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PRIME AND SEMIPRIME IDEALS IN A Γ-SEMIRING

TILAK RAJ SHARMA[1] and SHWETA GUPTA[2]

[1,2]Department of Mathematics
Himachal Pradesh University
Regional Centre Khaniyara, Dharamshala
Himachal Pradesh, India-176218
E-mail: trpangotra@gmail.com

Abstract

From an algebraic perspective, semirings give the most natural basic speculation of the hypothesis of rings and the vast majority of the methods utilized in analyzing semiring are taken from ring theory and group theory. In this paper, the semiring hypothetical consequences of [4] and [10] concerning the prime and semiprime ideals of semirings to Γ-semirings are generalized.

1. Introduction

The set of non negative integers N with addition and multiplication gives a characteristic illustration of a semiring. There are numerous different examples of semirings, for example, for a given integer n, the set $\{\langle a_{ij}\rangle_{n\times n}\}$ over a semiring R structures a semiring with usual addition and multiplication over R. In any case, the circumstances for the arrangement of the set of all negative integers and for the set of all $\{\langle a_{ij}\rangle_{n\times n}\}$ over a semiring R are different. They do not shape semirings with the above operations, since multiplication in the above sense are no longer binary compositions. This thought gives another sort of algebraic structure, what is known as a Γ-semiring.

"The idea of Γ-semiring was presented by [7] in 1995" as a speculation of semiring as well as Γ-ring (it may be reviewed here that the thought of Γ – was first presented in algebra by N. Nobusawa in 1964). Later it was

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discovered that Γ-semiring additionally give an algebraic home to the negative cones of totally ordered rings and the set of rectangular matrices over a semiring.

In this paper, we set up certain outcomes with respect to prime and semiprime ideals of a Γ-semiring. We at that point get various characterizations of prime and semiprime ideals of Γ-semiring R.

2. Preliminaries

For preliminaries of Γ-semirings, we refer to [1], [2], [3] and [6]. Some of the following definitions are crucial in this paper. Throughout this paper, R represents a Γ-semiring.

Definition 2.1[9]**.** Let R and Γ be two additive commutative semigroup. Then R is called a Γ-semiring if there exists a mapping $R \times \Gamma \times R \to R$ denoted by $x\alpha y$ for all $x, y \in R$ and $\alpha \in \Gamma$ satisfying the following conditions:

(i) $(x + y)\alpha z = x\alpha z + y\alpha z$.

(ii) $x(\alpha + \beta)z = x\alpha z + x\beta z$.

(iii) $x\alpha(y + z) = x\alpha y + x\alpha z$.

(iv) $(x\alpha y)\beta z = x\alpha(y\beta z)$ for all $x, y, z \in R$ and $\alpha, \beta \in \Gamma$.

Let $A \subseteq R, B \subseteq R$ and $\Delta \subseteq \Gamma$. Let us denote $A\Delta B \subseteq R$ by $A\Delta B = \{\Sigma a_i \gamma_i b_i$, where $a_i \in A$, $b_i \in B$ and $\gamma_i \in \Gamma\}$ be a set of finite sums.

Definition 2.2[7]**.** "0 is the element of R if and only if $0\gamma x = 0 = x\gamma 0$ and $x + 0 = x = 0 + x$ and '1' is the identity element if $1\gamma x = x = x\gamma 1$ for all $x \in R$ and $\gamma \in P$."

Definition 2.3[7]**.** "If $x\gamma y = y\gamma x$ for all $x, y \in R$ and $\gamma \in \Gamma$, then R is a commutative Γ-semiring".

Definition 2.4[3]**.** "A non empty sub set I of R is said to be left (right) ideal of R if I is subsemigroup of $(R, +)$ and $x\gamma y \in I, (y\gamma x \in I)$ for all $x \in R$, $y \in I$ and $\gamma \in \Gamma$. If I is both left and right ideal of a Γ-semiring R then I is called an ideal of R".

Definition 2.5[4]. "A proper ideal M of R is said to be maximal ideal if there does not exist any other proper ideal of R containing M properly".

