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THERMAL EFFECT OF SUSPENDED PARTICLES AND ROTATION ON THE MICROPOLAR FLUID FLOW SATURATING A POROUS MEDIUM

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Abstract

The effect of suspended particles and rotation on a micropolar fluid layer heated from below in the presence of uniform vertical magnetic field in a porous medium is studied, using normal mode, the problem has been analyzed numerically and it is found that the medium permeability and suspended particles have destabilizing effect. The rotation, magnetic field and micro-polar parameters have stabilizing effect. The effect of suspended particle on the system is very important result. The sufficient condition for the non-existence of over stability has also been obtained.

1. Introduction

There are some important classes of fluid in technology areas, one of them being micropolar fluid. In order to study of micropolar fluids, the general theory of micropolar fluid was introduced and micro rotational inertia and micro rotational effects of micropolar fluid were shown according to Eringen [3, 4]. Qin and Kaloni [14] studied the problem of thermal instability in a rotating micropolar fluid. According to Olajuwon et al. [10] introduced the heat and mass transfer of a hydro magnetic flow when study the two-dimensional flow of a micropolar fluid over a porous medium having uniform magnetic field in the presence of thermal radiation. Kumar and Mehta [5] demonstrate the effects of permeability, hall parameter, magnetic fields in a porous medium. Wooding [21] discusses the Rayleigh instability of flow through a porous medium. According to Perez-Garcia et al. [13] extended

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effect of the microstructures in the absence of Rayleigh-Benard instability and coupling between thermal and micropolar effect.

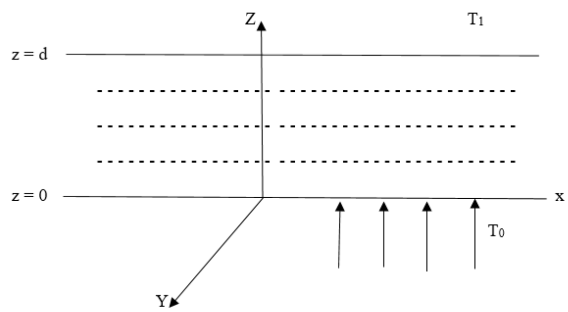
Sharma and Khanduri [16] discuss the entropy analysis for MHD flow with thermal conductivity. According to Sharma et al. [17] the effect of heat source on the hydro magnetic mixed convection flow of an electrically conducting micropolar fluid introduced through a vertical plate to a porous medium with the soret effect and taking a first order homogeneous chemical reaction. Two-dimensional mixed convection Casson fluid flow past an infinite plate under the influence of a uniform magnetic field as Patel [11]. Chand et al. [1] investigated thermal convection in a horizontal layer of micropolar nano fluid with the linear stability. Chandrawat et al. [2] an unsteady flow of two immiscible micropolar fluid is discussed and dusty fluid is assumed to pass through a horizontal channel and the work analysis of the interface. Patra et al. [12] have studied the effect of marginal stability on ferro fluid layer with heat and mass transfer in a porous medium. Miqdady and Idris [9] have studied the effect of cubic temperature gradient and linear feedback control on the onset of Rayleigh-Benard-Marangoni-Magnetoconvection in a micropolar fluid. Reena and Rana [15] have investigated the double-diffusive convection in a micropolar fluid flow.

Mittal and Rana [8] have investigated the effect of dust particles on thermal convection in micropolar ferromagnetic fluid that saturating a porous medium with a uniform magnetic field and the magnetic thermal Rayleigh number numerically for the onset of instability. According to Singh [20] discussed the effect of suspended particles and rotation on a ferromagnetic micro-polar fluid layer heated from below saturated in a porous medium. Sharma and Gupta [18] have discussed the convection of a saturated micropolar fluids heated from below in the presence of dusty. Sharma and Gupta [19] have studied the numerically effect on hydrodynamics flow of a suspended and rotating micropolar.

In view of the above discussion, application of the micropolar fluid in geophysics, chemical technology, astrophysics, biomechanics and industry. In this paper I attempt to study the effect of suspended particles on thermal convection in a micropolar rotating fluid in a porous medium. To the best of my knowledge this problem has not been investigated so far using the generalized Darcy's model.

2. Mathematical Formulation

In the present problem, we have considered an infinite, horizontal and incompressible micropolar fluid layer of thickness d. This fluid layer is assumed to be the following through suspended particles in porous medium of porosity ϵ and medium permeability k_1. The lower limit at $z = 0$ and upper limit at $z = d$ are maintained at constant but varying temperatures T_0 and T_1 such that a study adverse temperature gradient $\beta = \left|\frac{dT}{dz}\right|$ has been maintained. The whole system is acted on by a uniform vertical rotation $\Omega = (0, 0, \Omega)$ and gravity $g = (0, 0, -g)$.



The mathematical equations governing the motion of a micropolar fluid saturating a porous medium following Boussinesq approximation for the above model as follows G. Lebon [6], Lukaszewicz [7] and Eringen [3].

The equations of continuity, momentum and angular momentum are

$$\nabla \cdot \vec{q} = 0. \tag{1}$$

$$\frac{\rho_0}{\epsilon}\left[\frac{\partial \vec{q}}{\partial t} + \frac{1}{\epsilon}(\vec{q} \cdot \nabla)\vec{q}\right] = -\nabla P - \frac{1}{k_1}(\mu + k)\vec{q} + k\nabla \times \vec{v} - \rho g \hat{e}_z \frac{2\rho_0}{\epsilon}(\vec{q} \times \Omega)$$

$$+ \frac{KN}{\epsilon}(\overrightarrow{q_d} - \vec{q}) + \frac{\mu_e}{4\pi}(\nabla \times \vec{H}) \times \vec{H} \tag{2}$$

$$\rho_0 J\left[\frac{\partial \vec{v}}{\partial t} + \frac{1}{\epsilon}(\vec{v} \cdot \nabla)\vec{v}\right] = (\epsilon' + \beta')\nabla(\nabla \vec{v}) + \gamma' \nabla^2 \vec{v} + \frac{k}{\epsilon}(\nabla \times \vec{q}) - 2k\vec{v} \tag{3}$$

where, ρ-Fluid density, ρ_0-Reference density, $\vec{q}$-Filter velocity, $\vec{v}$-Spin (micro rotation), μ-Shear kinematic viscosity coefficient, k-Coupling viscosity coefficient, P-Pressure, μ_e-Magnetic permeability, $\hat{e}_z$-Unit vector in z-direction, ϵ'-Bulk spin viscosity coefficient, β'-Shear spin viscosity coefficient, γ'-Micropolar coefficient of viscosity, J-Micro inertia constant, t-time, $\vec{q}_d(x, t)$ -Filter velocity of suspended particles and $N(x, y)$-Number density of suspended particles in the micropolar fluid respectively.

