Advances and Applications in Mathematical Sciences
Volume 22, Issue 9, July 2023, Pages 2045-2056

ROLE OF SUSPENDED PARTICLES IN COOLING A STRETCHING FILM AT A DESIRED RATE

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Abstract

The main goal of this research is to look into velocity and heat transfer in an electrically conducting Newtonian fluid flowing over a quadratically extending sheet with Navier slip affecting velocity at the boundary. The flow events are described by a nonlinear partial differential equation which are translated to an ordinary differential equation using well established similarity transformations. We have adopted differential transformation method with Pade approximations to get numerical convergent series solution. The graphical and tabular results for velocity and heat transfer with slip effects are displayed for different dimensionless parameters for a range of values.

Introduction

Extrusion, melt-spinning, hot rolling, wire drawing, glass-fiber production, plastic and rubber sheet manufacturing and cooling of a huge metallic plate are all examples of engineering processes where flow across a stretched surface is a critical issue. Thin-film production is a difficult procedure that requires cooling a stretched film at a specific rate to offer

2020 Mathematics Subject Classification: 35Q79.
Keywords: stretching flow, Newtonian fluid, slip condition, numerical solution, Pade approximation.
Received May 5, 2022; Accepted September 1, 2022

unidirectional orientation to the finished product. Because the whole system is contained in cooling liquids, selecting a coolant and stretching approach that involves stretching thin films by equal and opposing pressures is critical. The thin film's properties are determined by the pace of cooling and the type of the coolant.

The concept of a boundary layer over a continuous solid surface moving at a same speed was initiated by Sakiadis [1]. Crane [2] assumed that the sheet's velocity varied linearly with axial distance. They focused on the flow field with the no-slip at the edge of the flow. In some cases, this boundary requirement should be substituted by the Navier slip boundary condition. Anderson [3] looked at the effects of a slip boundary condition on Newtonian fluid flow past a stretched sheet. Bidin et al. [4] investigated the flow over a stretching sheet with a convective boundary condition and slip effect. Nandeppanavar et al. [5] debriefed heat transfer over a stretching sheet with nonlinear navier second order slip flow boundary condition. In the presence of a transverse magnetic field and viscous dissipation, M.R. Krishnamurthy et al. [6] resolved the problem of steady, boundary layer flow and heat transfer of a nanofluid with fluid-particle suspension across an exponentially extending surface. The convective heat transfer properties of an incompressible viscous dusty fluid across an exponentially stretched surface with an exponential temperature distribution were investigated by Siti Nur HaseelaIzani et al. [7]. Najeeb Alam Khan et al. [8] explained the velocity and heat transfer at the edge of flow considering fourth-grade fluid over an exponentially stretching sheet and Fazle Mabood et al. T. Gangaiah et al. [9] described the effects of thermal radiation and heat source/sink parameters on the mixed convective MHD flow of a Casson nanofluid with zero normal flux of nanoparticles over an exponentially stretching sheet. Basant K. Jha and Dauda Gambo [10] investigated nature of flow wrt applied magnetic field on unsteady MHD Coutte flow of dusty fluid in an annulus.

Mathematical relationship of the investigation:

Taking into account a two-dimensional uninterrupted flow of an incompressible, electrically conducting Boussinesq-Stokes suspension fluid (couple stress fluid) across an exponentially extending sheet. Using two equal and opposing pressures along the x-axis, the boundary sheet is made to move axially with an exponential velocity. The y-axis is transversal to it. The guiding equations are:

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0,\ x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\vartheta\frac{\partial^2 u}{\partial x^2}-\frac{\mu_m^2\sigma H_0^2}{\rho}u-\vartheta'\frac{\partial^4 u}{\partial x^4},$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial u}{\partial y}=\frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^2$$

With $U_w(x)=U_0\exp\left(\frac{x}{l}\right)+\chi\gamma\frac{\partial u}{\partial y},\ \frac{\partial^2 u}{\partial x^2}=0,\ v=0$ at $y=0$

