Br(C) \cup \mathcal{F}_{\mathcal{RTM}}Br(D)$$

Hence $\mathcal{F}_{\mathcal{RTM}}Br(C \cup D) \subseteq \mathcal{F}_{\mathcal{RTM}}Br(C') \cup \mathcal{F}_{\mathcal{RTM}}Br(D')$.

Proposition 3.4. *Let S and T be any two fuzzy rough algebraic in fuzzy rough TM system (X, TM). Then*

$$\mathcal{F}_{\mathcal{RTM}}Br(S \cap T) \supseteq \mathcal{F}_{\mathcal{RTM}}Br(S) \cap \mathcal{F}_{\mathcal{RTM}}Br(T).$$

Proof.

$$\mathcal{F}_{\mathcal{RTM}}Br(S \cap T) = (S \cap T) \cap \mathcal{F}_{\mathcal{RTM}}cl(S \cap T)'$$

$$= (S \cap T) \cap (\mathcal{F}_{\mathcal{RTM}}cl(S' \cup T'))$$

$$= (S \cap T) \cap (\mathcal{F}_{\mathcal{RTM}}cl(S') \cup \mathcal{F}_{\mathcal{RTM}}cl(T'))$$

$$\supseteq (S \cap \mathcal{F}_{\mathcal{RTM}}cl(S')) \cap (T \cap \mathcal{F}_{\mathcal{RTM}}cl(T'))$$

$$= \mathcal{F}_{\mathcal{RTM}}Br(S) \cap \mathcal{F}_{\mathcal{RTM}}Br(T).$$

Hence $\mathcal{F}_{\mathcal{RTM}}Br(S \cap T) \supseteq \mathcal{F}_{\mathcal{RTM}}Br(S) \cap \mathcal{F}_{\mathcal{RTM}}Br(T)$.

Proposition 3.5. *For any fuzzy rough algebraic A in* (X, TM),

$$\mathcal{F}_{\mathcal{RTM}}Br(A) \subseteq \mathcal{F}_{\mathcal{RTM}}Bd(A).$$

Proof.

$$\mathcal{F}_{\mathcal{RTM}}Br(A) = A \cap \mathcal{F}_{\mathcal{RTM}}cl(A')$$

$$\subseteq \mathcal{F}_{\mathcal{RTM}}cl(A) \cap \mathcal{F}_{\mathcal{RTM}}cl(A')$$

$$= \mathcal{F}_{\mathcal{RTM}}Bd(A).$$

Therefore, $\mathcal{F}_{\mathcal{RTM}}Br(A) \subseteq \mathcal{F}_{\mathcal{RTM}}Bd(A)$.

Remark 3.1. If A is any fuzzy rough algebraic closed in (X, TM) then $\mathcal{F}_{\mathcal{RTM}}Br(A) \subseteq \mathcal{F}_{\mathcal{RTM}}Bd(A)$.

Proof. Since A is a fuzzy rough algebraic closed, $\mathcal{F}_{\mathcal{RTM}}cl(A) = A$.

Now, $\mathcal{F}_{\mathcal{RTM}}Br(A) = A \cap \mathcal{F}_{\mathcal{RTM}}cl(A')$

$$= \mathcal{F}_{\mathcal{RTM}}cl(A) \cap \mathcal{F}_{\mathcal{RTM}}cl(A').$$

$$= \mathcal{F}_{\mathcal{RTM}}Bd(A).$$

Therefore, $\mathcal{F}_{\mathcal{RTM}}Br(A) = \mathcal{F}_{\mathcal{RTM}}Bd(A)$.

Definition 3.3. A fuzzy rough algebraic D in (X, TM) is said to be fuzzy rough algebraic exterior of D is denoted and defined by

$$\mathcal{F}_{\mathcal{RTM}}Er(D) = \mathcal{F}_{\mathcal{RTM}}\,\mathrm{int}(D').$$

Proposition 3.6. *Let D be a fuzzy rough algebraic in fuzzy rough algebraic TM system* (X, TM). *Then*

(i) $\mathcal{F}_{\mathcal{RTM}}Er(D) \subseteq (\mathcal{F}_{\mathcal{RTM}}cl(D))'$.

(ii) $\mathcal{F}_{\mathcal{RTM}}Er(\mathcal{F}_{\mathcal{RTM}}Er(D)) = \mathcal{F}_{\mathcal{RTM}}\,\mathrm{int}(\mathcal{F}_{\mathcal{RTM}}cl(D))$.

