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PICTURE FUZZY IDEALS IN GAMMA SEMIGROUPS

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Abstract

The picture fuzzy ideals in gamma semigroups have been introduced and some of their properties have been investigated. In this paper, the theory is illustrated by some examples connected with the notion of picture fuzzy bi-gamma ideals in gamma semigroups.

1. Introduction

The fuzzy set theories were developed by Zadeh [20]. The study of fuzzy algebraic structures started with the introduction of the concepts of fuzzy (left, right) ideal, which were pioneered by Rosenfeld [13] while Kuroki [8, 9] introduced and studied fuzzy (left, right) ideals and fuzzy bi-ideals in semigroups. Γ-groups and Γ-regular semigroups and established a relation between Γ-groups and Γ-regular semigroups were studied and characterized by Sen and Saha [17], Saha [14, 15]. The Γ-semigroups were defined by Sen [16]. Mustafa et al. [19] studied the intuitionistic fuzzification of several types

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of Γ-ideal in a Γ-semigroup and their properties. The concept of a bi-Γ-ideal in a Γ-semigroup was introduced by Chinram and Jirojkul [4]. Prince Williams et al. [12] investigated the fuzzification of bi-Γ-ideals in Γ-semigroups. The intuitionistic fuzzy set (IFS) was proposed by Atanassov [2, 3]. IFS have been applied in different areas by various researchers. It is seen that one of the important concept of neutrality degree is lacking in IFS theory. Concept of neutrality degree can be seen in situations when we involving more answers of type yes, abstain, no, refusal (Palash Dutta and Silpashree Ganju [10]. For example, in a democratic election station, so the picture fuzzy set was introduced by B. C. Cuong [7]. Some aspects of picture fuzzy sets were studied by Palash Dutta and Silpashree Ganju [10] in medical diagnosis degree of neutrality which can be considered. Phong et al. [11] studied some compositions of picture fuzzy relations. B. C. Cuong and Kreinovich [6] introduced Picture Fuzzy Set (PFS) which is a direct extension of fuzzy set and intuitionistic fuzzy set by incorporating the concept of positive, negative and neutral membership degree of an element and suggested distance measures between PFSs. In this paper, Sujit Kumar et al. [18] were introduced the notion "Atanassov's intuitionistic fuzzy ideals of Γ-semigroups" based on the concept of the paper, we introduce picture fuzzy ideal of gamma semigroups and investigate some of properties connected with level cut. Also we illustrate picture fuzzy bi-gamma ideal of gamma semigroups by suitable examples.

2. Preliminaries

Definition 2.1[5]**.** Let S be a semigroup and a non-empty subset A of S is a sub semigroup if $A^2 \subseteq A$.

Definition 2.2[14]**.** A fuzzy set μ in S is called a fuzzy subsemigroup of S if $\mu(xy) \geq \min\{\mu(x), \mu(y)\}$ for all $x, y \in S$.

Definition 2.3[1]**.** A subsemigroup A of a semigroup S is called bi-ideal if $ASA \subseteq A$.

Definition 2.4[14]**.** A fuzzy subsemigroup μ of a semigroup is called a fuzzy bi-ideal of S if

$$\mu(xyz) \geq \min\{\mu(x), \mu(z)\} \text{ for all, } x, y \text{ and } z \in S.$$

Definition 2.5[1]**.** Let $S = \{x, y, z, \ldots\}$ and $\Gamma = \{\alpha, \beta, \gamma, \ldots\}$ be two non-empty sets. Then S is called a Γ-semigroups if it satisfies

(i) $x\gamma y \in S$

(ii) $(x\beta y)\gamma z = x\beta(\gamma yz)$ for all $x, y, x \in S$ and $\beta, \gamma \in \Gamma$.

Definition 2.6[1]**.** Let S be a Γ-semigroup, A non-empty subset A of a Γ-semigroup S is said to be a Γ-subsemigroup of S if $A\Gamma A \subseteq A$.