$T=T_w=T_\infty+(T_w-T_\infty)\exp\left(\frac{x}{l}\right)$ in $PEST$, $-\kappa\frac{\partial T}{\partial y}=T_1 e^{\frac{3x}{2l}}$ in $PEHF$ at $y=0$

With the concept of nondimensionalisation and stream function

$$(X,Y)=\left(\frac{x}{l},\frac{y}{l}\right),(U,V)=\left(\frac{u}{\sqrt{U_0\vartheta}},\frac{v}{\sqrt{U_0\vartheta}}\right),\ U=\frac{\partial\varphi}{\partial Y},\ V=-\frac{\partial\varphi}{\partial X}$$

$$C\frac{\partial^5\varphi}{\partial Y^5}-\frac{\partial^5\varphi}{\partial Y^3}+\frac{\partial\varphi}{\partial Y}\frac{\partial^2\varphi}{\partial X\partial Y}-\frac{\partial\varphi}{\partial X}\frac{\partial^2\varphi}{\partial Y^2}+Q\frac{\partial\varphi}{\partial Y}=0,$$

$$\frac{\partial\varphi}{\partial Y}\frac{\partial T}{\partial X}-\frac{\partial\varphi}{\partial X}\frac{\partial T}{\partial Y}=\frac{1}{\Pr}\frac{\partial^2 T}{\partial Y^2}+\frac{\mu c}{\rho C_p l^2}\left(\frac{\partial\varphi}{\partial Y}\right)^2$$

with $\frac{\partial\varphi}{\partial Y}=\sqrt{\frac{U_0}{\gamma}}e^X+\chi\gamma\frac{\partial^2\varphi}{\partial Y^2},\ \frac{\partial\varphi}{\partial X}=0,\ \frac{\partial^3\varphi}{\partial Y^3}=0$ at $y=0$

$T=T_w=T_\infty+(T_w-T_\infty)e^X$ in $PEST$, $-\kappa\frac{\partial T}{\partial y}=T_1 e^{\frac{3x}{2l}}$ in $PEHF$ at $y=0$

$$\frac{\partial\varphi}{\partial Y}\to 0,\ \frac{\partial^3\varphi}{\partial Y^3}\to 0,\ T\to T_\infty \text{ at } Y\to\infty.$$

Using following similarity transformation:

$\gamma(X,Y)=\sqrt{2\,\mathrm{Re}}f(\eta)e^{\frac{X}{2}}$ where $\eta=Y\sqrt{\frac{\mathrm{Re}}{2}}e^{\frac{X}{2}}$ is the similarity variable.

Using above list of equations, momentum equation becomes

$$Cf_{\eta\eta\eta\eta\eta} - f_{\eta\eta\eta} + 2ff_{\eta\eta} + 2Qf_{\eta} = 0 \text{ with}$$

$$f = 0,\ f_{\eta} - 1 = Kf_{\eta\eta},\ f_{\eta\eta\eta} = 0 \text{ at } \eta = 0$$

$$f_{\eta} \to 0,\ f_{\eta\eta} \to 0 \text{ as } \eta \to \infty \text{ where } K = \frac{\chi U_0 \sqrt{\mathrm{Re}}}{\sqrt{2}}$$

Temperature distribution analysis:

PEST: $\theta(\eta) = \dfrac{T - T_{\infty}}{T_w - T_{\infty}}$

$$\theta_{\eta\eta} + \Pr f\theta_{\eta} - 2\Pr f_{\eta}\theta + \Pr Ef^2{}_{\eta\eta} = 0 \text{ with}$$

$$\theta = 1 \text{ at } \eta = 0,\ \theta \to 0 \text{ as } \eta \to \infty$$

PEHF: $\phi(\eta) = \dfrac{T - T_{\infty}}{T_w - T_{\infty}}$ where $T - T_{\infty} = \dfrac{T_1}{k}\sqrt{\dfrac{2}{\mathrm{Re}}}e^{\frac{3X}{2}}\phi(\eta)$ and $T_w - T_{\infty} = \dfrac{T_1 l}{k}\sqrt{\dfrac{2}{\mathrm{Re}}}e^{\frac{3X}{2}}$.