Definition 2.6[9]. "An ideal P of R is k-ideal if $y \in P$, $x + y \in P$, $x \in R$ implies that $\gamma \in \Gamma$".

Definition 2.7[6]. "Let R_1 and R_2 be two Γ-semirings. Then $f : R_1 \to R_2$ be a Γ-homomorphism if $f(x + y) = f(x) + f(y)$ and $f(x\gamma y) = f(x)\gamma f(y)$ for all $x, y \in R_1$ and $\gamma \in \Gamma$."

OR

"Let R_1 be Γ_1-semiring and R_2 be Γ_2-semiring. Then $(f, g) : (R_1, \Gamma_1) \to (R_2, \Gamma_2)$ is called homomorphism if $f : R_1 \to R_2$ and $g : \Gamma_1 \to \Gamma_2$ are homomorphisms of semigroup such that $f(x + y) = f(x) + f(y)$ and $f(x\gamma y) = f(x)g(\gamma)f(y)$ for all $x, y \in R_1$ and $\gamma \in \Gamma_1$".

Lemma 2.8[6]. "*Let* Λ *be a non empty index set and* $\{I_\lambda\}_{\lambda \in A}$ *be a family of ideals of a* Γ*-semiring* R. *Then* $\bigcap_{\lambda \in \Lambda} I_\lambda$ *is an ideal of* Γ*-semiring* R".

All through this paper, R will signify a Γ-semiring with zero element '0' and identity component '1' except if in any case expressed.

3. Prime and Semiprime Ideals in a Γ-Semiring

In this segment, we set up certain outcomes with respect to prime and semiprime ideals of a Γ-semiring R. Further, we obtain different characterizations of prime and semiprime ideals of R.

Remark [10] "Let R_1 and R_2 be two Γ-semirings and $T : R_1 \to R_2$ be an onto Γ-homomorphism. Let $K_T = \{x \in R_1 \mid \text{there exist } y, z \text{ in } R_1 \text{ such that } x = y + z \text{ and } T(y) = T(z)\}$. Then K_T is an ideal of R_1 containing $Ker\, T$, where $Ker\, T = \{x \in R_1 \mid T(x) = 0\}$".

Definition 3.1[3]. "An ideal P of R is prime if for any two ideals A and B of R, $A\Gamma B \subseteq P$ we have, either $A \subseteq P$ or $B \subseteq P$".

Theorem 3.2. *Let* R_1 *and* R_2 *be two* Γ*-semirings and* $T : R_1 \to R_2$ *be an onto homomorphism. Let* A_1 *be an ideal of* R_1 *and* A_2 *an ideal of* R_2. *Then*

(i) A_2 *is prime if and only if* $T^{-1}(A_2)$ *is prime.*

(ii) *If* A_1 *is a k-ideal and* $A_1 \supseteq K_T$, *then* A_1 *is prime if and only if* $T(A_1)$ *is prime.*

(iii) *If* $T^{-1}(A_2)$ *is maximal, then* A_2 *is maximal.*

(iv) *Let* A_1 *be k-ideal containing* K_T. *If* A_1 *is maximal, then* $T(A_1)$ *is maximal.*

Proof. (i) Let A_2 be given prime in R_2. Let P_1 and Q_1 be two ideals of R_1 such that $P_1 \Delta Q_1 \subseteq T^{-1}(A_2)$. Therefore $T(P_1)\Delta T(Q_1) = T(P_1 \Delta Q_1) \subseteq T(T^{-1}(A_2)) = A_2$ [c.f. [11], Lemma 3.3 (i), (iii), (vii)]. But A_2 is prime, so either $T(P_1) \subseteq A_2$ or $T(Q_1) \subseteq A_2$. This implies that either $P_1 \subseteq T^{-1}(A_2)$ or $Q_1 \subseteq T^{-1}(A_2)$. Thus $T^{-1}(A_2)$ is prime. Conversely, let P_2 and Q_2 be two ideals of R_2 such that $P_2 \Delta Q_2 \subseteq A_2$. Using [c.f. [11], Lemma 3.3 (vi), (x)], we have $T^{-1}(P_2)\Delta T^{-1}(Q_2) \subseteq T^{-1}(P_2 \Delta Q_2) \subseteq (T^{-1}(A_2))$. But $T^{-1}(A_2)$ is prime, therefore either $(T^{-1}(P_2)) \subseteq (T^{-1}(A_2))$ or $(T^{-1}(Q_2)) \subseteq (T^{-1}(A_2))$ so by [c.f. [11], lemma (i); (vii)])] we have either $P_2 \subseteq T(T^{-1}(A_2))$ or $Q_2 \subseteq T(T^{-1}(A_2))$. So either $P_2 \subseteq A_2$ or $Q_2 \subseteq A_2$.