The equations of energy, basic state and Maxwell's are

$$[\rho_0 C_v \epsilon + \rho_s C_s (1-\in)]\frac{\partial T}{\partial t} + \rho_0 C_v (\vec{q}\cdot\nabla)T + mNC_{Pt}\left(\epsilon\frac{\partial T}{\partial t} + \vec{q}_d\cdot\nabla T\right)$$

$$= \chi\nabla^2 T + \delta'(\nabla\times\vec{v})\cdot\nabla T \tag{4}$$

$$\rho = \rho_0[1-\alpha(T-T_a)] \tag{5}$$

$$\epsilon\frac{\partial\vec{H}}{\partial t} = \nabla(\vec{q}\times\vec{H}) + \epsilon\eta\nabla^2\vec{H} \tag{6}$$

and

$$\nabla\cdot\vec{H} = 0 \tag{7}$$

where, $\vec{H} = (0, 0, H_z), H_z$ -Constant, η-Electrical resistivity, C_v - Specific heat at constant value, C_s - Specific heat of solid (Porous Material Matrix), C_{Pt} - Specific heat of suspended particles, ρ_s -Density of solid matrix, χ-Thermal conductivity, T-Temperature, δ'-Micropolar heat conduction coefficient, α-Coefficient of thermal expansion, T_a-Average temperature given by $T_a = \frac{(T_0+T_1)}{2}$ (T_0 and T_1 are the constant average temperature of the lower and upper surface of the fluid layer) and mN -The mass of suspended particles per unit volume.

The equations of motion and continuity for the particles are

$$mN\left[\frac{\partial\vec{q}_d}{\partial t} + \frac{1}{\in}(\vec{q}_d\cdot\nabla)\vec{q}_d\right] = KN(\vec{q}-\vec{q}_d) \tag{8}$$

$$\in\frac{\partial N}{\partial t} + \nabla\cdot(N\vec{q}_d) = 0 \tag{9}$$

Basic State of the problem

$\vec{q} = \vec{q}_b(0, 0, 0)$, $\vec{q}_d = (\vec{q}_d)_b(0, 0, 0)$, $\vec{v} = \vec{v}_b(0, 0, 0)$, $\rho = \rho_b(z)$, and $P = P_b(z)$

Under this basic state, equations (1) to (9) become

$$\frac{dP_b}{dz} + \rho_b g = 0 \tag{10}$$

$$T = T_b(z) = -\beta z + T_a; \text{ where } \beta = \frac{(T_1 - T_0)}{d} \text{ and } N = N_b = N_0 \text{ (constant).} \tag{11}$$

$$\rho_b = \rho_0(1 + \alpha\beta z) \tag{12}$$

3. Linearize Perturbation Equations

$$\nabla \cdot \overline{q'} = 0 \tag{13}$$

$$L\left[\frac{\rho_0}{\epsilon}\frac{\partial \overrightarrow{q'}}{\partial t}\right] = L\left[-\nabla P' - \frac{1}{k_1}(\mu + k)\overrightarrow{q'} - \rho_0\alpha\theta g\hat{e}_z + k\nabla \times \overrightarrow{v'} + \frac{2\rho_0}{\epsilon}(\overrightarrow{q'} \times \Omega)\right.$$

$$\left. + \frac{\mu_e H_z}{4\pi}(\nabla \times \vec{h}) \times \hat{e}_z\right] - \frac{mN_0}{\epsilon}\frac{\partial \vec{q}'}{\partial t} \tag{14}$$

$$\rho_0 J\frac{\partial \overrightarrow{v'}}{\partial t} = (\epsilon' + \beta')\nabla(\nabla \cdot \overrightarrow{v'}) + \gamma'\nabla^2\overrightarrow{v'} + \frac{k}{\epsilon}(\nabla \times \overrightarrow{q'}) - 2k\overrightarrow{v'} \tag{15}$$

$$L[E + h_T\epsilon]\frac{\partial\theta}{\partial t} = L\left[k_T\nabla^2\theta - \frac{\delta'}{\rho_0 C_v}(\nabla \times \overrightarrow{v'})_z\beta + \beta(\overrightarrow{q'})_z\right] + h_T\beta(\overrightarrow{q'})_z \tag{16}$$

$$\in\frac{\partial \vec{h}}{\partial t} = H_z\nabla \times (\overrightarrow{q'} \times \hat{e}_z) + \epsilon\eta\nabla^2\vec{h} \tag{17}$$

$$\nabla . \vec{h} = 0 \tag{18}$$

$$\rho' = -\rho_0\alpha\theta \tag{19}$$

where, $k_T = \dfrac{\chi}{\rho_0 C_v}$ and $h_T = \dfrac{mN_0 C_{Pt}}{\rho_0 C_v}$ are the thermal diffusivity, $L = \left[\dfrac{m}{K}\dfrac{\partial}{\partial t} + 1\right]$.