Using the listed transformations, we get

$$\varphi_{\eta\eta} + \Pr f\varphi_{\eta} - 2\Pr f_{\eta} + \Pr Ef_{\eta\eta}^2 = 0 \text{ with}$$

$$\varphi_{\eta} = -1 \text{ at } \eta = 0,\ \varphi \to 0 \text{ as } \eta \to \infty.$$

Research Methodology:

The shooting technique is used to convert transformed nonlinear ordinary differential equations with defined boundary conditions into an initial value problem. The Runge-Kutta-Fehlberg 45 technique is then used to solve IVP.

Momentum equation:

$$\frac{dF_1}{d\eta} = F_2 \qquad F_1(0) = 0, \qquad \frac{dF_2}{d\eta} = F_3 \qquad F_2(0) = 1 + KF_3$$

$$\frac{dF_3}{d\eta} = F_4 \qquad F_3(0) = \alpha_4 \qquad \frac{dF_4}{d\eta} = F_5 \qquad F_4(0) = 0$$

$$\frac{dF_5}{d\eta} = \frac{1}{C}[F_4 - 2F_2^2 + F_1F_3 - 2QF_2] \qquad F_5(0) = \beta_4$$

PEST:

$$\frac{dF_6}{d\eta} = F_7 \qquad F_6(0) = 1,$$

$$\frac{dF_7}{d\eta} = -\Pr F_1F_7 + 2\Pr F_2F_6 - \Pr EF_3^2 \quad F_7(0) = \gamma_4$$

PEHF:

$$\frac{dF_8}{d\eta} = F_9 \qquad F_8(0) = \lambda_4$$

$$\frac{dF_7}{d\eta} = -\Pr F_1F_7 + 2\Pr F_2F_6 - \Pr E_sF_3^2 \quad F_9(0) = -1$$

Momentum Equation:

$$C(k+1)(k+2)(k+3)(k+4)(k+5)F[k+5] - (k+1)(k+2)(k+3)F[k+3]$$

$$-2\sum_{r=0}^{k}(r+1)F[r+1](k+1-r)F[k+1-r]$$

$$-\sum_{r=0}^{k}(r+1)(r+2)F[r+2]F[k-r] + 2Q(k+1)F[k+1] = 0$$

$$F[0] = 0,\ F[1] = 1 + k\alpha_4,\ F[2] = \frac{\alpha_4}{2},\ F[3] = 0,\ F[4] = \frac{\beta_4}{24}$$

PEST:

$$(k+1)(k+2)G[k+2]$$

$$+\Pr\sum_{r=0}^{k}(r+1)G[r+1]F[k-r]$$

$$-2\sum_{r=0}^{k}(r+1)F[r+1]G[k-r]$$

$$+ E \Pr \sum_{r=0}^{k} (r+1)(r+2)F[r+2](k-r+1)(k-r+2)$$

$G[0] = 1,\ G[1] = \gamma_4$

PEHF:

$$(k+1)(k+2)H[k+2]$$

$$+ \Pr \sum_{r=0}^{k} (r+1)H[r+1]F[k-r]$$

$$- 2\sum_{r=0}^{k} (r+1)F[r+1]H[k-r]$$

$$+ E \Pr \sum_{r=0}^{k} (r+1)(r+2)F[r+2](k-r+1)(k-r+2)$$

$H[0] = \lambda_4,\ H[1] = -1$

Then, using the inverse differential transform, we arrive at

$$f(\eta) = F[0] + F[1]\eta + F[2]\eta^2 + F[3]\eta^3 + \ldots$$

$$f'(\eta) = F[1] + 2F[2]\eta + 3F[3]\eta^2 + \ldots$$

$$g(\eta) = g[0] + g[1]\eta + g[2]\eta^2 + g[3]\eta^3 + \ldots$$

$$H(\eta) = H[0] + H[1]\eta + H[2]\eta^2 + H[3]\eta^3 + \ldots$$

In MATHEMATICA 7.0, with the help of command "PadeApproximant", we can obtain Pade approximation of $f(x)$ around the point $x = 0$.