(ii) Let $T(A_1) = A_2$. Since A_1 is k-ideal containing K_T, so $T^{-1}(A_2) = T^{-1}(T(A_1)) = A_1$ ([c.f. [11], Lemma 3.3 (vi); (viii)]. Then by (i) A_1 is prime if and only if $T(A_1)$ is prime.

(iii) Suppose $T^{-1}(A_2)$ is maximal. Let Q_2 be any other ideal of R_2 such that $A_2 \subseteq Q_2 \subseteq R_2$ then by [c.f. [11], Lemma 3.3 (vi)] $T^{-1}(A_2) \subseteq T^{-1}(Q_2) \subseteq R_1$. Since $T^{-1}(A_2)$ is maximal so either $T^{-1}(A_2) = T^{-1}(Q_2)$ or $T^{-1}(A_2) = R_1$. Now result follows easily using the fact that $T(T^{-1}(A_2)) = A_2$ for every ideal A_2 of R_2.

(iv) Follows exactly as (ii) and (iii). □

Definition 3.3[3]. "Let P be an ideal of R. Then P is semiprime if for any two ideals A of R, $A\Gamma A \subseteq P$ gives that $A \subseteq P$".

The following theorem is proved in [1].

Theorem 3.4[1]. "*Let R be a Γ-semiring. For an ideal $P \subsetneq R$ the following statements are equivalent*

(i) *P is prime.*

(ii) *For $a, b \in R$, $a\Gamma R\Gamma b \subseteq P$ if and only if $a \in P$ or $b \in P$.*

(iii) *For $a, b \in R$, $(a)\Gamma(b) \subseteq P$ if and only if $a \in P$ or $b \in P$.*

(iv) *For any left (right) ideals H, K of R, $H\Gamma K \subseteq P$ implies that either $H \subseteq P$ or $K \subseteq P$".*

Theorem 3.5. *Let $P \subsetneq R$ be an ideal of R. Then the following statements are equivalent*

(i) *P is semiprime.*

(ii) *If $a \in R$, $(a)\Gamma(a) \subseteq P$ then $a \in P$.*

(iii) *$a \in R$, $a\Gamma R\Gamma a \subseteq P$ if and only if $a \in P$.*

(iv) *If H is any left (right) ideal of R, $H\Gamma H \subseteq P$ then $H \subseteq P$.*

Proof. We omit, because it is a matter of routine verification. □

Definition 3.6[12]. "An element $x \in R$ is multiplicative Γ-idempotent if there $\gamma \in \Gamma$, such that $x = x\gamma x$. R is called multiplicative Γ-idempotent Γ-semiring if every element of R is multiplicative Γ-idempotent. It is denoted by $I^{\times}(\Gamma R)$".

Definition 3.7[7]. "Let $C(R) = \{x \in R \mid x\gamma x = y\gamma x,$ for all $y \in R, \gamma \in \Gamma\}$. Then $C(R)$ is called centre of Γ-semiring R".

Definition 3.8[5]. $x \in R$ is a unit if there exists an element $y \in R$ and $\gamma \in \Gamma$ such that $x\gamma y = 1 = y\gamma x$. The element $y \in R$ is called the inverse of x

in R. The set of all elements of R having units is denoted by $U(\Gamma R)$. Clearly, $U(\Gamma R) \neq \phi$, since $1 \in U(\Gamma R)$ and is not all of R.

