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PROPERTIES OF WπGR CLOSED SETS IN TOPOLOGICAL SPACES

S. R SREENA, E. N SATHEESH and VARUGHESE MATHEW

Department of Mathematics
Mar Thoma College
Thiruvalla, India
E-mail: sreenasr2022@gmail.com
satheeshkizhakkemadom@gmail.com
principal@marthomacollege.org

Abstract

In this paper, we introduce and study a new class of generalized regular closed sets called wπgr closed sets in topological spaces and investigate some of its basic properties. We analyse the relation between wπgr closed sets with already existing closed sets.

1. Introduction

Topology is one of the important areas in Mathematics. It stands one among the most recent in whole of Mathematics. Topology has become an important component of applied mathematics and pure mathematics. The concept of generalized closed set and generalized open set was first introduced by N. Levine in topological space. Later on N. Palaniappan studies the concept of regular generalized closed set in topological space. Zaitsev introduced the concept of closed sets in topological space Dontchev. J and Noiri. T. introduced the concept of πg closed set in topological space. Andrijevic D, Levine, Nagaveni, K. Balachandran and Arokiarani Gnanambal Mashour. A. S. et al. and Maki et al. Janaki C. [1, 12, 15, 4, 8, 14, 13, 10] introduced the concepts of semi open sets, generalized closed sets generalized pre closed sets pre-regular closed sets pre closed sets and α-closed sets, πgα-closed sets respectively.

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2. Preliminaries

Throughout this paper X, Y and Z denote the topological spaces (X, τ), (Y, δ) and (Z, η) respectively on which no separation axioms are assumed unless explicitly stated. Let A be the subset of space X. We denote the closure of A and interior of A by $Cl(A)$ and $\text{int}(A)$ respectively.

Definition 2.1. A subset A of a topological space X is said to be

(i) a pre-open if $A \subseteq \text{int}(cl(A))$ and pre-closed if $cl(\text{int}(A)) \subseteq A$

(ii) a semi open if $A \subseteq cl(\text{int}(A))$ and semi-closed if $\text{int}(cl(A)) \subseteq A$

(iii) a regular open if $A = \text{int}(cl(A))$ and regular closed if $A = cl(\text{int}(A))$

(iv) a α-open if $A \subseteq \text{int}(cl(\text{int}(A)))$ and α-closed if $cl(\text{int}(cl(A))) \subseteq A$

(v) π-open if A is the finite union of regular open sets and the compliment of π-open is π-closed set in X.

(vi) a semi pre-open set if $A \subseteq cl(\text{int}(cl(A)))$ and a semi pre closed set if $\text{int}(cl(\text{int}(A))) \subseteq A$

(vii) a b-open set if $A \subseteq cl(\text{int}(A))\, U\, \text{int}(cl(A))$ and its compliment is b-closed.

Definition 2.2. A subset A of topological space X is said to be

(1) a ω-closed if $cl(A) \subseteq U$ whenever $A \subseteq U$ and $U \in SO(X)$.

(2) a generalized closed set if $cl(A) \subseteq U$ whenever $A \subseteq U$ and $U \in O(X)$.

(3) a regular generalized closed set if(briefly rg-closed set) if $(cl(A) \subseteq U$ whenever $A \subseteq U$ and $U \in RO(X)$.

(4) a generalized pre regular closed set (briefly gpr-closed set) if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and $U \in O(X)$.

(5) a weakly generalized closed (briefly wg-closed) if $cl(\text{int}(A)) \subseteq U$ whenever $A \subseteq U$ and $U \in O(X)$.

(6) a π-generalized closed (briefly πg-closed set) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and $U \in \pi o(X)$.

(7) a πgα-closed set if $\alpha cl(A) \subseteq U$, whenever $A \subseteq U$ and $U \in \pi O(X)$.

(8) a regular semi open set if there is a regular open set U such that $U \subseteq A \subseteq \alpha cl(U)$.

(9) a rω-closed set if $cl(A) \subseteq U$, whenever $A \subseteq U$ and U is regular semi open set in X.

(10) a regular α-open in X if there is a regular open set U such that $U \subseteq A \subseteq \alpha cl(U)$.

(11) a regular generalized α-closed set (briefly rgα closed set) if $\alpha cl(A) U \subseteq$, whenever $A \subseteq U$ and $U \in R\alpha O(X)$.

(12) a regular weakly generalized closed set (briefly rwg closed) if $cl(\text{int}(A)) \subseteq U$ whenever $A \subseteq U$ and $U \in RO(X)$.