Findings and Analysis

1. Increasing the Chandrasekhar number (Q) causes the flow to be resisted, resulting in a drop in the momentum boundary layer.

2. Increasing the value of the couple stress (C), widens the momentum boundary layer and reduces the thermal boundary layer.

3. As the Prandtl number (Pr) is raised, the thermal boundary layer thickness reduces.

4. The PEHF boundary condition is appropriate for the adequate cooling of the stretching sheet.

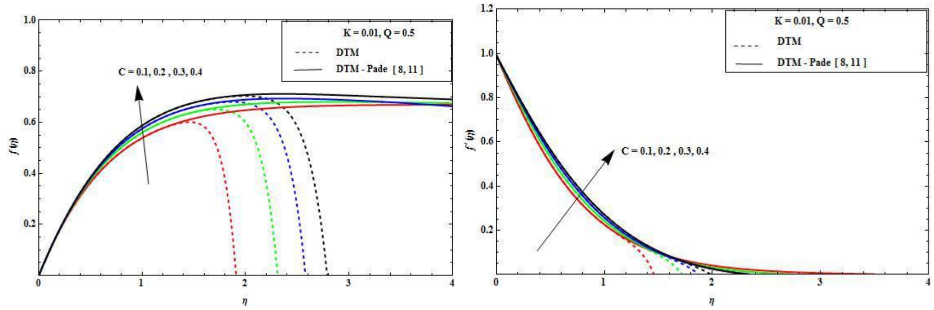

Figure 1.1. Illustration of $f(\eta)$ verses η for range of C values.

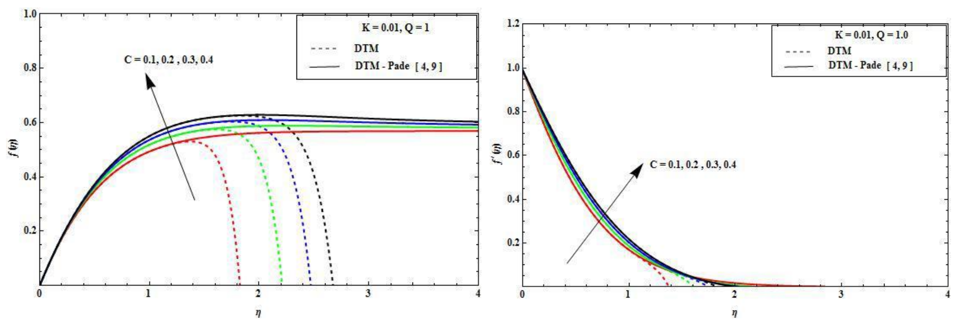

Figure 1.2. Illustration of $f(\eta)$ verses η for range C values.

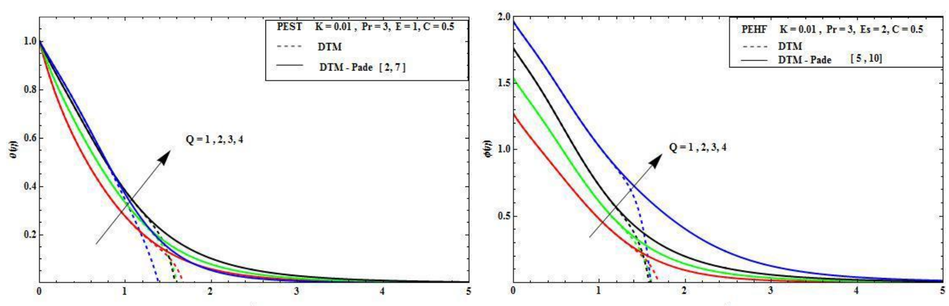

Figure 1.3. Illustration of heat transfer for range of Q values.