Definition 2.7[1]**.** A left (right) ideal of a Γ-semigroup S is a non-empty subset A of such that $S\Gamma A \subseteq A(A\Gamma S \subseteq A)$.

Definition 2.8[1]**.** If A is both a left and a right ideal of a Γ-semigroup then we say that A is a Γ-ideal of S.

Definition 2.9[1]**.** Let M be a Γ-semigroup. A Γ-semigroup of is called a bi-Γ-ideal of M if $A\Gamma M\Gamma A \subseteq A$.

Definition 2.10[20]**.** A fuzzy set μ of a Γ-semigroup is called a fuzzy Γ-subsemigroup of M if $\mu(x\gamma y) \geq \min\{\mu(x), \mu(y)\}$ for all $x, y \in M$ and $\gamma \in \Gamma$.

Definition 2.11. A PFS $P = (\zeta_P, \eta_p, \upsilon_p)$ of S is called picture fuzzy subsemigroup if it satisfies

(i) $\zeta_P(x_1 \alpha x_2) \geq \min\{\zeta_P, (x_1), \zeta_P, (x_2)\}$

(ii) $\eta_p(x_1\alpha x_2) \leq \max\{\eta_p, (x_1), \eta_p, (x_2)\}$

(iii) $\upsilon_p(x_1\alpha x_2) \leq \max\{\upsilon_p, (x_1), \upsilon_p, (x_2)\}$ for all $x_1, x_2 \in S, \alpha \in \Gamma$.

Definition 2.12[3]**.** An picture fuzzy set (PFS)P is an object having the form $P = \{z, (\zeta_P(z), \eta_p(z), \upsilon_p(z)) : z \in M\}$ where the functions $\zeta_P : M \to [0, 1]$ and $\eta_p : M \to [0, 1]$ $\upsilon_p : M \to [0, 1]$ and denote the degree of membership the degree of neutral membership and the degree of non membership of each element $z \in M$ to the set P respectively, and

$$0 \leq \zeta_P(z) + \eta_p(z) + \upsilon_p(z) \leq 1 \text{ for all } (z \in M).$$

For the sake of simplicity, we shall use the symbol $P = (\zeta_P, \eta_p, \upsilon_p)$ for the PFS

$$PFS = \{(z, (\zeta_P(z), \eta_p(z), \upsilon_p(z))) : z \in M\}.$$

Definition 2.13[18]**.** Let P and Q be to $P(X)$ then the following expressions are defined as

1. $P \subseteq Q$ iff $\zeta_P(x) \leq \zeta_Q(x)$, $\eta_p(x) \geq \eta_Q(x)$ and $\upsilon_p(x) \geq \upsilon_Q(x)$

2. $P = Q$ iff $P \subseteq Q$ and $Q \subseteq P$

3. $P \cap Q$

$= \{x, \min\{\zeta_P(x), \zeta_Q(x)\}, \max\{\eta_p(x), \eta_Q(x)\}, \max\{\upsilon_p(x), \upsilon_Q(x)\}/\ x \in X\}$

4. $P \cup Q$

$= \{x, \max\{\zeta_P(x), \zeta_Q(x)\}, \min\{\eta_p(x), \eta_Q(x)\}, \min\{\upsilon_p(x), \upsilon_Q(x)\}/x \in X\}.$

3. Picture Fuzzy Ideal of Γ-Semigroups

Definition 3.1. Let S be a Γ-semigroup. A PFS, $P^* = (\zeta_P, \eta_p, \upsilon_p)$ of S is a picture fuzzy left Ideal of S (PFLI(S)) if it satisfies

1. $\zeta_P(x\gamma y) \geq \zeta_P(y)$

2. $\eta_p(x\gamma y) \leq \eta_p(y)$

3. $\upsilon_p(x\gamma y) \leq \upsilon_p(y)$, for all $x, y \in S$ and for all $\gamma \in \Gamma$.