Definition 3.9[3]. "Let $H \neq \phi$. Then H is an m-system of R if $c\alpha_1 r\alpha_2 d \in H$, for any $c, d \in H$, $r \in R$ and $\alpha_1, \alpha_2 \in \Gamma$".

Example 3.10. Let S be a submonoid of R with identity 1, then S is an m-system. Specifically, $U(\Gamma R)$, $C(R)$ and $I^{\times}(\Gamma R) \cap C(R)$ are m-systems and $I^{\times}(\Gamma R)$ is an m-system if R is commutative.

Definition 3.11[2]. "$\phi \neq A \subset R$ is an n-system of R if $a\beta r\gamma a \in A$, for $a \in A$, $r \in R$ and $\beta, \gamma \in \Gamma$".

The following results are proved in [1] and [3], however for the completeness we state the following.

Theorem 3.12[1]. "*An ideal P of a* Γ-*semiring R is prime if and only if* $R \setminus P$ *is an m-system of R*".

Theorem 3.13[1]. *Let P be an ideal of a commutative* Γ-*semiring R. Then P is prime if* $a\Gamma b \subseteq P$ *then either* $a \in P$ *or* $b \in P$".

Theorem 3.14[1]. "*An ideal P of a* Γ-*semiring R is semiprime if and only if for* $a \in R$, $(a)\Gamma(a) \subseteq P$ implies that $a \in P$.

Corollary 3.15[3]. "*Let R be a* Γ-*semiring. An ideal I of a* Γ-*semiring R is semiprime if and only if* $R \setminus I$ *is an n-system*".

Theorem 3.16[3]. *A non-empty subset A of a* Γ-*semiring R is an n-system if and only if it is union of m-systems*".

Now we have.

Theorem 3.17. *Let A be an m-system of elements of a* Γ-*semiring R. On the off chance that if P is maximal ideal among each one of those of R which are disjoint from A then P is prime.*

Proof. Let $I \not\subseteq P$ and $J \not\subseteq P$ satisfying $I\Gamma J \subseteq P$, where I and J are ideals of R. Then $P \subseteq I + P$ and $P \subseteq J + P$. Therefore $(I + P) \cap A \neq \phi$ and $(J + P) \cap A \neq \phi$. Specifically, there exist finite subsets

$\{p_1, p_2, \ldots, p_n, q_1, q_2, \ldots, q_m\}$ of P, $\{i_1, i_2, \ldots, i_n\}$ of I and $\{j_1, j_2, \ldots, j_m\}$ of J, $1 \leqslant k \leqslant n, 1 \leqslant s \leqslant m$ so that $p = \sum_{k=1}^{n} (i_k + p_k) \in A \bigcap (I + P)$ and $q = \sum_{s=1}^{m} (j_s + q_s) \in A \bigcap (J + P)$. But A is an m-system, therefore for $r \in R, \alpha, \beta \in \Gamma$ we have $a\alpha r\beta b \in A$. In any case, $p\alpha r\beta q$
$= \sum_{s=1}^{m} [\sum_{k=1}^{n} (p_k + i_k)\alpha r\beta q_s + \sum_{k=1}^{n} (p_k + i_k)\alpha r\gamma j_s]$
$= \sum_{s=1}^{m} [\sum_{k=1}^{n} p_k\alpha r\beta q_s + \sum_{k=1}^{n} p_k\alpha r\gamma j_s + \sum_{k=1}^{n} i_k\alpha r\beta q_s + \sum_{k=1}^{n} i_k\alpha r\gamma j_s]$
$\in P + P\Gamma J + I\Gamma P + I\Gamma J \subseteq P + I\Gamma J \subseteq P$, since $I\Gamma J \subseteq P$, logical inconsistency to the theory that $P \bigcap A = \phi$. Hence P is prime. □

Corollary 3.18. *If P be any maximal ideal of R. Then P is prime.*

Proof. The result follows from Theorem 3.17 and example 3.10 and on the off chance that an ideal of R is maximal among each one of those ideals of R which are disjoint from $U(\Gamma R)$.