(13) a π^*g-closed set if $cl(\text{int}(A)) \subseteq U$ whenever $A \subseteq U$ and $U \in \pi O(X)$.

(14) a πgb-closed set if $Pcl(A) \subseteq U$ whenever $A \subseteq U$ and $U \in \pi O(X)$.

(15) a πgp-closed set if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and $U \in \pi O(X)$.

(16) a πgs- closed set if $scl(A) \subseteq U$ whenever $A \subseteq U$ and $U \in \pi O(X)$.

(17) a πgsp- closed set if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and $U \in \pi O(X)$.

(18) a Pr-closed set in X if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular semi open in X.

(19) a rgw-closed set in X if $cl(\text{int}(A)) \subseteq U$ whenever $A \subseteq U$ and U is regular semi open in X.

(20) a generalized regular closed set (briefly g*r- closed set) if $rcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open.

3. Basic Properties of wπgr Closed Sets

Definition 3.1. A subset A of X is called wπgr closed set if $cl(\text{int } A) \subseteq U$ whenever $A \subseteq U$ and U is πgr open in X. The complement of wπgr-closed set is called wπgr-open set in X.

Theorem 3.2. *Every closed set is wπgr closed but not conversely.*

Proof. Let A be a closed set and $A \subseteq U, U$ is πgr open, Then $cl(\text{int } A) \subseteq U$, Whenever $A \subseteq U$ and U is πgr open. We know that $\text{int}(A) \subseteq A$ then $cl\, \text{int}(A) \subseteq cl(A) = A \Rightarrow cl\, \text{int}(A) = A \subseteq U$ Whenever $A \subseteq U$ and U is πgr open $\Rightarrow A$ is wπgr closed .

Example 3.3. Let $X = \{a, b, c, d\}$, $\tau = \{\varphi, \{a\}, \{c, d\}, \{a, c, d\}, X\}$, $\tau^c = \{x, \{b, c, d\}, \{a, b\}, \{b\}, \varphi\}$. Wπgr-closed sets $= \{X, \{b\}, \{c\}, \{d\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, \varphi\}$, $A = \{c\}$ is wπgr closed but not closed in X.

Remark 3.4. The concepts of rg-closed and wπgr closed sets are independent.

Example 3.5. Let $X = \{a, b, c, d\}$, $\tau = \{\varphi, \{a\}, \{c, d\}, \{a, c, d\}, X\}$, $\tau^c = \{x, \{b, c, d\}, \{a, b\}, \{b\}, \varphi\}$, Wπgr-closed sets $= \{X, \{b\}, \{c\}, \{d\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, \varphi\}$, rg-closed sets $= \{\varphi, \{b\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, X\}$. Hence the set $A = \{c\}$ is wπgr closed but not rg closed and the set $B = \{a, c\}$ is rg closed but not wπgr closed.

Remark 3.6. Every regular closed set is wπgr closed.

Example 3.7. Let $X = \{a, b, c, d\}$, $\tau = \{\varphi, \{a\}, \{c, d\}, \{a, c, d\}, X\}$, $\tau^c = \{x, \{b, c, d\}, \{a, b\}, \{b\}, \varphi\}$, regular open sets $= \{X, \{a\}, \{c, d\}, \varphi\}$, regular closed sets $= \{\varphi, \{b, c, d\}, \{a, b\}, \{b\}, X\}$, wπgr-closed sets $= \{X, \{b\}, \{c\}, \{d\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, \varphi\}$.

Remark 3.8. The concept of πgr-closed set and wπgr-closed set are independent and are shown in the following examples.

Example 3.9. Let $X = \{a, b, c, d\}$, $\tau = \{\varphi, \{a\}, \{c, d\}, \{a, c, d\}, X\}$, $\tau^c = \{x, \{b, c, d\}, \{a, b\}, \{b\}, \varphi\}$, πgr-closed sets $= \{\varphi, \{b\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, X\}$. wπgr-closed sets $= \{X, \{b\}, \{c\}, \{d\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, \varphi\}$. Let $A = \{c\}$. The set A is wпgr-closed but not пgr-closed in X.

Example 3.10. Let $X = \{a, b, c, d\}$, $\tau = \{\varphi, \{a\}, \{b\}, \{a, b\}, X\}$, $\tau^c = \{x, \{b, c, d\}, \{a, c, d\}, \{c, d\}, \varphi\}$, πgr-closed sets $= \{\varphi, \{c\}, \{d\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{b, c, d\}, \{a, b, d\}, \{a, c, d\}, X\}$, wπgr-closed sets $= \{X, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}, \varphi\}$. Let $A = \{a, c\}$. The set A is πgr-closed but not wпgr-closed in X.