Converting equations (13) to (19) to non-dimensional form by the following transformation and dropping the strings,

$x = d\,x^*,\quad y = d\,y^*,\quad z = d\,z^*,\quad \vec{q}' = \frac{k_T}{d} q^*,\quad \theta = \beta d \theta^*,\quad P' = \frac{\mu k_T}{d^2} P^*,\quad \vec{v}' = \frac{k_T}{d^2} \vec{v}^*,\quad t = \frac{\rho_0 d^2}{\mu} t^*,$

$\Omega = \frac{\mu}{\rho_0 d^2} \Omega^*,\ \vec{h} = H_z H^*,\ \nabla = \frac{\nabla^*}{d},\ K = \frac{k}{\mu},\ L^* = \tau \frac{\partial}{\partial t^*} + 1$ and $\tau = \frac{m\mu}{K \rho_0 d^2}$, then we have

$$\nabla \cdot \vec{q} = 0 \tag{20}$$

$$L \frac{1}{\in} \frac{\partial q}{\partial t} = L\left[-\nabla P - \frac{1}{K_1}(1+K)\vec{q} + R\theta \hat{e}_z + K(\nabla \times \vec{v}) + \frac{2}{\in}(\vec{q} \times \Omega) + Q(\nabla \times \vec{h}) \times \vec{e}_z \right] - \frac{F}{\in} \frac{\partial \vec{q}}{\partial t} \tag{21}$$

$$\vec{J} \frac{\partial \vec{v}}{\partial t} = C_1 \nabla(\nabla \cdot \vec{v}) - C_0 \nabla \times (\nabla \times \vec{v}) + K\left(\frac{1}{\in}(\nabla \times \vec{q}) - 2\vec{v} \right) \tag{22}$$

$$L P_r E_r \frac{\partial \theta}{\partial t} = L[\nabla^2 \theta - \bar{\delta}(\nabla \times \vec{v})_z + (\vec{q})_z] + h_r (\vec{q})_z \tag{23}$$

$$E P_r \frac{\partial \theta}{\partial t} = \nabla^2 \theta + W \tag{24}$$

$$\in P_r \frac{\partial \vec{h}}{\partial t} = \frac{\partial \vec{q}}{\partial z} + \frac{\in P_r}{P_m} \nabla^2 \vec{h} \tag{25}$$

where, $R = \frac{\rho_0 g \alpha \beta d^4}{\mu k_T}$ is the thermal Rayleigh number, $P_r = \frac{\mu}{\rho_0 k_T}$ is the Prandtl number, $P_m = \frac{\mu}{\rho_0 \eta}$ is the magnetic Prandtl number, $Q = \frac{H_z^2 d^2}{4\pi \mu k_T}$ is the Chandrasekhar number, $\bar{J} = \frac{J}{d^2},\ K_1 = \frac{k_1}{d^2},\ \bar{\delta} = \frac{\delta'}{\rho_0 C_v d^2},\ C_0 = \frac{\gamma'}{\mu d^2},\ C_1 = \frac{\alpha' + \beta' + \gamma'}{\mu d^2},$ $F = \frac{m N_0}{\rho_0},\ E_r = E + h_T \epsilon,\ W = \vec{q} \cdot \hat{e}_z.$

The boundary condition is $W = \frac{d^2 W}{dz^2} = 0,\ \theta = 0,$ at $z = 0$ and $z = 1$. (26)

4. Dispersion Relation

Taking curl on both sides of equation (21) then we have

$$\left[\left\{ \frac{1}{\in} \frac{\partial}{\partial t} + \frac{1}{K_1}(1+K) \right\} L + \frac{F}{\in} \frac{\partial}{\partial t} \right] (\nabla \times \vec{q}) = L\left[R\left(\frac{\partial \theta}{\partial y} \hat{e}_x + \frac{\partial \theta}{\partial x} \hat{e}_y \right) + K\nabla + (\nabla \times \vec{v}) \right.$$

$$+\frac{2}{\in}\nabla\times\left(\vec{q}\times\Omega\right)+q\frac{\partial}{\partial z}\left(\nabla\times\vec{h}\right)\Bigg] \tag{27}$$

Let $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}, \nabla_1^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}, D=\frac{\partial}{\partial z}, \zeta_z=\left(\nabla\times\vec{q}\right)_z, \quad \Omega'_z=\left(\nabla\times\vec{v}\right)_z, \quad m_z=\left(\nabla\times\vec{h}\right)\hat{e}_z$

and $\vec{h}_z=\vec{h}\cdot\hat{e}_z$ is the z-component of vorticity. Again, applying curl on both sides of equation (27) and taking z-component on both side then we have

$$\left[\left\{\frac{1}{\in}\frac{\partial}{\partial t}+\frac{1}{K_1}(1+K)\right\}L+\frac{F}{\in}\frac{\partial}{\partial t}\right]\nabla^2 W=L\left[R\nabla_1^2\theta+K\nabla^2\Omega'_z\hat{e}_z-\frac{2}{\in}\Omega\left(D\zeta_z\right)+QD\left(\nabla^2\vec{h}_z\right)\right] \tag{28}$$

Now, taking z-component on both sides of equation (27)

$$\left[\left\{\frac{1}{\in}\frac{\partial}{\partial t}+\frac{1}{K_1}(1+K)\right\}L+\frac{F}{\in}\frac{\partial}{\partial t}\right]\zeta_z=L\left[\frac{2\Omega}{\epsilon}DW+QDm_z\right] \tag{29}$$

Taking curl on both side of equation (22) and taking z-component on the both side then we have

$$\bar{J}\frac{\partial\Omega_{z'}}{\partial t}=C_0\nabla^2\Omega'_z-K\left(\frac{1}{\in}\nabla^2 W+2\Omega'_z\right) \tag{30}$$

where K and C_0 account for coupling between vorticity and spin diffusion, spin effect respectively.

Taking z-component on both sides of equation (23) then we have

$$LP_rE_r\frac{\partial\theta}{\partial t}=L[\nabla^2\theta-\bar{\delta}\Omega'_z+W]+h_TW \tag{31}$$

Taking curl and z-component in equation (25)

$$\varepsilon P_r\frac{\partial m_z}{\partial t}=D\zeta_z+\varepsilon\frac{P_r}{P_m}\nabla^2 m_z \tag{32}$$

Taking z-component in equation (25)

$$\varepsilon P_r\frac{\partial h_z}{\partial t}=DW+\varepsilon\frac{P_r}{P_m}\nabla^2 h_z \tag{33}$$

Boundary condition (26) becomes

$$W=D^2W=0=\zeta_z=D\zeta_z=\Omega'_z=\theta \text{ at } z=0 \text{ and } z=1. \tag{34}$$

5. Normal Mode Analysis

Let $[W, \zeta_z, \theta, \Omega'_z, h_z, m_z] = [W(z), X(z), \Theta(z), G(z), B(z), M(z)] \ \exp[ik_x x + ik_y y + \sigma t]$.