Picture fuzzy right ideal of S (PFRI(S)) is defined in an analogous way.

A PFS $P = (\zeta_P, \eta_p, \upsilon_p)$ in S is called picture fuzzy ideal of S(PFI(S)) if it is both a picture fuzzy left ideal and picture fuzzy right ideal of S.

It is clear that any picture fuzzy left (right) ideal of S is a picture fuzzy Γ-semigroup of S.

Definition 3.2. For any $t \in [0, 1]$ and a fuzzy subset μ of S, the set

$U(\mu, t) = \{x \in X, \mu(x) \geq t\}$ (resp. $L(\mu, t) = \{x \in X, \mu(x) \leq t\}$) is called an upper (resp. lower) t-level cut of μ.

Definition 3.3. Let S be a Γ-semigroup. Let $M = (\zeta_M, \eta_M, \upsilon_M)$ and

$N = (\zeta_N, \eta_N, \upsilon_N) \in PFLI(S)$ (PFRI(S), PFI(S)). Then the product $M \circ N$ of M and N is defined as

$$\zeta_{M \circ N}(x) = \begin{cases} \sup\{\min\{\zeta_M(u), \zeta_N(v)\} : u, v \in S, \gamma \in \Gamma\} \\ x = u\gamma v \\ 0 \text{ if for every } u, v \in S; \gamma \in \Gamma, x \neq u\gamma v \end{cases}$$

$$\eta_{M \circ N}(x) = \begin{cases} \inf\{\max\{\eta_M(u), \eta_N(v)\}\} : u, v \in S, \gamma \in \Gamma\} \\ x = u\gamma v \\ 0 \text{ if for every } u, v \in S, \gamma \in \Gamma, x \neq u\gamma v \end{cases}$$

$$\upsilon_{M \circ N}(x) = \begin{cases} \inf\{\max\{\upsilon_M(u), \upsilon_N(v)\} : u, v \in S, \gamma \in \Gamma\} \\ x = u\gamma v \\ 0 \text{ if for every } u, v \in S, \gamma \in \Gamma, x \neq u\gamma v. \end{cases}$$

Theorem 3.1. *If $P^* = (\zeta_P, \eta_p, \upsilon_p)$ is a PFLI(S) (PFRI(S), PFI(S)) then the upper and the lower level cuts $U(\zeta_P, t)$, $L(\eta_p, t)$ and $L(\upsilon_p, t)$ are LI(S) (RI(S), I(S)) for every $t \in \operatorname{Im}(\zeta_P) \cap \operatorname{Im}(\eta_p) \cap \operatorname{Im}(\upsilon_p)$.*

Proof. Let $t \in \operatorname{Im}(\zeta_P) \cap \operatorname{Im}(\eta_p) \cap \operatorname{Im}(\upsilon_p)$. Let $x \in S, \gamma \in \Gamma$ and $y \in \operatorname{Im}(\zeta_P)$. Then $\zeta_P(y) \geq t$.

Since $P^* = (\zeta_P, \eta_p, \upsilon_p)$ is a PFLI(S), then $\zeta_P(x\gamma y) \geq \zeta_P(y) \geq t$. Thus $x\gamma y \in U(\zeta_P, t)$ now, let $x \in S, \gamma \in \Gamma$ and $y \in L(\eta_p, t)$. Then $\eta_p(y) \leq t$. Since $P^* = (\zeta_P, \eta_p, \upsilon_p)$ is a PFLI(S), then $\eta_P(x\gamma y) \leq \eta_p(y) \leq t$ consequently $x\gamma y \in L(\eta_p, t)$. Again let $x \in S, \gamma \in \Gamma$ and $y \in L(\upsilon_p, t)$. Then $\upsilon_p(y) \leq t$. Since $P^* = (\zeta_P, \eta_p, \upsilon_p)$ is a PFLI(S). Hence $\upsilon_P(x\gamma y) \leq \upsilon_p(y) \leq t$. Thus $x\gamma y \in L(\upsilon_p, t)$. Therefore $U(\zeta_P, t)$, $L(\eta_p, t)$, $L(\upsilon_p, t)$ are LI(S) (RI(S), I(S)).

