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MINIMUM VERTEX COVER OF ω-PENTAGONAL FUZZY LINEAR SUM BOTTLENECK ASSIGNMENT PROBLEM

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Abstract

In this paper we presented a spread of minimum solution of fuzzy optimization matching procedure in the bipartite graph. It provides minimum vertex cover with edge set E for solving ω-Pentagonal Fuzzy Linear Sum Bottleneck Assignment Problems [ω-PFLSAP]. The ω-pentagonal fuzzy linear sum bottleneck assignment problem is minimum cost and maximum matching in the bipartite graph. The Linear Sum Bottleneck Assignment Cost [LSBAC] we taken as ω-Pentagonal Fuzzy Numbers (ω-PFN).

1. Introduction

In 1987, Zadeh [20] introduced fuzzy set as a basis for a theory of possibility. In 1996, Channas and Kuchta [6] discussed a concept of optimal solution of the transportation problem with fuzzy cost coefficient. Kuhn [12] presented the Hungarian method for the assignment problem in the year of 2005. In 2010, Mukherjee and Basu [13] discussed application of fuzzy ranking method for solving assignment problem with fuzzy costs.

In 2012, Amit Kumar and Anila Gupta [1] exposed assignment and travelling salesman problems with coefficients as LR fuzzy parameters and Isabel and Uthra [9] discussed an application of linguistic variables in assignment problem with fuzzy costs and De and Yadav [7] presented a

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general approach for solving assignment problems involving with fuzzy cost coefficients. In 2014, Berghman, Leus and Spieksma [4] presented optimal solutions for a dock assignment problem with trailer transportation.

In 2015, Panda and Pal [16] exposed a study on pentagonal fuzzy number and its corresponding matrices and Jatinder Pal Singh and Neha Ishesh Thakur [10] exposed a novel method to solve assignment problem in fuzzy environment. In the year of 2016, Ghadle and Pathade [8] introduced optimal solution of balanced and unbalanced fuzzy transportation problem using hexagonal fuzzy number and Anchal Choudhary et al. [2] presented a new algorithm for solving fuzzy assignment problem using branch and bound method. In the year of 2017 Avinash Kamble [3] discussed some notes on pentagonal fuzzy numbers.

In this paper, we discussed minimum vertex cover of perfect matching solution (φ) for solving optimal solution of ω-pentagonal fuzzy linear sum bottleneck assignment problem.

2. Preliminaries

In this chapter discussed basic concepts of fuzzy sets and ω-trapezoidal fuzzy numbers, pentagonal fuzzy numbers and ω-pentagonal fuzzy numbers, spread of solution.

2.1 Definition. A fuzzy set is defined as a pair of $(X, \mu(x))$ in which X is a set and $\mu(x) = X \to [0, 1]$ is a membership function. For each $x \in X,$ the set X is referred to as the universe of discourse. The value $\mu(x)$ is called the grade of membership of x in $(X, \mu(x)).$ The function $\mu(x)$ is called the membership function of the fuzzy set $\widetilde{P} = (X, \mu(x)).$

2.2 Definition. A fuzzy number $\widetilde{P} = (p_1, p_2, p_3, p_4, \omega)$ is said to be a ω-trapezoidal fuzzy number if its membership function

$$\mu_{\widetilde{P}}(x) = \begin{cases} \omega\left(\dfrac{x - p_1}{p_2 - p_1}\right) & \text{for } p_1 \le x \le p_2 \\ \omega & \text{for } p_2 \le x \le p_3 \\ \omega\left(\dfrac{p_4 - x}{p_4 - p_3}\right) & \text{for } p_3 \le x \le p_4 \end{cases} \quad \text{where } \omega \in (0, 1)$$