Applying above normal mode analysis to equations (28) to (33), we have

$$\left[\left\{\frac{\sigma}{\epsilon} + \frac{1}{K_1}(1 + K)\right\}(1 + \tau\sigma) + \frac{F}{\epsilon}\sigma\right](D^2 - a^2)W$$

$$= (1 + \tau\sigma)\left[- Ra^2\Theta + K(D^2 - a^2)G - \frac{2}{\epsilon}\Omega DX + QD(D^2 - a^2)B\right] \quad (35)$$

$$\left[\left\{\frac{\sigma}{\epsilon} + \frac{1}{K_2}(1 + K)\right\}(1 + \tau\sigma) + \frac{F}{\epsilon}\sigma\right]X = (1 + \tau\sigma)\left[\frac{2}{\epsilon}\Omega DW + QDM\right] \quad (36)$$

$$[m\sigma + 2A - (D^2 - a^2)]G = -\frac{A}{\epsilon}(D^2 - a^2)W \quad (37)$$

$$[\{P_r E_r \sigma - (D^2 - a^2)\}(1 + \tau\sigma)]\Theta = (1 + \tau\sigma)[W - \bar{\delta}G] + h_T W \quad (38)$$

$$\left[\varepsilon P_r \sigma - \varepsilon\frac{P_r}{P_m}(D^2 - a^2)\right]M = DX \quad (39)$$

$$\left[\varepsilon P_r \sigma - \varepsilon\frac{P_r}{P_m}(D^2 - a^2)\right]B = DW \quad (40)$$

where, $a^2 = k_x^2 + k_y^2$ is the wave number, $\sigma = \sigma_r + i\sigma_r$ is the stability parameter and $m = \frac{\bar{J}A}{K}$, $A = \frac{K}{C_0}$, A is the ratio between the micropolar viscous effect and micropolar diffusion effects.

Now, the boundary condition becomes

$$W = D^2W = 0 = X = DX = G = M = DM,\ \Theta = 0 \text{ at } z = 0 \text{ and } z = 1. \quad (41)$$

$D^{2n}W = 0$ at $z = 0$ and $z = 1$, where n is positive integer.

Thus, the proper solution satisfying (41) can be taken as

$$W = W_0 \sin \pi z, \text{ where } W_0 \text{ is a constant.} \quad (42)$$

Eliminating Θ, G, M, B and X from (35) to (40) and put the value of W and

$b = \pi^2 + a^2$, then we have

$$b\left[\frac{\left[\left\{\frac{\sigma}{\epsilon}+\frac{1}{K_1}(1+K)\right\}(1+\tau\sigma)+\frac{F}{\epsilon}\sigma\right][P_rE_r\sigma+b][m\sigma+2A+b]\left[P_r\sigma+\frac{P_r}{P_m}b\right]}{\left[\left(\left\{\frac{\sigma}{\epsilon}+\frac{1}{K_1}(1+K)\right\}(1+\tau\sigma)+\frac{F}{\epsilon}\sigma\right)\left[P_r\sigma+\frac{P_r}{P_m}b\right]+(1+\tau\sigma)Q\frac{\pi^2}{\epsilon}\right]}\right]$$

$$= Ra^2\left[\frac{\left[\left(\left\{\frac{\sigma}{\epsilon}+\frac{1}{K_1}(1+K)\right\}(1+\tau\sigma)+\frac{F}{\epsilon}\sigma\right)\left[P_r\sigma+\frac{P_r}{P_m}b\right]+(1+\tau\sigma)Q\frac{\pi^2}{\epsilon}\right]}{(1+\tau\sigma)\left[\{m\sigma+2A+b\}-\frac{\bar{\delta}A}{\epsilon}b+h_T\{m\sigma+2A+b\}\right]\left[P_r\sigma+\frac{P_r}{P_m}b\right]}\right]$$

$$-\frac{4\Omega^2\pi^2}{\epsilon^2}(1+\tau\sigma)\left[[m\sigma+2A+b][P_rE_r\sigma+b]\left[P_r\sigma+\frac{P_r}{P_m}b\right]^2\right]$$

$$+\frac{KA}{\varepsilon}b^2(1+\tau\sigma)\left[\frac{[P_rE_r\sigma+b]\left[P_r\sigma+\frac{P_r}{P_m}b\right]}{\left[\left(\left\{\frac{\sigma}{\epsilon}+\frac{1}{K_1}(1+K)\right\}(1+\tau\sigma)+\frac{F}{\epsilon}\sigma\right)\left[P_r\sigma+\frac{P_r}{P_m}b\right]+(1+\tau\sigma)Q\frac{\pi^2}{\epsilon}\right]}\right]$$

$$-\frac{Qb\pi^2(1+\tau\sigma)}{\epsilon}\left[\frac{[m\sigma+2A+b][P_rE_r\sigma+b]}{\left[\left(\left\{\frac{\sigma}{\epsilon}+\frac{1}{K_1}(1+K)\right\}(1+\tau\sigma)+\frac{F}{\epsilon}\sigma\right)\left[P_r\sigma+\frac{P_r}{P_m}b\right]+(1+\tau\sigma)Q\frac{\pi^2}{\epsilon}\right]}\right]. \tag{43}$$

6. Stationary Convection

Put the $\sigma = 0$ in equation (43), then we get

$$R = \frac{1}{a^2}\left[\frac{b^2(2A+b)\left[\frac{1}{K_1}(1+K)\right]-\frac{KA}{\varepsilon}b^3+Q\frac{\pi^2}{\epsilon}b\frac{P_m}{P_r}(2A+b)}{\left[(2A+b)(1+h_T)-\frac{\bar{\delta}Ab}{\epsilon}\right]} + \frac{\frac{4\Omega^2\pi^2b^2}{\epsilon^2}\frac{P_r}{P_m}(2A+b)}{\left[\left\{\frac{1}{K_1}(1+K)\right\}\frac{P_r}{P_m}b+Q\frac{\pi^2}{\epsilon}\right]\left[(2A+b)(1+h_T)-\frac{\bar{\delta}Ab}{\epsilon}\right]}\right] \tag{44}$$