Theorem 3.15. *Every ω-closed set is wπgr closed but not conversely.*

Example 3.11. Let $X = \{a, b, c, d\}, \tau = \{\varphi, \{a\}, \{c, d\}, \{a, c, d\}, X\}$, $\tau^c = \{x, \{b, c, d\}, \{a, b\}, \{b\}, \varphi\}$, wπgr-closed sets $= \{X, \{b\}, \{c\}, \{d\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, \varphi\}$ ω-closed sets $= \{\varphi, \{b\}, \{a, b\}, \{b, d\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, X\}$. Hence the set $\{c\}$ is wпgr closed but not w closed.

Remark 3.12. Every wπgr closed set is wg closed set but not conversely.

Example 3.13. Let $X = \{a, b, c, d\}$, $\tau = \{\varphi, \{a\}, \{b\}, \{a, b\}, X\}$, $\tau^c = \{x, \{b, c, d\}, \{a, c, d\}, \{c, d\}, \varphi\}$, wπgr-closed sets $= \{X, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}, \varphi\}$, wg-closed sets $= \{X, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \varphi\}$, Hence the set $A = \{a, c\}$ is wg closed but not wпgr closed.

Remark 3.14. The concept of πg closed set and wπgr closed are independent and is shown in the following examples.

Example 3.15. Let $X = \{a, b, c, d\}, \tau = \{\varphi, \{a\}, \{c, d\}, \{a, c, d\}, X\}$, $\tau^c = \{x, \{b, c, d\}, \{a, b\}, \{b\}, \varphi\}$, wπgr-closed sets $= \{X, \{b\}, \{c\}, \{d\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, \varphi\}$ πg-closed sets $= \{\varphi, \{b\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, X\}$. Let $A = \{c\}$. The A is wπgr-closed but not πg-closed.

Example 3.16. Let $X = \{a, b, c, d\}$, $\tau = \{\varphi, \{a\}, \{b\}, \{a, b\}, X\}$, $\tau^c = \{x, \{b, c, d\}, \{a, c, d\}, \{c, d\}, \varphi\}$, πg-closed sets $= \{\varphi, \{c\}, \{d\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{b, c, d\}, \{a, b, d\}, \{a, c, d\}, X\}$, wπgr-closed sets $= \{X, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}, \varphi\}$. Let $A = \{a, c\}$. The set A is πg-closed but not wпgr-closed in X.

Remark 3.17. Every $\pi^* g$ closed sets are wпgr closed.

Example 3.18. Let $X = \{a, b, c, d\}$, $\tau = \{\varphi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, b, c\}, X\}$, $\tau^c = \{x, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}, \{c, d\}, \{b, d\}, \{a, d\}, \{d\}, \varphi\}$ wπgr-closed sets $= \{X, \{d\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, c, d\}, \{a, b, d\}, \{b, c, d\}, \varphi\}$ $\pi^* g$ -closed sets $= \{X, \{d\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, c, d\}, \{a, b, d\}, \{b, c, d\}, \varphi\}$.

Remark 3.19. The concepts of πgp closed sets and wπgr closed are independent.

Example 3.20. Let $X = \{a, b, c, d\}$, $\tau = \{\varphi, \{a\}, \{c, d\}, \{a, c, d\}, X\}$, $\tau^c = \{x, \{b, c, d\}, \{a, b\}, \{b\}, \varphi\}$, wπgr-closed sets $= \{X, \{b\}, \{c\}, \{d\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, \varphi\}$ πgp-closed sets $= \{\varphi, \{b\}, \{c\}, \{d\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, X\}$.

Theorem 3.21. *Every wπgr closed set is rwg closed but not conversely.*

Proof. Let A be wпgr closed set and $A \subseteq U$ and U is regular open .Then $cl\,\text{int}(A) \subseteq U$ Whenever $A \subseteq U$ and U is regular open $\Rightarrow A$ is rwg closed.

Remark 3.22. The converse of the above theorem need not be true.

Example 3.23. Let $X = \{a, b, c, d\}$, $\tau = \{\varphi, \{a\}, \{c, d\}, \{a, c, d\}, X\}$, $\tau^c = \{x, \{b, c, d\}, \{a, b\}, \{b\}, \varphi\}$, wπgr-closed sets $= \{X, \{b\}, \{c\}, \{d\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, \varphi\}$ rwg-closed sets $= \{\varphi, \{b\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, X\}$.