(iii) $\mathcal{F}_{\mathcal{RTM}}Er(\widetilde{1}) = \widetilde{0}$.

(iv) $\mathcal{F}_{\mathcal{RTM}}Er(\widetilde{0}) = \widetilde{1}$.

(v) $\mathcal{F}_{\mathcal{RTM}}\operatorname{int}(D) \subseteq \mathcal{F}_{\mathcal{RTM}}Er(\mathcal{F}_{\mathcal{RTM}}cl(D))$.

Proof.

(i) The proof follows directly from the Definition 2.17.

(ii) $\mathcal{F}_{\mathcal{RTM}}Er(\mathcal{F}_{\mathcal{RTM}}Er(D)) = \mathcal{F}_{\mathcal{RTM}}Er(\mathcal{F}_{\mathcal{RTM}}Er(D'))$ [By Definition 2.17]

$$= \mathcal{F}_{\mathcal{RTM}}\operatorname{int}(\mathcal{F}_{\mathcal{RTM}}\operatorname{int}(D'))'$$

$$= \mathcal{F}_{\mathcal{RTM}}\operatorname{int}(\mathcal{F}_{\mathcal{RTM}}cl(D)).$$

Hence, $\mathcal{F}_{\mathcal{RTM}}Er(\mathcal{F}_{\mathcal{RTM}}Er(D)) = \mathcal{F}_{\mathcal{RTM}}\operatorname{int}(\mathcal{F}_{\mathcal{RTM}}cl(D))$.

(iii) By (ii),

$$\mathcal{F}_{\mathcal{RTM}}Er(\widetilde{1}) = \mathcal{F}_{\mathcal{RTM}}Er(\widetilde{1}') = \widetilde{0}.$$

(iv) By (ii),

$$\mathcal{F}_{\mathcal{RTM}}Er(\widetilde{0}) = \mathcal{F}_{\mathcal{RTM}}Er(\widetilde{0}') = \widetilde{1}.$$

(v) Since $A \subseteq \mathcal{F}_{\mathcal{RTM}}cl(A)$.

$$\Rightarrow \mathcal{F}_{\mathcal{RTM}}\operatorname{int}(A) \subseteq \mathcal{F}_{\mathcal{RTM}}\operatorname{int}(\mathcal{F}_{\mathcal{RTM}}cl(A))$$

$$\Rightarrow \mathcal{F}_{\mathcal{RTM}}\operatorname{int}(A) \subseteq \mathcal{F}_{\mathcal{RTM}}Er(\mathcal{F}_{\mathcal{RTM}}Er(A)) \text{ by (iii).}$$

Proposition 3.7. *Let R and S be any two fuzzy rough algebraic in a fuzzy rough algebraic TM system* (X, TM). *Then the following conditions hold.*

(i) *If* $R \subseteq S$ *then* $\mathcal{F}_{\mathcal{RTM}}Er(R) \supseteq \mathcal{F}_{\mathcal{RTM}}Er(S)$.

(ii) $\mathcal{F}_{\mathcal{RTM}}Er(R \cup S) = \mathcal{F}_{\mathcal{RTM}}Er(R) \cap \mathcal{F}_{\mathcal{RTM}}Er(S)$.

(iii) $\mathcal{F}_{\mathcal{RTM}}Er(R \cap S) = \mathcal{F}_{\mathcal{RTM}}Er(R) \cup \mathcal{F}_{\mathcal{RTM}}Er(S)$.

Proof.

(i) Since $R \subseteq S$, $\mathcal{F}_{\mathcal{RTM}}Er(R) \subseteq \mathcal{F}_{\mathcal{RTM}}Er(S)$. Hence the proof is obvious by Definition 2.17.

(ii) $\mathcal{F}_{\mathcal{RTM}}Er(R \cup S) = \mathcal{F}_{\mathcal{RTM}} \operatorname{int}(R \cup S)'.$

$$= \mathcal{F}_{\mathcal{RTM}} \operatorname{int}(R' \cup S')$$

$$= \mathcal{F}_{\mathcal{RTM}} \operatorname{int}(R') \cap \mathcal{F}_{\mathcal{RTM}} \operatorname{int}(S')$$

$$= \mathcal{F}_{\mathcal{RTM}} \operatorname{int}(R) \cap \mathcal{F}_{\mathcal{RTM}} \operatorname{int}(S).$$

Hence, $\mathcal{F}_{\mathcal{RTM}}Er(R \cup S) = \mathcal{F}_{\mathcal{RTM}}Er(R) \cap \mathcal{F}_{\mathcal{RTM}}Er(S).$

(iii) The proof is similar to (ii).