Theorem 3.2. *Let $M = (\zeta_M, \eta_M, \upsilon_M)$ be a PFRI(S) and $N = (\zeta_N, \eta_N, \upsilon_N)$ be a PFLI(S). Then $M \circ N \subseteq M \cap N$.*

Proof. Let $M = (\zeta_M, \eta_M, \upsilon_M)$ be a PFRI(S) and $N = (\zeta_N, \eta_N, \upsilon_N)$ be a PFLI(S)

Let $z \in S$. Suppose there exist $u, v \in S$ and $\gamma \in \Gamma$ such that $z = u\gamma v$. Then, we have

$$(\zeta_M \circ \zeta_N)(z) = \sup_{z = u\gamma v} \{\min \{\zeta_M(u), \zeta_N(v)\}\}$$

$$\leq \sup_{z = u\gamma v} \{\min \{\zeta_M(u\gamma v), \zeta_N(u\gamma v)\}\}$$

$$= \min \{\zeta_M(z), \zeta_N(z)\}$$

$$= (\zeta_M \cap \zeta_N)(z)$$

$$(\eta_M \circ \eta_N)(z) = \inf_{z = u\gamma v} \{\max \{\eta_M(u), \eta_N(v)\}\}$$

$$\geq \inf_{z = u\gamma v} \{\max \{\eta_M(u\gamma v), \eta_N(u\gamma v)\}\}$$

$$= \max \{\eta_M(z), \eta_N(z)\}$$

$$= (\eta_M \cap \eta_N)(z)$$

and

$$(\upsilon_M \circ \upsilon_N)(z) = \inf_{z = u\gamma v} \{\max \{\upsilon_M(u), \upsilon_N(v)\}\}$$

$$\geq \inf_{z = u\gamma v} \{\max \{\upsilon_M(u\gamma v), \upsilon_N(u\gamma v)\}\}$$

$$= \max \{\upsilon_M(z), \upsilon_N(z)\}$$

$$= (\upsilon_M \cap \upsilon_N)(z).$$

Suppose, there do not exist $u, v \in S$ and $\gamma \in \Gamma$ such that $z = u\gamma v$.

Then $(\zeta_M \circ \zeta_N)(z) = 0 \leq (\zeta_M \cap \zeta_N)(z), (\eta_M \circ \eta_N)(z) = 0 \geq (\eta_M \cup \eta_N)(z)$ and $(\upsilon_M \circ \upsilon_N)(z) = 0 \geq (\upsilon_M \cup \upsilon_N)(z)$.

Hence the proof.

From the above theorem and definition $M \cap N$ of the following theorem follows easily.

Theorem 3.3. *Let* $M = (\zeta_M, \eta_M, \upsilon_M)$ *be a PFRI*(S) *and* $N = (\zeta_N, \eta_N, \upsilon_N) \in PFI(S)$. *Then* $M \circ N \subseteq M \cap N \subseteq M$ *and* N.

Theorem 3.4. *Let S be a regular* Γ*-semigroup and* $M = (\zeta_M, \eta_M, \upsilon_M)$ *and* $N = (\zeta_N, \eta_N, \upsilon_N)$ *be two PFS*(S). *Then* $M \circ N \supseteq M \cap N$.