Theorem 3.19. *Let P be a prime ideal of R. Then every prime ideal P contains a minimal prime ideal.*

Proof. Let $\{K_i \mid i \in \Lambda\}$ be such that for $i \geqslant j$ in Λ we have, $K_i \subseteq K_j$, where each K_i is a descending chain of prime ideals of R. Let $K = \bigcap_{i \in \Lambda} K_i$. Therefore, K is an ideal of R [c.f. Lemma 2.10]. Further let $a, b \in R$, $\alpha, \beta \in \Gamma$ be such that $\{a\alpha r\beta b \mid r \in R\} \subseteq K$. Let $a \notin K$. Then for any $m \in \Lambda$ we have $a \notin K_m$. So by Theorem 3.4, $b \in K_m$. Thus, $b \in K_i$ for all $i \leqslant m (K_i \supseteq K_m)$. Further, if $i > m$ then $K_i \subseteq K_m$ and so $a \notin K_i$. So by Theorem 3.4, $b \in K_i$ for all $i \in \Lambda$. Thus $b \in K$. Hence by Theorem 3.4, K is prime. Now the result is self evident by Zorn's lemma to the set of all prime ideals of R contained in P. □

Theorem 3.20. *An ideal H of a* Γ*-semiring R is semi prime if and only if H is the intersection of all prime ideals of R containing H.*

Proof. Let H be a semi prime ideal of R. Then by Corollary 3.15 $R \setminus H$ is an n-system. Also by Theorem 3.16, $R \setminus H = \bigcup_{i \in \Lambda} K_i$ where each K_i is an m-

system and $K_i \subseteq R \mid H$. But $H \bigcap K_i = \phi$ for each $i \in \Lambda$, so by Zorn's Lemma, $H \subseteq L_i$, where L_i is maximal disjoint from K_i. This implies that, each such L_i is prime [c.f. Theorem 3.17]. Thus $H \subseteq \bigcap_{i \in \Lambda} L_i$ $\subseteq \bigcap_{i \in \Lambda}(R \setminus K_i) = H$. Hence H is the intersection of all prime ideals of R containing H. Conversely, the result follows exactly from Theorem 3.14, Corollary 3.15 and Theorem 3.16. □

Theorem 3.21. *Let Q be an ideal of R. On the off chance that if H is an ideal of R minimal among each one of those ideals of R, properly containing Q then $P = \{x \in R \mid x\Gamma H \subseteq Q\}$ is a prime ideal of R.*

Proof. Let $x, y \in P$. Therefore, $x, y \in R$ and $x\Gamma H \subseteq Q, y\Gamma H \subseteq Q$. Now $(x + y)\Gamma H = x\Gamma H + y\Gamma H \subseteq Q$. This implies that $x + y \in P$. Let $x \in P$. Then $x \in R, x\Gamma H \subseteq Q$. Now $x\gamma r \in R, \gamma \in \Gamma$, we have $(x\gamma r)\gamma H = x\gamma(r\gamma H) \subseteq x\Gamma Q$ $\subseteq Q$. Similarly, $(r\gamma x)\Gamma H = r\gamma(x\Gamma H) \subseteq r\Gamma Q \subseteq Q$. Thus P is an ideal of R. Let $A\Gamma B \subseteq P$, where A and B be two ideals of R and assume that $B \not\subseteq P$. Now we claim that $A \subseteq P$. Since $A\Gamma B \subseteq P$ and $B \not\subseteq P$, we have $A\Gamma B\Gamma H \subseteq Q$ and $B\Gamma H \not\subseteq P$. Therefore $Q \subset Q + B\Gamma H \subseteq H$, since H is minimal, so we have $Q + B\Gamma H = H$. Thus $A\Gamma(Q + B\Gamma H) = A\Gamma H \subseteq Q$. Hence $A \subseteq P$.