To investigate the behavior of medium permeability, rotation, suspended particle, magnetic field, coupling parameter, micropolar coefficient and micropolar heat conduction parameter, we find the nature of $\frac{dR}{dK_1}, \frac{dR}{d\Omega}, \frac{dR}{dh_T}, \frac{dR}{dQ}, \frac{dR}{dK}, \frac{dR}{dA}$ and $\frac{dR}{d\bar{\delta}}$ respectively, then from equations (44)

$$\frac{dR}{dK_1} = \frac{-b^2(2A+b)\left(\frac{1+K}{K_1^2}\right)}{a^2\left[(2A+b)(1+h_r)-\frac{\delta Ab}{\varepsilon}\right]}\left[1-\frac{\frac{4\Omega^2\pi^2 b}{\varepsilon^2}\left(\frac{P_r}{P_m}\right)^2}{\left[\left(\frac{1+K}{K_1}\right)\frac{P_r}{P_m}b+Q\frac{\pi^2}{\varepsilon}\right]^2}\right] \tag{45}$$

$$\frac{dR}{dK_1} < 0 \text{ if } Q < \frac{2\Omega\sqrt{b}}{\pi}\frac{P_r}{P_m} \text{ and } \bar{\delta} < \frac{\epsilon h_T}{A}.$$

From equation (45), we can say that the medium permeability has destabilizing effect when $Q < \frac{2\Omega\sqrt{b}}{\pi}\frac{P_r}{P_m}$.

$$\frac{dR}{d\Omega} = \frac{\frac{8\Omega\pi^2 b^2}{\epsilon^2}\frac{P_r}{P_m}(2A+b)}{a^2\left[\left\{\frac{1}{K_1}(1+K)\right\}\frac{P_r}{P_m}b+Q\frac{\pi^2}{\epsilon}\right]\left[(2A+b)(1+h_T)-\frac{\bar{\delta}Ab}{\epsilon}\right]} \tag{46}$$

$$\frac{dR}{d\Omega} > 0 \text{ if } \bar{\delta} < \frac{\epsilon h_T}{A}.$$

From equation (46), we can say that the rotation has stabilizing effect when the $\bar{\delta} < \frac{\in h_T}{A}$.

$$\frac{dR}{dh_r} = \frac{-(2A+b)}{a^2\left[(2A+b)(1+h_T)-\frac{\bar{\delta}Ab}{\epsilon}\right]^2}\cdot$$

$$\left[\begin{array}{l}\left\{b^2(2A+b)\left[\frac{1}{K_1}(1+K)\right]-\frac{KA}{\varepsilon}b^3+Q\frac{\pi^2}{\varepsilon}b\frac{P_m}{P_r}(2A+b)\right\} \\ +\left\{\frac{\frac{4\Omega^2\pi^2 b^2}{\epsilon^2}\frac{P_r}{P_m}(2A+b)}{\left[\left\{\frac{1}{K_1}(1+K)\right\}\frac{P_r}{P_m}b+Q\frac{\pi^2}{\epsilon}\right]}\right\}\end{array}\right] \tag{47}$$

$$\frac{dR}{dh_T} < 0 \text{ if } \frac{1}{K_1} > \frac{A}{\epsilon} \text{ and } \bar{\delta} < \frac{\epsilon h_T}{A}.$$

From equation (47), we can say that the suspended particles have destabilizing effect when $\frac{1}{K_1} > \frac{A}{\in}$.

$$\frac{dR}{dQ} = \frac{(2A+b)\pi^2 b \frac{P_m}{P_r}}{a^2\epsilon\left[(2A+b)(1+h_T) - \frac{\bar{\delta}Ab}{\epsilon}\right]}\left[1 - \frac{\frac{4\Omega^2\pi^2 b}{\epsilon^2}\left(\frac{P_r}{P_m}\right)^2}{\left[\left\{\frac{1}{K_1}(1+K)\right\}\frac{P_r}{P_m}b + Q\frac{\pi^2}{\epsilon}\right]^2}\right] \quad (48)$$

$$\frac{dR}{dQ} > 0 \text{ if } Q < \frac{2\Omega\sqrt{b}}{\pi}\frac{P_r}{P_m} \text{ and } \bar{\delta} < \frac{\epsilon h_T}{A}.$$

From equation (48), we can say that the magnetic field has stabilizing effect when $Q < \frac{2\Omega\sqrt{b}}{\pi}\frac{P_r}{P_m}$.

$$\frac{dR}{dK} = \frac{1}{a^2\left[(2A+b)(1+h_T) - \frac{\bar{\delta}Ab}{\in}\right]}\left[Ab^2\left(\frac{2}{K_1} - \frac{b}{\in}\right) + \frac{b^3}{K_1}\left\{1 - \frac{\frac{4\Omega^2\pi^2 b}{\in^2}\left(\frac{P_r}{P_m}\right)^2(2A+b)}{\left[\left\{\frac{1}{K_1}(1+K)\right\}\frac{P_r}{P_m}b + Q\frac{\pi^2}{\in}\right]^2}\right\}\right] \quad (49)$$

$\frac{dR}{dK} > 0$ if $\frac{1}{K_1} > \frac{b}{\in}, \bar{\delta} < \frac{\in h_T}{A}$ and $K > \frac{K_1\pi}{b\in}\left[2\Omega\sqrt{(2A+b)} - Q\pi\frac{P_m}{P_r}\right] - 1$ or and $K > \frac{K_1\pi}{b\in}\left(2\Omega\sqrt{(2A+b)}\right)$.

From equation (49), we can say that the coupling parameter has a stabilizing effect when $K > \frac{K_1\pi}{b\in}\left(2\Omega\sqrt{(2A+b)}\right)$, if $\frac{1}{K_1} > \frac{b}{\epsilon}$ and $\bar{\delta} < \frac{\epsilon h_T}{A}$.

$$\frac{dR}{dA} = \frac{1}{a^2\left[(2A+b)(1+h_T) - \frac{\bar{\delta}Ab}{\in}\right]^2}\left[\begin{array}{l} b^4\left(\frac{1+K}{K_1}\right)\left\{\bar{\delta} - \left(\frac{K}{1+K}\right)K_1(1+h_T)\right\} + Q\frac{\pi^2 b^3\bar{\delta}}{\in^2}\frac{P_m}{P_r} \\ + \left\{\frac{\frac{4\Omega^2\pi^2 b^4\bar{\delta}}{\in^2}\frac{P_r}{P_m}}{\left[\left[\left\{\frac{1}{K_1}(1+K)\right\}\frac{P_r}{P_m}b + Q\frac{\pi^2}{\in}\right]\right]}\right\} \end{array}\right] \quad (50)$$

$$\frac{dR}{dA} > 0 \text{ if } \bar{\delta} > (1 + h_T)K_1.$$

From equation (50), we can say that the micropolar coefficient has a stabilizing effect when $\bar{\delta} > (1 + h_T)K_1$.