2.3 Definition. ω-Pentagonal fuzzy number ωPFN of a fuzzy set $\widetilde{P}$ is denoted as $\widetilde{P} = (P_1, P_2, P_3, P_4, P_5, \omega)$ where P_3 is the middle point and (P_1, P_2) and (P_4, P_5) are the left and right side points of P_3, respectively and its membership function is given by

$$\mu_{\widetilde{P}}(x) = \begin{cases} \frac{1}{2}\left(\frac{x - P_1}{P_2 - P_1}\right) & \text{for } P_1 \le x \le P_2 \\ \frac{1}{2}\left(\frac{1}{2} - \omega\right)\left(\frac{x - P_2}{P_3 - P_2}\right) & \text{for } P_2 \le x \le P_3 \\ \omega & \text{for } x = P_3 \qquad \text{where } \omega \in (0.5, 1) \\ \frac{1}{2} - \left(\frac{1}{2} - \omega\right)\left(\frac{P_4 - x}{P_4 - P_3}\right) & \text{for } P_5 \le x \le P_4 \\ \frac{1}{2}\left(\frac{P_5 - x}{P_5 - P_4}\right) & \text{for } P_4 \le x \le P_5 \end{cases}$$

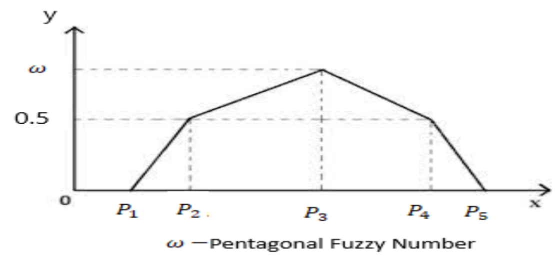

Figure 1.

2.4 Definition. Let $\tilde{C} = c_{ij}$ be a given $(m \times n)$ ω-Pentagonal Fuzzy Linear Sum Bottleneck Assignment Problem (ω-PFLSBAP) and φ be an arbitrary permutation of the set then $spr(\varphi) = \frac{\max}{i}\{\widetilde{c}_{i\varphi(i)}\} - \frac{\min}{i}\{\widetilde{c}_{i\varphi(i)}\}$ is called spread of solution (φ).

2.5 Definition. Let φ be a matching solution of the bipartite graph. If the solution is maximum matching and minimum spread of solution (φ) or $spr(\varphi) = 0$, then the solution is optimal or minimum vertex cover of spread of ω-pentagonal fuzzy linear sum bottleneck assignment problem.

3. Arithmetic Operations of ω-Pentagonal Fuzzy Numbers

Let $\widetilde{P} = (p_1, p_2, p_3, p_4, p_5, \omega_1)$ and $\widetilde{Q} = (q_1, q_2, q_3, q_4, q_5, \omega_2)$ be a two ω-Pentagonal Fuzzy Numbers (ω-PFN) then the following arithmetic operations:

1. $\widetilde{P} + \widetilde{Q} = (p_1 + q_1,\ p_2 + q_2,\ p_3 + q_3,\ p_4 + q_4,\ p_5 + q_5,\ \min(\omega_1, \omega_2))$

2. $\widetilde{P} - \widetilde{Q} = (p_1 - q_5,\ p_2 - q_4,\ p_3 - q_3,\ p_4 - q_2,\ p_5 - q_1;\ \min(\omega_1 - \omega_2))$

3. $\widetilde{P} + (-\widetilde{Q}) = (p_1, p_2, p_3, p_4, p_5) + (-q_1, -q_2, -q_3, -q_4, -q_5), (\omega_1, \omega_2)$

   $= (p_1 - q_1,\ p_2 - q_2,\ p_3 - q_3,\ p_4 - q_4,\ p_5 - q_5,\ \min(\omega_1 - \omega_2))$

4. $\lambda\widetilde{P} = \{\lambda p_1, \lambda p_2, \lambda p_3, \lambda p_4, \lambda p_5\}, \lambda > 0$

   and $\{\lambda p_5, \lambda p_4, \lambda p_3, \lambda p_2, \lambda p_1\}, \lambda < 0$