Proof. Let $a \in S$. Since, S is regular, then there exists an element $x \in S$ and $\gamma_1, \gamma_2 \in \Gamma$ such that $a = a\gamma_1 x\gamma_2 a = a\gamma a$ where $\gamma = \gamma_1 x\gamma_2 = \Gamma$. Then

$$(\zeta_M \circ \zeta_N)(a) = \sup_{a = u\gamma v} \{\min\{\zeta_M(u), \zeta_N(v)\}\}$$

$$\geq \min\{\zeta_M(a), \zeta_N(a)\}$$

$$= (\zeta_M \cap \zeta_N)(a)$$

$$(\eta_M \circ \eta_N)(a) = \inf_{a = u\gamma v} \{\max\{\eta_M(u), \eta_N(v)\}\}$$

$$= \max\{\eta_M(a), \eta_N(a)\}$$

$$= (\eta_M \cup \eta_N)(a)$$

and

$$(\upsilon_M \circ \upsilon_N)(a) = \inf_{a = u\gamma v} \{\max\{\upsilon_M(u), \upsilon_N(v)\}\}$$

$$= \max\{\upsilon_M(a), \upsilon_N(a)\}$$

$$= (\upsilon_M \cap \upsilon_N)(a).$$

Hence $M \circ N \supseteq M \cap N$.

4. Picture Fuzzy bi-Γ-Ideal of Γ-Semigroups

Let M denote a Γ-semigroup unless otherwise specified.

Definition 4.1. A PFS $P = (\zeta_P, \eta_p, \upsilon_p)$ of M is called a picture fuzzy bi-Γ-ideal of M if it satisfies:

(i) $\zeta_P(x_1\, a\, x_2\, \beta\, x_3) \geq \min\{\zeta_P(x_1), \zeta_P(x_3)\}$

(ii) $\eta_p(x_1\, a\, x_2\, \beta\, x_3) \leq \max\{\eta_p(x_1), \eta_p(x_3)\}$

(iii) $\upsilon_p(x_1\, a\, x_2\, \beta\, x_3) \leq \max\{\upsilon_p(x_1), \upsilon_p(x_3)\}$ for all $x_1, x_2, x_3 \in M$ and $\alpha, \beta \in \Gamma$.

Example 4.1[12]**.** Let $M = \{0, a, b, c\}$ and $\Gamma = (\alpha, \beta)$ be a non empty set of binary operations defined below

α	0	a	b	c
0	0	0	0	0
a	a	a	a	a
b	0	0	0	b
c	0	0	0	c

β	0	a	b	c
0	0	0	0	0
a	0	b	0	a
b	0	b	b	0
c	0	0	0	b

Clearly M is a Γ-semigroup. Moreover the picture fuzzy set $P = (\zeta_P, \eta_p, \upsilon_p)$ is defined by

$\zeta_P, (0) = 0.7, \zeta_P, (a) = 0.6, \zeta_P, (b) = \zeta_P, (c) = 0.3\, \eta_p(0) = 0.1, \eta_p(a) = 0.1,$ $(b) = \eta_p(c) = 0.3$ and $\upsilon_p(0) = 0.3, \upsilon_p(a) = 0.4, \upsilon_p(b) = 0.5, \upsilon_p(c) = 0.6.$

Let $x_1 = 0, x_2 = a, x_3 = b$. Take LHS $\zeta_P(x_1\, \alpha\, x_2\, \beta\, x_3) = \zeta_P(0\, \alpha\, a\, \beta\, b)$ $= \zeta_P(0\, \beta\, b) = \zeta_P(0) = 0.7.$

Then RHS $\min\{\zeta_P(x_1), (\zeta_P(x_3)\} = \min\{\zeta_P(0), \zeta_P(b)\} = \min\{0.7, 0.6\}$ $= 0.6.$

Therefore $\zeta_P(x_1 \alpha x_2 \beta x_3) \geq \min\{\zeta_P(x_1), \zeta_P(x_3)\}$. Similarly we calculate other conditions also.

Hence, $P = (\zeta_P, \eta_p, \upsilon_p)$ picture fuzzy bi-Γ-ideal of *M*.