$$\frac{dR}{d\bar{\delta}} = \frac{1}{a^2\left[(2A+b)(1+h_T) - \frac{\bar{\delta}Ab}{\epsilon}\right]^2}$$

$$\left[\begin{array}{c} \frac{Ab}{\epsilon}\left[b^3\left\{\frac{1}{K_1}(1+K) - \frac{KA}{\epsilon}\right\} + 2Ab^2\frac{1}{K_1}(1+K) + \frac{Q\pi^2 b}{\epsilon}\frac{P_m}{P_r}(2A+b)\right] \\ + \frac{\frac{4\Omega^2\pi b^3 A}{\epsilon^3}\frac{P_r}{P_m}(2A+b)}{\left[\left\{\frac{1}{K_1}(1+K)\right\}\frac{P_r}{P_m}b + Q\frac{\pi^2}{\epsilon}\right]} \end{array}\right] \quad (51)$$

$$\frac{dR}{d\bar{\delta}} > 0 \text{ if } \frac{1}{K_1} > \frac{A}{\epsilon}.$$

From equation (51), we can say that the micropolar heat conduction parameter has a stabilizing effect when $\frac{1}{K_1} > \frac{A}{\in}$.

7. Oscillatory Convection

Putting $\sigma = i\sigma_i$ in equation (43) then we get real and imaginary part and eliminating R between them, then we have

$$f_0\sigma_i^{12} + f_1\sigma_i^{10} + f_2\sigma_i^8 + f_3\sigma_i^6 + f_4\sigma_i^4 + f_5\sigma_i^2 + f_5 = 0$$

Put $s = \sigma_i^2$, $\quad f_0 s^6 + f_1 s^5 + f_2 s^3 + f_3 s^4 + f_4 s^2 + f_5 s + f_6 = 0 \quad (52)$

where $f_0 = a_1q_1 - P_1b_1 > 0$, $f_1 = a_2q_1 + a_1q_2 - P_2b_1 - P_1b_2 > 0$, $f_2 = a_3q_1 + a_2q_2 + a_1q_3 - P_3b_1 - P_2b_2 - P_1b_3$
$f_3 = a_4q_1 + a_3q_2 + a_2q_3 - P_4b_1 - P_3b_2 - P_2b_3 - P_1b_4$, $f_4 = a_5q_1 + a_4q_2 + a_3q_3 - P_4b_2 - P_3b_3 - P_2b_4$
$f_5 = a_5q_2 + a_4q_3 - P_4b_3 - P_3b_4$, $f_6 = a_5q_3 - P_4b_4$

$$a_1 = b\left(\frac{\tau}{\epsilon^2}E_rP_r^2m\right),\ b_1 = -a^2\tau m h_T$$

$$a_2 = -b\left[\begin{array}{l}\left[\frac{\tau}{\epsilon}P_r\left\{(E_rP_rC_2 + bC_1)m + \left(E_rP_rC_1 + \frac{\tau b}{\epsilon}\right)\left(\frac{bm}{P_m} + (2A+b)\right)\right.\right. \\ \left.\left. + \frac{\tau E_rP_rb(2A+b)}{\epsilon P_m}\right\}\right] + \left\{\left(E_rP_rC_1 + \frac{\tau b}{\epsilon}\right)m + \left(\frac{\tau E_rP_r}{\epsilon}\right)\right. \\ \left.\left(\frac{bm}{P_m} + (2A+b)\right)\right\}\left(\frac{\tau bP_r}{P_m} + C_1P_r\right) + \frac{\tau E_rP_rm}{\epsilon}\left(\frac{P_rb}{P_m} + P_rC_2 + \frac{\tau Q\pi^2}{\epsilon}\right)\end{array}\right]$$

$$+\frac{KAb^2}{\epsilon}\frac{E_rP_r^2\tau^2}{\epsilon} - \frac{Qb\pi^2}{\epsilon}\left[\frac{E_rP_r^2m\tau^2}{\epsilon}\right]$$

$$b_2 = a^2\left[\begin{array}{l}\tau mh_T\left(\frac{P_rC_1b}{P_m} + P_rC_2 + \frac{\tau Q\pi^2}{\epsilon}\right) + \left[mh_T + \frac{\tau mh_Tb}{P_m} + \tau\right. \\ \left.\left\{m + (2A+b)(1+h_T) - \frac{\bar{\delta}Ab}{\epsilon}\right\}\right]\left(\frac{\tau bP_r}{P_m} + C_1P_r\right) + \frac{\tau P_r}{\epsilon}\left[\left(1 + \frac{\tau b}{P_m}\right)\right. \\ \left.\left\{m + (2A+b)(1+h_T) - \frac{\bar{\delta}Ab}{\epsilon}\right\} + \frac{mh_Tb}{P_m}\right]\end{array}\right]$$

$$P_2 = b\left[\begin{array}{l}\frac{\tau P_r}{\in}\left\{bC_2m + (E_rP_rC_2 + bC_1)\frac{bm}{P_m} + (E_rP_rC_2 + bC_1)(2A+b)\right. \\ \left. + \left(E_rP_rC_1 + \frac{\tau b}{\in}\right)\left(\frac{(2A+b)b}{P_m}\right)\right\} + \left\{(E_rP_rC_2 + bC_1)m + \left(E_rP_rC_1 + \frac{\tau b}{\in}\right)\right. \\ \left.\left(\frac{bm}{P_m} + (2A+b)\right) + \frac{\tau E_rP_rb(2A+b)}{\in P_m}\right\}\left(\frac{\tau bP_r}{P_m} + C_1P_r\right) \\ + \left\{\left(E_rP_rC_1 + \frac{\tau b}{\in}\right)m + \left(\frac{\tau E_rP_r}{\in}\right)\left(\frac{bm}{P_m} + (2A+b)\right)\right\}\left(\frac{P_rb}{P_m} + P_rC_2 + \frac{\tau Q\pi^2}{\in}\right) \\ + \frac{\tau E_rP_rm}{\in}\left(\frac{P_rC_2b}{P_m} + \frac{Q\pi^2}{\in}\right)\end{array}\right]$$