5. $\widetilde{P}\widetilde{Q} = (p_1q_1,\ p_2q_2,\ p_3q_3,\ p_4q_4,\ p_5q_5,\ \min(\omega_1, \omega_2))$

6. $\dfrac{\widetilde{P}}{\widetilde{Q}} = \widetilde{P}\overline{\widetilde{Q}} = \left(\dfrac{p_1}{q_5}, \dfrac{p_2}{q_4}, \dfrac{p_3}{q_3}, \dfrac{p_4}{q_2}, \dfrac{p_5}{q_1}\right).$

7. If $\widetilde{P} = (p_1, p_2, p_3, p_4, p_5, \omega)$ be a ω-pentagonal fuzzy numbers;

$$\widetilde{P}^{-1} = \frac{1}{\widetilde{P}} = \left(\frac{1}{p_5}, \frac{1}{p_4}, \frac{1}{p_3}, \frac{1}{p_2}, \frac{1}{p_1}, \frac{1}{2\omega}\right).$$

4. Mathematical Formulation of ω-Pentagonal Fuzzy Linear Sum Bottleneck Assignment Problem (ω-PFLSBAP)

Let $\tilde{C} = c_{ij}$ be a given $(n \times n)$ ω-Pentagonal Fuzzy Linear Sum Bottleneck Assignment Problem (ωPFLSBAP) then the following mathematical formulation:

$$\text{Objective } \min_{\varphi} spr(\varphi)$$

Subject to the constraints

$$\sum_{j=1}^{n} x_{ij} = 1, i = (1, 2, \ldots, n)$$

$$\sum_{i=1}^{n} x_{ij} = 1,\ j = (1, 2, \ldots, n)$$

$x_{ij} = 0$ (or) 1.

4.1 Matching solution of the ω-Pentagonal Fuzzy Linear Sum Bottleneck Assignment Problem (ω-PFLSBAP)

- The matching solution $\varphi = (\varphi_1, \varphi_2, \ldots, \varphi_n)$ and row (j)

$$row\,(j) = \begin{cases} i & \text{if column } j \text{ is matched to row } i. \\ 0 & \text{if column } j \text{ is not matched to row } i \end{cases} (j = 1, 2, \ldots, n)$$

- The matching solution $\varphi = (\varphi_1, \varphi_2, \ldots, \varphi_n)$ and implements the inverse of row

$$\varphi = \begin{cases} j & \text{if row } i \text{ is matched to column } j. \\ 0 & \text{if row } i \text{ is not matched to column } j \end{cases} (i = 1, 2, \ldots, n)$$

5. A Spread of Minimum Vertex Cover of Optimization Matching Techniques for Solving ω-Pentagonal Fuzzy Linear Sum Bottleneck Assignment Problems

Find minimum vertex cover of optimization matching techniques in the bipartite graph for solving ω-pentagonal fuzzy linear sum bottleneck assignment problems. The Linear Sum Bottleneck Assignment Cost [LSBAC] we taken as ω-Pentagonal Fuzzy Numbers (ω-PFN).

Step 1. First check whether the given ω-Pentagonal Fuzzy Linear Sum Bottleneck Assignment Problem (ω-PFLSBAP) is balanced or not,

- If the total number of persons is equal to the total number of jobs, then ω-PFLSBAP is balanced, go to step 3.
- If the total number of persons is not equal to the total number of jobs, then ω-PFLSBAP is unbalanced, go to step 2.

Step 2. Add a dummy row/column of ω-pentagonal fuzzy linear sum bottleneck assignment cost. Entries with a cost of dummy row/dummy column are always zero.