Definition 3.22. A Γ-semiring R is said to be left noetherian if and only if it satisfies the ascending chain conditions on left ideals. Similarly we can define right noetherian.

Theorem 3.23. *Let S be the set of all ideals of a commutative Γ-semiring R which are not finitely generated. If P is a maximal k-ideal such that $P \subseteq S$ then P is prime.*

Proof. Let $a\gamma b \in P, a, b \in R \setminus P, \gamma \in \Gamma$. Then clearly $P + (a)$ and $P + (b)$ are ideals of R. Therefore, $P \subset P + (a), P \subset P + (b)$ and so both are finitely generated. Let $P + (a) = (\{p_1 + r_1\gamma_1 a, p_2 + r_2\gamma_2 a, \ldots, p_n + r_n\gamma_n a\})$ and $P + (b) = (\{p'_1 + r'_1\gamma'_1 b, p'_2 + r'_2\gamma'_2 b, \ldots, p'_m + r'_m\gamma_m b\}), p_i, p'_j \in P \subseteq R, \gamma_i, \gamma'_j \in \Gamma$, $1 \leq i \leq n, 1 \leq j \leq m$. Let $K = \{r \in R \mid r\gamma a \in P, \gamma \in \Gamma\}$. Thus K is an ideal of

R. If $1 \leq j \leq m$ then $(p'_j + r'_j\gamma_j b)\beta a = p'_j\beta a + (r'_j\gamma_j b)\beta a = p'_j\beta a + r'_j\gamma_j(b\beta a)$ $= p'_j\beta a + r'_j\gamma_j(a\beta b) \in K$. Therefore $P \subset P + (b) \subseteq K$. Since P is maximal so K is finitely generated. Let $K = \{k_1, k_2, \ldots, k_q\}$. This implies that $\sum_{i=1}^{q} t_i\gamma_i k_i \in K, t_i \in R, 1 \leq i \leq q$. If $p \in P$ then for $s_1, s_2, \ldots, s_n$ of R we have $p = \sum_{i=1}^{n} s_i\alpha_i(p_i + r_i\beta a) = \sum_{i=1}^{n} s_i\alpha_i p_i + \sum_{i=1}^{n}(s_i\alpha_i r_i)\beta a$, for all $\alpha_i, \beta \in \Gamma$. Since P is k-ideal, so $\sum_{i=1}^{n}(s_i\alpha_i r_i)\beta a \in P$. Thus $\sum_{i=1}^{n}(s_i\alpha_i r_i) \in K$. Therefore, there exist $t_1, t_2, \ldots, t_q \in R$ such that $\sum_{i=1}^{n} s_i\alpha_i r_i = \sum_{i=1}^{q} t_i\gamma_i k_i$. So $P = \sum_{i=1}^{n} s_i\alpha_i p_i + \sum_{i=1}^{q} t_i\gamma_i k_i\beta a$. Thus P is generated by $\{p_1, p_2, \ldots, p_n, k_1\beta a, \ldots, k_q\beta a\}$, which is logical inconsistency to the theory that P is not finitely generated. Hence $a\gamma b \in P$ gives that either $a \in P$ or $b \in P$. Hence P is prime. □

Theorem 3.24. *Let P be a k-prime ideal of a commutative* Γ*-semiring R. Then P is finitely generated if and only if R is noetherian.*

Proof. Let R be noetherian, then P is finitely generated [c.f. [11], proposition 4.2]. Conversely, let P be finitely generated and M be the set of all ideals of R which are not finitely generated. We will show that $M = \phi$ [c.f. [11], proposition 4.2]. If possible let $M \neq \phi$. Let $H = \bigcup_{i \in \Lambda} H_i$, where each $H_i, i \in \Lambda$ is a chain of elements of M. Then H cannot be finitely generated and is an ideal of R. Otherwise, it must be one of H_i, which is logical inconsistency to the theory that none of H_i is finitely generated. The result is self evident by Zorn's Lemma that, M has maximal element. Thus by theorem 3.23, P is prime, which is a contradiction to given. Hence R is noetherian.

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