$$-\frac{KAb^2}{\epsilon}\left[E_rP_r\tau\left(\frac{\tau bP_r}{P_m} + C_1P_r\right) + \frac{\tau P_r}{\epsilon}\left\{E_rP_r + b\tau + \frac{E_rP_rb^2\tau}{P_m}\right\}\right] + \frac{4\Omega^2\pi^2}{\epsilon^2}E_rP_r^2\tau$$

$$+\frac{Qb\pi^2}{\epsilon}\left[E_rP_rm\tau\left(\frac{\tau bP_r}{P_m} + C_1P_r\right) + \frac{\tau b}{P_m}\{m\tau + E_rP_rm + (2A+b)E_rP_r\tau\}\right]$$

$$q_2 = -a^2 \left[\begin{array}{l} \tau m h_T \left(\dfrac{P_r C_2 b}{P_m} + \dfrac{Q\pi^2}{\in} \right) + \left[m h_T + \dfrac{\tau m h_T b}{P_m} \right. \\ \left. + \tau \left\{ m + (2A+b)(1+h_T) - \dfrac{\bar{\delta} A b}{\in} \right\} \right] \left(\dfrac{P_r C_1 b}{P_m} + P_r C_2 + \dfrac{\tau Q \pi^2}{\in} \right) \\ \left[\left(1 + \dfrac{\tau b}{P_m} \right) \left\{ m + (2A+b)(1+h_T) - \dfrac{\bar{\delta} A b}{\in} \right\} + \dfrac{m h_T b}{P_m} \right] \left(\dfrac{\tau b P_r}{P_m} + C_1 P_r \right) \\ + \dfrac{\tau P_r b}{\in P_m} \left\{ m + (2A+b)(1+h_T) - \dfrac{\bar{\delta} A b}{\in} \right\} \end{array} \right]$$

$$P_1 = -b \left[\frac{\tau P_r}{\in} \left\{ \left(E_r P_r C_1 + \frac{\tau b}{\in} \right) m + \left(\frac{\tau E_r P_r}{\in} \right) \left(\frac{bm}{P_m} + (2A+b) \right) \right\} + \frac{\tau E_r P_r m}{\in} \left(\frac{\tau b P_r}{P_m} + C_1 P_r \right) \right]$$

$$q_1 = a^2 \left[\tau m h_T \left(\frac{\tau b P_r}{P_m} + C_1 P_r \right) + \frac{\tau P_r}{\in} \left[m h_T + \frac{\tau m h_T b}{P_m} + \tau \left\{ m + (2A+b)(1+h_T) - \frac{\bar{\delta} A b}{\in} \right\} \right] \right]$$

From (52), we observed that $s = \sigma_i^2$ which is always positive, therefore the sum of roots of equation (52) is positive but this is impossible if $f_0 > 0$ and $f_1 > 0$, the sum of roots of equation (52) is $\frac{-(f_0)}{(f_1)}$. Thus, $f_0 > 0$ and $f_1 > 0$ are the sufficient condition for the non-existence of over stability.

Now $f_0 > 0$ and $f_1 > 0$ when $C_v \left(1 - \frac{\bar{\delta} A}{\in} \right) > C_{pt}, \frac{h_T K_1}{\in} < K E_r K_1 < 2, 0 < \frac{\bar{\delta} A}{\in} < 1$ and $4\Omega^2 \pi < (\pi^2 + a^2) \ Q < 8\Omega^2 \pi.$

8. Numerical Calculation

Now we show numerically the effect of medium permeability, rotation, suspended particles, magnetic field, coupling parameter, micropolar coefficient and micropolar heat conduction coefficient.

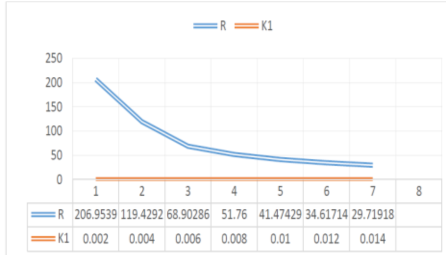


Figure 1.

where, $A = 0.1$, $a = 1$, $\Omega = 10$, $K = 0.2$, $Q = 1$, $P_r = 2$, $P_m = 4$, $\bar{\delta} = 0.5$, $h_T = 2$, $\tau = 1$, $\delta = 1$, $\in = 0.5$ and K_1 = 0.002, 0.004, 0.006, 0.008, 0.01, 0.012, 0.14.

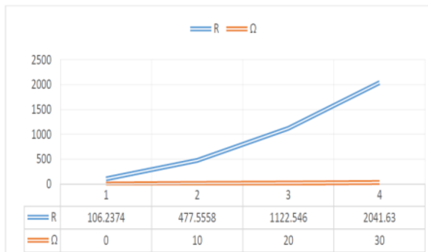


Figure 2.

where, $A = 0.1$, $a = 1$, $K = 0.2$, $Q = 1$, $P_r = 2$, $P_m = 4$, $\bar{\delta} = 0.5$, $h_T = 2$, $\tau = 1$, $\bar{\delta} = 0.5$, $\in = 0.5$ and Ω = 0, 10, 30.

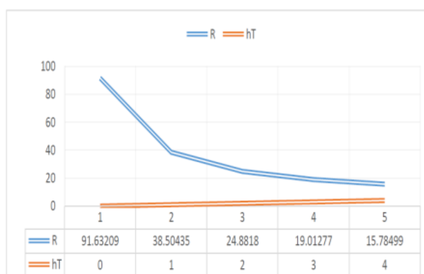


Figure 3.

where, $A = 0.1$, $\Omega = 10$ $a = 1$, $K = 0.2$, $Q = 1$, $P_r = 2$, $P_m = 4$, $\bar{\delta} = 0.5$, $\tau = 1$, $\delta = 1$, $\in = 0.5$ and h_T = 0, 1, 2, 3, 4.