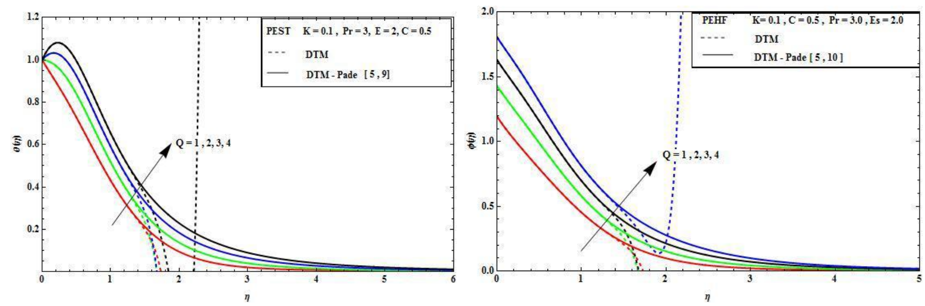

Figure 1.4. Illustration of temperature profiles for range of Q values.

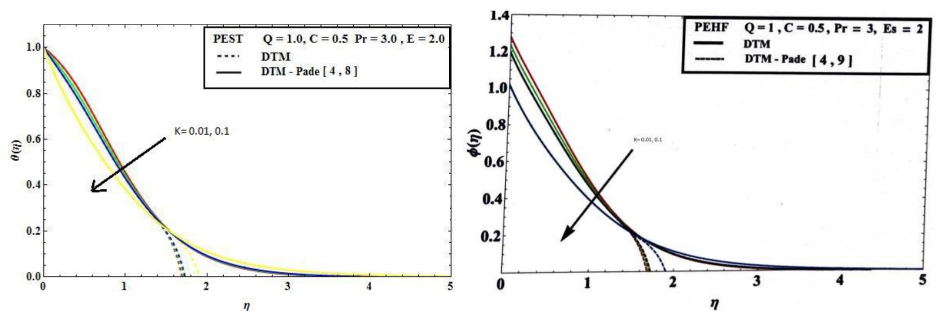

Figure 1.5. Illustration of heat transfer profiles for range of slip parameter K values.

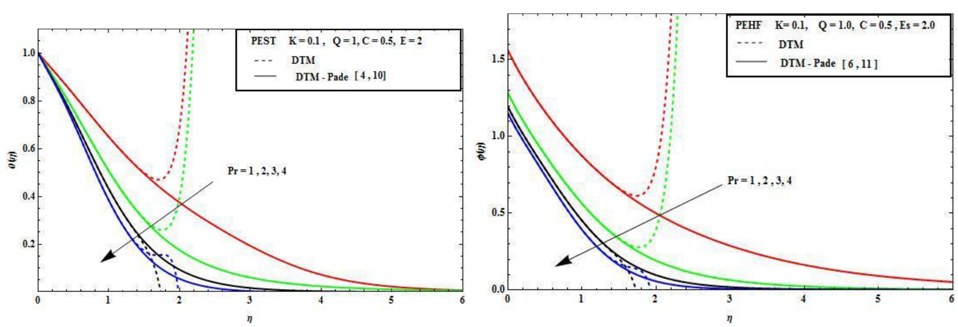

Figure 1.6. Illustration of heat transfer profiles for range of $\Pr$ values.