Step 3. Calculate the matching solution (φ) of ω-PFLSBAP (row minimum)

$$\phi = \min(C_{ij}) \text{ for } i = 1, 2, \ldots, n$$

Step 4. Calculate $spr(\varphi)$

Let φ be a matching solution of ω-PFLSBAP with cost matrix C_{ij}

$$\prod = \max_i(\widetilde{C}_{i\varphi(i)})$$

$$\lambda = \min_i(\widetilde{C}_{i\varphi(i)})$$

$$Spr(\phi) = \Pi + (-\lambda), T = \{(i, j) : \lambda < C_{ij}\}$$

Step 5. Find minimum $spr(\varphi)$ or $spr(\varphi) = 0$

In the bipartite graph $(n \times n)$ with edge set E, find the minimum vertex cover (σ). If $|\sigma| = n$, then φ be a perfect matching solution of ω-PFLSBAP.

$$\prod = \max_i(\widetilde{C}_{i\varphi(i)}),\ \lambda = \min_i(\widetilde{C}_{i\varphi(i)}),\ Spr(\varphi) = \pi + (-\lambda)$$

Step 6. Minimum vertex cover of cardinality n with perfect matching.

Let us take uncovered vertex $\overline{T} = \{(i, j) \in T, (i, j)$ is uncovered$\}$. If $\overline{T}$ is not equal to ϕ then find $\Pi = \min\{C_{ij} : (i, j) \in \overline{T}\}$ and $\lambda = \Pi + (-Spr(\varphi))$. By completing nonperfect matching through augmenting techniques, the new vertex cover is obtained from the current partial solution.

Step 7. (Apply optimal test of ω-PFLSBAP)

- If each person and each job contain exactly one matching solution with $Spr(\varphi) = 0$ or minimum $Spr(\varphi)$, then the current ω-PFLSBAP is optimal.
- If each person and each job contain exactly one matching solution with maximum $Spr(\varphi)$, then the current ω-PFLSBAP is not optimal but feasible and perfect matching solution.

Step 8. Finally obtained the graph has minimum vertex cover of cardinality n with perfect matching. Repeat the procedure (1) to (7), until an optimum ω-PFLSBA is attained.

Step 9. STOP.

6. Numerical Example

A company wants to assign four persons A, B, C and D to four jobs 1, 2, 3, 4 one for each job, with no person working on more than one job. The assignment cost is considered as ω-pentagonal fuzzy number. Find minimum vertex of cardinality n with perfect matching of ω-pentagonal fuzzy linear sum bottleneck assignment problem.

Solution.

Consider the following ω-pentagonal fuzzy linear sum bottleneck assignment table.

Table 1.

	J_1	J_2	J_3	J_4
P_1	(20,26,32,38,44;0.9)	(25,32,39,46,53;0.92)	(4,7,10,13,16;0.65)	(2,4,6,8,10;0.63)
P_2	(6,10,14,18,22;0.75)	(10,15,20,25,30)	(4,7,10,13,16;0.65;0.95)	(20,26,32,38,44;0.9)
P_3	(0,1,2,3,4;0.6)	(20,26,32,38,44;0.9)	(35,40,45,50,55;.90)	(6,10,14,18,22;0.75)
P_4	(2,4,6,8,10;0.63)	(8,12,16,20,24;0.8)	(20,26,32,38,44;0.9)	(4,7,10,13,16;0.65)

Table 2.

(20,26,32,38,44;0.9)	(25,32,39,46,53;0.92)	(4,7,10,13,16;0.65)	(2,4,6,8,10;0.63)
(6,10,14,18,22;0.75)	(10,15,20,25,30)	(4,7,10,13,16;0.65;0.95)	(20,26,32,38,44;0.9)
(0,1,2,3,4;0.6)	(20,26,32,38,44;0.9)	(35,40,45,50,55;.90)	(6,10,14,18,22;0.75)
(2,4,6,8,10;0.63)	(8,12,16,20,24;0.8)	(20,26,32,38,44;0.9)	(4,7,10,13,16;0.65)

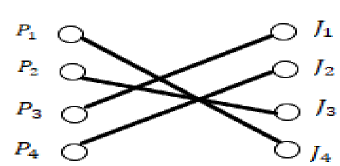

Figure 2.