Remark 3.24. The concepts of the sets R^g closed and wпgr closed sets are independent.

Example 3.25. $X = \{a, b, c, d\}$, $\tau = \{\varphi, \{a\}, \{d\}, \{c, d\}, \{a, c, d\}, X\}$, $\tau^c = \{x, \{b, c, d\}, \{a, b, c\}, \{b, c\}, \{a, b\}, \{c\}, \varphi\}$, wπgr–closed sets $= \{\varphi, \{b\}, \{c\}, \{a, c\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{b, c, d\}, \{a, b, d\}, X\}$ R^g-closed sets $= \{\varphi, \{b, c\}, \{c\}, \{a, c\}, \{a, b\}, \{a, b, c\}, \{b, c, d\}, \{a, b, d\}, X\}$.

Let $A = \{b\}$ and $B = \{a, b\}$. Here the set A is wпgr-closed but not R^g-closed.

Remark 3.26. The concepts of wпgr and πgα are independent.

Example 3.27. Let $X = \{a, b, c, d\}$, $\tau = \{\varphi, \{a\}, \{b\}, \{a, b\}, X\}$, $\tau^c = \{x, \{b, c, d\}, \{a, c, d\}, \{c, d\}, \varphi\}$, πgα-closed sets $= \{\varphi, \{c\}, \{d\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{b, c, d\}, \{a, b, d\}, \{a, c, d\}, X\}$, wπgr-closed sets $= \{X, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}, \varphi\}$. Let $A = \{a, c\}$. The set A is πgα-closed but not wпgr-closed in X.

Example 3.28. Let $X = \{a, b, c, d\}$, $\tau = \{\varphi, \{a\}, \{c, d\}, \{a, c, d\}, X\}$, $\tau^c = \{x, \{b, c, d\}, \{a, b\}, \{b\}, \varphi\}$, wπgr-closed sets $= \{X, \{b\}, \{c\}, \{d\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, \varphi\}$ πgα-closed sets $= \{\varphi, \{b\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, X\}$. The set $\{c\}$ is wпgr closed but not πgα-closed.

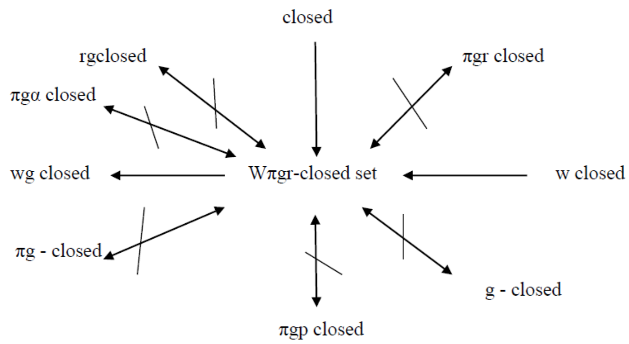
Remark 3.29. The concepts of g closed and wπgr closed are independent.

Example 3.30. Let $X = \{a, b, c, d\}$, $\tau = \{\varphi, \{a\}, \{b\}, \{a, b\}, X\}$, $\tau^c = \{x, \{b, c, d\}, \{a, c, d\}, \{c, d\}, \varphi\}$, g-closed sets $= \{\varphi, \{c\}, \{d\}, \{a, c\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{b, c, d\}, \{a, b, d\}, \{a, c, d\}, X\}$, wπgr-closed sets $= \{X, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}, \varphi\}$. Let $A = \{a, c\}$. The set A is g-closed but not wпgr-closed in X.

Example 3.31. Let $X = \{a, b, c, d\}$, $\tau = \{\varphi, \{a\}, \{c, d\}, \{a, c, d\}, X\}$, $\tau^c = \{x, \{b, c, d\}, \{a, b\}, \{b\}, \varphi\}$, wπgr-closed sets $= \{X, \{b\}, \{c\}, \{d\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, \varphi\}$, g-closed sets $= \{\varphi, \{b\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, X\}$.

The set $\{c\}$ is wπgr closed but not g closed.

Remark 3.32. The above relations are diagrammatically represented as follows:



4. Conclusion

The concept of new closed set namely wπgr closed set is introduced and studied. The relationship of wπgr closed sets using existing closed set is established. Finally, some of its fundamental properties are studied. Hence we can conclude that the defined set forms a topology which results this work may be entered widely.

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