Theorem 4.1. *If* $\{P_i\}_i \in \wedge$ *is a family of picture fuzzy bi-*Γ*-ideal of M, then* $\cap P_i$ *is a picture fuzzy bi-*Γ*-ideal of M where* $\cap P_i = \{\wedge \zeta_{Pi}, \vee \eta_{pi}, \vee \upsilon_{pi}\}$ *and* $\wedge\zeta_{Pi}, \vee \eta_{pi}$ *and* $\vee\upsilon_{pi}$ *are defined as follows*:

$$\wedge\zeta_{Pi}(x_1) = \inf\{\zeta_{Pi}(x_1)/i \in \wedge, x_1 \in M\}$$

$$\vee\eta_{Pi}(x_1) = \sup\{\eta_{Pi}(x_1)/i \in \wedge, x_1 \in M\}$$

$$\vee\upsilon_{Pi}(x_1) = \sup\{\upsilon_{Pi}(x_1)/i \in \wedge, x_1 \in M\}.$$

Proof. Let $x_1, x_2, x_3 \in M$ and $\alpha, \beta \in \Gamma$.

Then we have

(i) $\wedge\zeta_{Pi}(x_1 \alpha x_2 \beta x_3) = \inf\{\min\{\zeta_{Pi}(x_1), \zeta_{Pi}(x_3)\}/i \in \wedge, x_1, x_3 \in M\}$

$= \min\{\inf\{\zeta_{Pi}(x_1), \zeta_{Pi}(x_3)/i \in \wedge, x_1, x_3 \in M\}$

$= \min\{\inf\{\zeta_{Pi}(x_1)/i \in \wedge x_1 \in M\}, \inf\{\zeta_{Pi}(x_3)/i \in \wedge, x_3 \in M\}\}$

$= \min\{\wedge \zeta_{Pi}(x_1), \wedge \zeta_{Pi}(x_3)\}$

(ii) $\vee\eta_{pi}(x_1 \alpha x_2 \beta x_3) = \sup\{\max\{\eta_{pi}(x_1), \eta_{pi}(x_3)\}/i \in \wedge, x_1, x_3 \in M\}$

$= \sup\{\max\{\eta_{pi}(x_1), \eta_{pi}(x_3)/i \in \wedge, x_1, x_3 \in M\}$

$= \max\{\sup\{\eta_{pi}(x_1), \eta_{pi}(x_3)/i \in \wedge, x_1, x_3 \in M\}$

$= \max\{\sup\{\eta_{pi}(x_1)/i \in \Lambda, x_1 \in M\}, \sup\{\eta_{pi}(x_3)/i \in \wedge, x_3 \in M\}\}$

$= \max\{\vee \eta_{pi}(x_1), \vee \eta_{pi}(x_3)\}$

(iii) $\vee\upsilon_{pi}(x_1 \alpha x_2 \beta x_3) = \sup\{\max\{\upsilon_{pi}(x_1), \upsilon_{pi}(x_3)\}/i \in \wedge, x_1, x_3 \in M\}$

$= \max\{\sup\{\upsilon_{pi}(x_1), \upsilon_{pi}(x_3)/i \in \wedge, x_1, x_3 \in M\}$

$= \max\{\sup\{\upsilon_{pi}(x_1)/i \in \Lambda, x_1 \in M\}, \sup\{\upsilon_{pi}(x_3)/i \in \wedge, x_3 \in M\}\}$

$= \max\{\vee\, \upsilon_{pi}(x_1), \vee\, \upsilon_{pi}(x_3)\}$

Hence $\bigcap P_i$ is a picture fuzzy bi-Γ-ideal of S.