Proposition 3.8. *Let A be a fuzzy rough algebraic in a fuzzy rough algebraic TM system (X, TM). Then $\mathcal{F}_{\mathcal{RTM}}Er(A) = A'$ if and only if A is fuzzy rough algebraic closed.*

Proof. If A is any fuzzy rough algebraic TM closed, then

$$A = \mathcal{F}_{\mathcal{RTM}}cl(A).$$

By Definition 2.17, $\mathcal{F}_{\mathcal{RTM}}Er(A) = \mathcal{F}_{\mathcal{RTM}} \operatorname{int}(A')$

$$= (\mathcal{F}_{\mathcal{RTM}}(A))'$$

$$= A'$$

Hence, $\mathcal{F}_{\mathcal{RTM}}Er(A) = A'.$

Conversely, $\mathcal{F}_{\mathcal{RTM}}Er(A) = A'$

$$\Rightarrow \mathcal{F}_{\mathcal{RTM}} \operatorname{int}(A') = A'.$$

Hence A' is a fuzzy rough algebraic open. Therefore, A is a fuzzy rough algebraic closed.

Proposition 3.9. *Let A be any fuzzy rough algebraic in a fuzzy rough algebraic TM system (X, TM). Then*

$$(\mathcal{F}_{\mathcal{RTM}}Bd(A))' = \mathcal{F}_{\mathcal{RTM}} \operatorname{int}(A) \cup \mathcal{F}_{\mathcal{RTM}}Er(A).$$

Proof.

Since, $\mathcal{F}_{\mathcal{RTM}}Bd(A) = \mathcal{F}_{\mathcal{RTM}}cl(A) \cap \mathcal{F}_{\mathcal{RTM}}cl(A')$.

Then, $(\mathcal{F}_{\mathcal{RTM}}Bd(A))' = (\mathcal{F}_{\mathcal{RTM}}cl(A) \cap \mathcal{F}_{\mathcal{RTM}}cl(A'))'$

$$= \mathcal{F}_{\mathcal{RTM}} \operatorname{int}(A') \cup \mathcal{F}_{\mathcal{RTM}} \operatorname{int}(A)$$

$$= \mathcal{F}_{\mathcal{RTM}} \operatorname{int}(A) \cup \mathcal{F}_{\mathcal{RTM}} \operatorname{int}(A')$$

$$= \mathcal{F}_{\mathcal{RTM}} \operatorname{int}(A) \cup \mathcal{F}_{\mathcal{RTM}}Er(A).$$

Therefore, $(\mathcal{F}_{\mathcal{RTM}}Bd(A))' = \mathcal{F}_{\mathcal{RTM}} \operatorname{int}(A) \cup \mathcal{F}_{\mathcal{RTM}}Er(A)$.

Proposition 3.10. *Let A be a fuzzy rough algebraic in a fuzzy rough algebraic TM system* (X, TM). *Then* $\mathcal{F}_{\mathcal{RTM}}Br(A) = (\mathcal{F}_{\mathcal{RTM}}Er(A'))'$.

Proof.

Since, $\mathcal{F}_{\mathcal{RTM}}Br(A) = A \cap \mathcal{F}_{\mathcal{RTM}}cl(A')$

Then $\mathcal{F}_{\mathcal{RTM}}Br(A) \subseteq A$

$$\subseteq \mathcal{F}_{\mathcal{RTM}}cl(A)$$

$$= (\mathcal{F}_{\mathcal{RTM}}Er(A))'.$$

Hence $\mathcal{F}_{\mathcal{RTM}}Br(A) = (\mathcal{F}_{\mathcal{RTM}}Er(A))'$.

Remark 3.2. From Propositions 3.5 to 3.10,

$$\mathcal{F}_{\mathcal{RTM}}Br(A) \not\subseteq \mathcal{F}_{\mathcal{RTM}}Br(A) \subseteq \mathcal{F}_{\mathcal{RTM}}Bd(A).$$

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