$\varphi = (4, 3, 1, 2)$, $\lambda = (0, 1, 2, 3, 4, 0.6)$, $\Pi = (8, 12, 16, 20, 24, 0.8)$,

$Spr(\varphi) = (8, 11, 14, 17, 20, 0.2)$.

Table 3.

(20,26,32,38,44;0.9)	(25,32,39,46,53;0.92)	(4,7,10,13,16;0.65)	-
(6,10,14,18,22;0.75)	(10,15,20,25,30)	-	(20,26,32,38,44;0.9)
-	(20,26,32,38,44;0.9)	(35,40,45,50,55;.90)	(6,10,14,18,22;0.75)
(2,4,6,8,10;0.63)	(8,12,16,20,24;0.8)	(20,26,32,38,44;0.9)	(4,7,10,13,16;0.65)

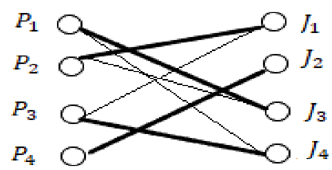

Figure 3.

$\varphi = (3, 1, 4, 2)$, $\lambda = (4, 7, 10, 13, 16, 0.65)$, $\Pi = (8, 12, 16, 20, 24, 0.8)$,

$Spr(\varphi) = (4, 5, 6, 7, 8, 0.15)$.

Table 4.

(20,26,32,38,44;0.9)	(25,32,39,46,53;0.92)	--	-
--	(10,15,20,25,30)	-	(20,26,32,38,44;0.9)
-	(20,26,32,38,44)	(35,40,45,50,55)	--
(2,4,6,8,10)	--	(20,26,32,38,44)	(4,7,10,13,16)

$\varphi = (1, 2, 3, 4)$, $\pi = (20, 26, 32, 38, 44, 0.9)$, $\lambda = \pi + (-Spr(\varphi))$

$= (16, 21, 26, 31, 36, .075)$.

Table 5.

(20,26,32,38,44)	(25,32,39,46,53)	--	-
--	(10,15,20,25,30)	-	(20,26,32,38,44)
-	(20,26,32,38,44)	(35,40,45,50,55)	--
(2,4,6,8,10)	--	(20,26,32,38,44)	(4,7,10,13,16)

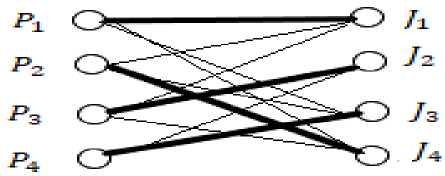

Figure 4.

$\varphi = (1, 4, 2, 3)$, $\pi = (20, 26, 32, 38, 44, 0.9)$, $\lambda = (20, 26, 32, 38, 44, 0.9)$,

$Spr(\varphi) = (0, 0, 0, 0)$.

$\therefore$ The optimal ω-PFLSBA schedule is $P_1 \to J_1$, $P_2 \to J_4$, $P_3 \to J_2$, $P_4 \to J_3$.

The optimal ω-PFLSBAP is (20, 26, 32, 38, 44, 0.9) + (20, 26, 32, 38, 44, 0.9) + (20, 26, 32, 38, 44, 0.9) + (20, 26, 32, 38, 44, 0.9) = (80, 104, 128, 152, 176, 0.9).

7. Conclusion

ω-Pentagonal fuzzy linear sum bottleneck assignment problem is minimum cost and maximum matching in the bipartite graph. If the matching solution $\varphi = (\varphi_1, \varphi_2, \ldots, \varphi_n)$ is not optimal, then the solution is maximum spread (φ), but feasible and perfect matching in the bipartite graph. If the matching solution $\varphi = (\varphi_1, \varphi_2, \ldots, \varphi_n)$ is optimal, then the solution is minimum spread (φ) (or) $spr(\varphi) = 0$ and perfect matching in the bipartite graph.

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