Theorem 4.2. *If a P_iFS $P = (\zeta_P, \eta_p, \upsilon_p)$ in M is a picture fuzzy bi-Γ-ideal of M, then so $P' = (\zeta_P, \zeta'_P)$, $\zeta'_P = 1 - \zeta_P$.*

Proof. It is sufficient to show that ζ'_P satisfies the condition (i) of picture fuzzy bi-Γ-ideal of M

For any $x_1, x_2, x_3 \in M$ and $\alpha, \beta \in \Gamma$, we have

$\zeta'_P(x_1\, \alpha\, x_2\, \beta\, x_3) = 1 - \zeta_P(x_1\, \alpha\, x_2\, \beta\, x_3)$

$\leq 1 - \min\{\zeta_P(x_1), \zeta_P(x_3)\}$

$= \max\{1 - \zeta_P(x_1), 1 - \zeta_P(x_3)\}$

$= \max\{\zeta'_P(x_1), \zeta_P(x_3)\}$.

Hence the proof.

Theorem 4.3. *Let $P = (\zeta_P, \eta_p, \upsilon_p)$ be a P_iFS in M. Then P is a picture fuzzy bi-Γ-ideal of M if and only if the fuzzy sets ζ_P, η'_p, and υ'_p are fuzzy bi-Γ-ideal of M.*

Proof. Let $P = (\zeta_P, \eta_p, \upsilon_p)$ in M is a picture fuzzy bi-Γ-ideal of M. Then clearly ζ_P is a picture fuzzy bi-Γ-ideal of M

Let $x_1, x_2, x_3 \in M$ and $\alpha, \beta \in \Gamma$,

$\eta'_p(x_1\, \alpha\, x_2\, \beta\, x_3) = 1 - \eta_p(x_1\, \alpha\, x_2\, \beta\, x_3)$

$\geq 1 - \max\{\eta_p(x_1), \eta_p(x_3)\}$

$= \min\{1 - \eta_p(x_1), 1 - \eta_p(x_3)\}$

$= \min\{\eta'_p(x_1), \eta'_p(x_3)\}$

Then $\upsilon'_p(x_1\, \alpha\, x_2\, \beta\, x_3) = -1 - \upsilon_p(x_1\, \alpha\, x_2\, \beta\, x_3)$

$\geq 1 - \max\{\upsilon_p(x_1), \upsilon_p(x_3)\}$

$= \min\{1 - \upsilon_p(x_1), 1 - \upsilon_p(x_3)\}$

$= \min\{\upsilon'_p(x_1), \upsilon'_p(x_3)\}$.

Hence η'_p, υ'_p is a fuzzy bi-Γ-ideal of M

Conversely,

ζ_P and η'_p, υ'_p are fuzzy bi-Γ-ideal of M

Let $x_1, x_2, x_3 \in M$ and $\alpha, \beta \in \Gamma$

Then $1 - \eta_p(x_1 \alpha x_2 \beta x_3) = \eta'_p(x_1 \alpha x_2 \beta x_3)$

$\geq \min\{\eta'_p(x_1), \eta'_p(x_3)\}$

$= \min\{1 - \eta_p(x_1), 1 - \eta_p(x_3)\}$

$= 1 - \min\{\eta_p(x_1), \eta_p(x_3)\}$

$\eta_p(x_1 \alpha x_2 \beta x_3) \leq \max\{\eta_p(x_1), \eta_p(x_3)\}$

and $1 - \upsilon_p(x_1 \alpha x_2 \beta x_3) = \upsilon'_p(x_1 \alpha x_2 \beta x_3)$

$\geq \min\{\upsilon'_p(x_1), \upsilon'_p(x_3)\}$

$= \min\{1 - \upsilon_p(x_1), 1 - \upsilon_p(x_3)\}$

$= 1 - \min\{\upsilon_p(x_1), \upsilon_p(x_3)\}$

which implies that $\upsilon_p(x_1 \alpha x_2 \beta x_3) \leq \max\{\upsilon_p(x_1), \upsilon_p(x_3)\}$

This completes the proof.