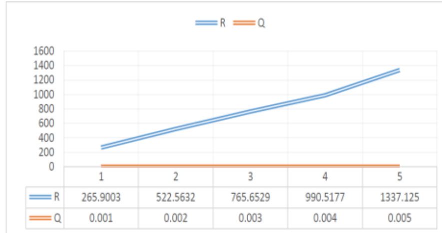


Figure 4.

where, $A = 0.1$, $a = 1$, $K = 0.2$, $\Omega = 10$ $P_r = 2$, $P_m = 4$, $\bar{\delta} = 0.5$, $h_T = 2$, $\tau = 1$, $\bar{\delta} = 0.5$, $\in = 0.5$ and Q = 0.001, 0.002, 0.003, 0.004, 0.005.

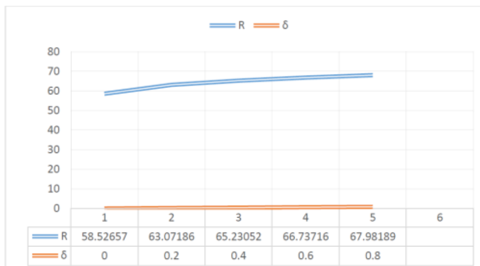


Figure 5.

where, $A = 0.1$, $a = 1$, $K = 0.2$, $\Omega = 10$ $Q = 1$, $P_r = 2$, $P_m = 4$, $\bar{\delta} = 0.5$, $h_T = 2$, $\tau = 1$, $\in = 0.5$ and $\bar{\delta}$ = 0, 0.02, 0.04, 0.06, 0.08.

9. Conclusions

According to sign of the derivatives and graphical representation we found that the effect of medium permeability and suspended particles are destabilizing. The effect of rotation, magnetic field, coupling parameter, micropolar coefficient and micropolar heat conduction coefficient are stabilizing. Among them the most important result that the effect of suspended particles destabilize on the system.

The sufficient condition for the non-existence of over stability is given by $C_v\left(1 - \frac{\bar{\delta} A}{\in}\right) > C_{pt}$, $\frac{h_T K_1}{\in} < K E_r K_1 < 2$, $0 < \frac{\bar{\delta} A}{\in} < 1$ and $4\Omega^2 \pi < (\pi^2 + a^2)$ $Q < 8\Omega^2 \pi$.

References

[1] Ramesh Chand, Dhananjay Yadav, K. Bhattacharyya and Mukesh Kumar Awasthi, Thermal convection in a layer of micropolar nanofluid, Asia-Pacific Journal of Chemical Engineering 16(5) (2021), e2681.

[2] R. K. Chandrawat, V. Joshi and S. Kanchan, Numerical simulation of interface tracking between two immiscible micropolar and dusty fluids, Mater. Today Proc., 2021, doi: 10.1016/j.matpr.2021.08.069.

[3] A. C. Eringen, Simple microfluids, International Journal of Engineering Science 2(2) (1964), 205-217.

[4] A. C. Eringen, Theory of micropolar fluid, International Journal of Engineering Science 16(1) (1966), 1-18.

[5] Rovin Kumar and Vijay Mehta, Hall Effect on MHD flow of visco-elastic fluid layer heated for below saturating a porous medium, International Journal of Statistics and Applied Mathematics 2(6) (2017), 250-260.

[6] G. Lebon and C. Perez-Garcia, Convective instability of a micro fluid layer by the method of energy, International Journal of Engineering Science 19 (1981), 1321-1329.

[7] Grzegorz Lukaszewicz, Micropolar fluid-Theory and applications. Birkha Boston; 1999.

[8] Reena Mittal and U. S. Rana, Effect of dust particles on a layer of micropolar ferromagnetic fluid heated from below saturating a porous medium, Applied Mathematics and Computation 215 (2009), 2591-2607.

[9] Abdallah Miqdady and Ruwaidiah Idris, Effect of controller and cubic temperature gradient on Rayleigh-Benard Marangoni-Magnetoconvection in a micropolar fluid with maxwell-cattaneo law, Review of International Geographical Education 11(4) (2021), 825-841.

[10] I. B. Olajuwan, I. J. Oahimire and A. M. Waheed, Convection heat and mass transfer in a hydromagnetic flow of a micropolar fluid over a porous medium, Theoretical and Applied Mechanics 41(2) (2014), 93-117.

[11] Patel R. Harshad, Effect of cross diffusion and heat generation on mixed convective MHD flow of Casson fluid through porous medium with non-linear thermal radiation, Elsevier 5(4) (2019), 01-26.

[12] B. K. Patra, S. Biswal and A. P. Mishra, Heat and mass transfer effect on the marginal stability of rotating dusty ferro fluid flowing through a porous medium, Ultra Scientist, 24(3) (2012), 469-482.

[13] C. Perez-Garcia, J. M. Rubi and J. Casas-Vazqez, On the stability of micropolar fluids, Journal of Non-Equilibriaum Thermodynamics 6 (1981), 65-78.

[14] Y. Qin and P. N. Kaloni, The thermal instability problem in a rotating micropolar fluid, International Journal of Engineering Science 30(9) (1992), 1117-1126.

[15] Reena and U. S. Rana, Thermosolutal convection of micropolar rotating fluids saturating a porous medium, International Journal of Engineering 22 (2009), 379-404.

[16] K. B. Sharma and Umesh Khanduri, Entropy analysis for MHD flow subject to temperature dependent viscosity and thermal conductivity, Nonlinear Dynamics and Applications (2022), 457-471.

[17] Bhupendra Kumar Sharma, Vikas Tailor and Mamta Goyal, Heat source and soret effect on magneto-micropolar fluid flow with variable permeability and chemical reaction, Globle Journal of Pure and Applied Mathematics 13 (2017), 5195-5212.

[18] Pushap Lata Sharma and Sumit Gupta, Effect of rotation and suspended particles on micropolar fluid heated and soluted from below saturating porous medium, International Journal of Mathematics Trends and Technology 65(9) (2019) 72-83.

[19] Pushap Lata Sharma and Sumit Gupta, Numerical investigations on rotatory micropolar fluid in hydromagnetics permited with suspended particles porous medium, Research Journal of Science and Technology 13(2) (2021), 57-69.

[20] Bhupander Singh, Effect of rotation on a layer of micro-polar ferromagnetic dusty fluid heated from below saturating a porous medium, International Journal of Engineering Research and Application 6(3) (2016), 04-28.

[21] R. A. Wooding, Rayleigh instability of a thermal boundary layer in flow through a porous medium, Journal of Fluid Mechanics 9(2) (1960), 183-192.