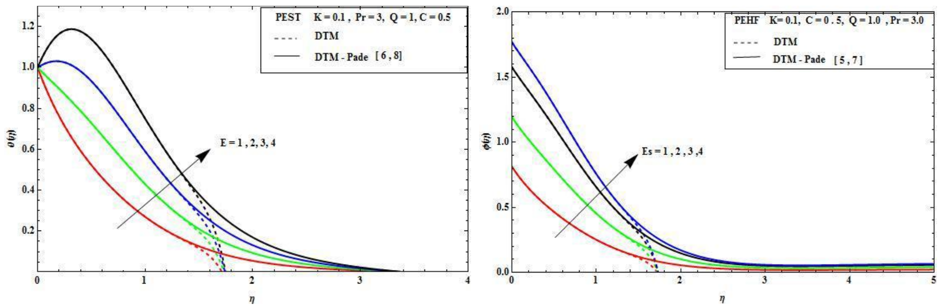

Figure 1.7. Illustration of heat transfer profiles for range of $E(Es)$ values.

Table 1. α_4 and β_4 for range of Q and C values.

Slip parameter K	Q	C	$C_f = \lvert f''(0) \rvert = \alpha_4$	$f'''(0) = \beta_4$
0.01	0.5	0.1	1.119926772	6.143065751
		0.2	1.023715885	3.927076649
		0.3	0.965503505	3.003995085
		0.4	0.923816576	2.477376318
0.01	0.5	0.1	1.263030327	7.956885243
		0.2	1.145729666	5.065095666
		0.3	1.075567284	3.865437398
		0.4	1.025698446	3.198751307
0.1	0	0.1	0.831284836	3.662315682
		0.2	0.775725168	2.378860715
		0.3	0.741562103	1.836375927
		0.4	0.717017904	1.523771535
0.1	0.5	0.1	0.988038328	5.311977266
		0.2	0.991451536	3.439556844
		0.3	0.874258662	2.651374678
		0.4	0.840072338	2.198575688

0.1	1.0	0.1	1.117388830	6.864520525
		0.2	1.025011109	4.430282239
		0.3	0.968670329	3.408740201
		0.4	0.928133292	2.823023705

Table 2. γ_4 and λ_4 for range of Q, C, Pr and $E(E_s)$ values.

Slip parameter	Q	C	Pr	$E(E_s)$	PEST $\theta'(0) = \lambda_4$	PEHF $\emptyset(0) = \lambda_4$
0.01	1	0.5	3	1	1.392967714	0.839263194
	2				1.052319124	0.978025785
	3				0.777138528	1.095719644
	4				0.542350085	1.200516246
0.01	1	0.5	1	1	0.743338084	0.839263194
			2		1.125953138	1.269481207
			3		1.392967714	1.699702309
			4		1.606361427	1.914813078
0.01	1	0.5	3	1	1.439787864	0.811513126
				2	0.546346308	1.194431102
				3	0.347097324	1.577351632
				4	0.240540874	1.960272652
0.01	1	0.1	3	2	0.517693860	1.645976811
		0.2			0.173479284	1.491501481
		0.3			0.048646915	1.394383539
		0.4			0.021226779	1.241094410

Table 3. γ_4 in PEST and λ_4 in PEHF for range of Q, C, Pr and $E(Es)$ values.

	Q	C	Pr	$E(Es)$	PEST $\theta'(0) = \lambda_4$	PEHF $\emptyset(0) = \lambda_4$
0.1	1	0.5	3	1	1.439787864	0.811513126
	2				1.138989365	0.938252582
	3				0.902484651	1.044646141
	4				0.704902578	1.138752686
0.1	1	0.5	1	1	0.743380841	1.211424418
			2		1.212370191	0.914746683
			3		1.385918477	0.811513126
			4		1.597262835	0.757265002
0.1	1	0.5	3	1	1.385918477	0.811513126
				2	0.313617540	1.194431021
				3	0.758684679	1.577351632
				4	0.790876161	1.960272652
0.1	1	0.1	3	2	0.517693865	1.645976810
		0.2			0.173479281	1.491501482
		0.3			0.048646910	1.394383539
		0.4			0.013585510	1.324109441

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