Theorem 4.4. *If a P_iFS $P = (\zeta_P, \eta_p, \upsilon_p)$ in S is a picture fuzzy bi-*Γ*-ideal of M if and only if $P' = (\zeta_P \zeta'_P)$, $P'' = (\eta_p, \eta'_p)$ and $P''' = (\upsilon_p, \upsilon'_p)$ are a picture fuzzy bi-*Γ*-ideal of M.*

Proof. The proof is straightforward by using above theorem.

5. Conclusion

In this paper, picture fuzzy ideal of gamma semigroups and picture fuzzy bi-gamma ideal of gamma semigroup have been defined. Subsequently, the theorem of PFLI(S) (PFRI(S), PFI(S)) and $\cap P_i$ have been proved respectively.

References

[1] Aiyared Iampan, Note on Bi-ideals in Γ-semigroups, International Journal of Algebra 3(4) (2009), 181-188.

[2] K. T. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems 20(1) (1986), 87-96.

[3] K. T. Atanassov, New operations defined over the intuitionistic fuzzy sets, Fuzzy Set and Systems 61(2) (1994), 137-142.

[4] R. Chinram and C. Jirojkul, On bi-Γ-ideals in Γ-semigroups, songklanakarin J. Sci. Technol. 29(1) (2007), 231-234.

[5] A. H. Clifford and G. B. Preston, The Algebraic Theory of Semigroups Vol. 1. American Mathematical Society, (1961).

[6] B. C. Cuong and V. Kreinovich, Picture fuzzy sets-A new concept for computational intelligence problems, in: Proceedings of the Third World Congress on Information and Communication Technologies WIICT, (2013), 1-6.

[7] B. C. Cuong, Picture fuzzy sets, Journal of Computer Science and Cybernetics 30(4) (2014), 409-420.

[8] N. Kuroki, Fuzzy bi-ideals in semigroups, Comment. Math. Univ. St. Paul. 28 (1980), 17-21.

[9] N. Kuroki, On fuzzy ideals and fuzzy bi-ideals in semigroups, Fuzzy Sets and Systems 5(2) (1981), 203-215.

[10] Palash Dutta and Silpashree Ganju, Some aspects of picture fuzzy set, Transactions of A. Razmadze Mathematical Institute 172(2) (2018), 164-175.

[11] P. H. Phong, D. T. Hieu, R. T. H. Ngan and P. T. Them, Some compositions of picture fuzzy relations, Proceedings of the 7th National Conference on Fundamental and Applied Information Technology Research, FAIR'7, Thai Nguyen (2014), 19-20.

[12] D. R. Prince Williams, K. B. Latha and E. Chandrasekeran, Fuzzy bi-Γ-ideals in Γ-semigroups, Hacettepe Journal of Mathematics and Statistics 38(1) (2009), 1-15.

[13] A. Rosenfeld, Fuzzy groups, J. Math. Anal. Appl. 35 (1971), 512-517.

[14] N. K. Saha, On Γ-semigroup-II, Bull. Calcutta Math. Soc. 79(6) (1987), 331-335.

[15] N. K. Saha, On Γ-semigroup-III, Bull. Calcutta Math. Soc. 80(1) (1988), 1-12.

[16] M. K. Sen, On Γ-semigroups, Proceedings of the International Conference on Algebra and its Application, Decker Publication, New York, 301 (1981).

[17] M. K. Sen and N. K. Saha, On Γ-semigroup-I, Bull. Calcutta Math. Soc. 78(3) (1986), 180-186.

[18] Sujit Kumar, Samit Kumar Mauumder and Manasi Mandal, Atanassovs intuitionistic fuzzy ideals of Γ-semigroups 16 Jan. (2011), 1-17. arXiv:1101.3080v1[math.GM]

[19] M. Uckun, M. A. Özturk and Y. B. Jun, Intuitionistic fuzzy sets in Γ-semigroups, Bull. Korean Math. Soc. 44(2) (2007), 359-367.

[20] L. A. Zadeh, Fuzzy Sets, Information and Control 8 (1965), 338-353.