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ON FUZZY F_σ-COMPLEMENTED SPACES

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Abstract

In this paper, the concept of fuzzy F_σ-complemented space is introduced and studied. The conditions under which fuzzy topological spaces become fuzzy F_σ-complemented spaces, are obtained. Also a condition under which weak fuzzy Oz-spaces become fuzzy F_σ-complemented spaces, is established. It is obtained that fuzzy F_σ-complemented spaces are neither fuzzy perfectly disconnected spaces nor fuzzy F'-spaces.

1. Introduction

The concept of fuzzy sets as a new approach for modelling uncertainties was introduced by L. A. Zadeh [19] in the year 1965. The potential of fuzzy notion was realized by the researchers and has successfully been applied in all branches of Mathematics. In 1968, C. L. Chang [4] introduced the concept of fuzzy topological space. The paper of Chang paved the way for the subsequent tremendous growth of numerous fuzzy topological concepts. In 2004, M. Henriksen and R. G. Woods [5] introduced the notion of cozero complemented space and several characterizations of these spaces are established. R. Levy and J. Shapiro [6] studied cozero complemented spaces under the name "*z*-good spaces". In [1], F. Azarpanah and M. Karavan studied cozero complemented spaces in the name "*m*-spaces".

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In the recent years, there has been a growing trend to introduce and study various types of fuzzy topological spaces. In this paper, the concept of fuzzy F_σ-complemented space is introduced and studied. Several characterizations of fuzzy F_σ-complemented spaces are established. The conditions under which fuzzy topological spaces become fuzzy F_σ-complemented spaces, are obtained. Also a condition under which weak fuzzy Oz-spaces become fuzzy F_σ-complemented spaces, is established. It is obtained that fuzzy F_σ-complemented spaces are neither fuzzy perfectly disconnected spaces nor fuzzy F'-spaces. It is obtained that fuzzy F_σ-complemented spaces in which fuzzy F_σ-sets are dense sets, are fuzzy Baire and fuzzy second category spaces. Also it is established that fuzzy almost P-spaces are not fuzzy F_σ-complemented spaces.

2. Preliminaries

Some basic notions and results used in the sequel, are given in order to make the exposition self-contained. In this work by (X, T) or simply by X, we will denote a fuzzy topological space due to Chang (1968). Let X be a nonempty set and I the unit interval [0, 1]. A fuzzy set λ in X is a mapping from X into I. The fuzzy set 0_X is defined as $0_X(x) = 0,$ for all $x \in X$ and the fuzzy set 1_X is defined as $1_X(x) = 1,$ for all $x \in X.$

Definition 2.1[4]**.** A fuzzy topology is a family T of fuzzy sets in X which satisfies the following conditions:

(a) $0_X \in T$ and $1_X \in T$

(b) If $A, B \in T,$ then $A \wedge B \in T,$

(c) If $A_i \in T$ for each $i \in J,$ then $\vee_i A_i \in T.$

T is called a fuzzy topology for X, and the pair (X, T) is a fuzzy topological space, or fts for short. Members of T are called fuzzy open sets of X and their complements fuzzy closed sets.

Definition 2.2[4]**.** Let (X, T) be a fuzzy topological space and λ be any fuzzy set in (X, T). The interior, the closure and the complement of λ are defined respectively as follows:

(i) $\text{int}(\lambda) = \vee\{\mu/\mu \leq \lambda, \mu \in T\}$,

(ii) $cl(\lambda) = \wedge\{\mu/\lambda \leq \mu, 1 - \mu \in T\}$.

(iii) $\lambda'(x) = 1 - \lambda(x)$, for all $x \in X$.

For a family $\{\lambda_i/i \in J\}$ of fuzzy sets in (X, T), the union $\psi = \vee_i(\lambda_i)$ and intersection $\delta = \wedge_i(\lambda_i)$, are defined respectively as

(iv) $\psi(x) = \sup_i\{\lambda_i(x)/x \in X\}$.

(v) $\delta(x) = \inf_i\{\lambda_i(x)/x \in X\}$.

Lemma 2.1[2]. *For a fuzzy set* λ *of a fuzzy topological space X*,

(i) $1 - \text{int}(\lambda) = cl(1 - \lambda)$ *and* (ii) $1 - cl(\lambda) = \text{int}(1 - \lambda)$.

Definition 2.3. A fuzzy set λ in a fuzzy topological space (X, T) is called a

(i) fuzzy regular-open set in (X, T) if $\lambda = \text{int}\,cl(\lambda)$; fuzzy regular-closed set in (X, T) if $\lambda = cl\,\text{int}(\lambda)$ [2].

(ii) fuzzy G_δ-set in (X, T) if $\lambda = \wedge_{i=1}^{\infty}(\lambda_i)$, where $\lambda_i \in T$ for $i \in I$; fuzzy F_σ-set in (X, T) if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where $1 - \lambda_i \in T$ for $i \in I$ [3].

(iii) fuzzy dense set if there exists no fuzzy closed set μ in (X, T) such that $\lambda < \mu < 1$. That is, $cl(\lambda) = 1$, in (X, T) [8].

(iv) fuzzy nowhere dense set if there exists no non-zero fuzzy open set μ in (X, T) such that $\mu < cl(\lambda)$. That is, $\text{int}\,cl(\lambda) = 0$, in (X, T) [8].

(v) fuzzy first category set if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are fuzzy nowhere dense sets in (X, T). Any other fuzzy set in (X, T) is said to be of fuzzy second category [8].

(vi) fuzzy residual set if $1 - \lambda$ is a fuzzy first category set in (X, T) [9].

(vii) fuzzy σ-boundary set if $\lambda = \vee_{i=1}^{\infty}(\mu_i)$, where $\mu_i = cl(\lambda_i) \wedge (1 - \lambda_i)$ and (λ_i)'s are fuzzy regular open sets in (X, T) [12].

(viii) fuzzy somewhere dense set if $\operatorname{int} cl(\lambda) \neq 0$, for a fuzzy set in (X, T) [17].

Definition 2.4. A fuzzy topological space (X, T) is called a

(i) fuzzy perfectly disconnected space if for any two non-zero fuzzy sets λ and μ defined on X such that $\lambda \leq 1 - \mu$, in (X, T) then $cl(\lambda) \leq 1 - cl(\mu)$, in (X, T) [14].

(ii) weak fuzzy Oz-space if for each fuzzy F_σ-set δ in (X, T), $cl(\delta)$ is a fuzzy G_δ-set in (X, T) [18].

(iii) fuzzy hyperconnected space if every non-null fuzzy open subset of (X, T) is fuzzy dense in (X, T) [7].

(iv) fuzzy submaximal space if for each fuzzy set λ in (X, T) such that $cl(\lambda) = 1$, $\lambda \in T$ [3].

(v) fuzzy globally disconnected space if each fuzzy semi-open set in (X, T) is fuzzy open in (X, T) [15].

(vi) fuzzy first category space if $1_X = \vee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are fuzzy nowhere dense sets in (X, T). A fuzzy topological space which is not of fuzzy first category is said to be of fuzzy second category [8].

(vii) fuzzy Baire space if int $(\vee_{i=1}^{\infty}(\lambda_i)) = 0$, where (λ_i)'s are fuzzy nowhere now here dense sets in (X, T) [9].

(viii) fuzzy almost P-space if for each non-zero fuzzy G_δ-set λ in (X, T), $\operatorname{int}(\lambda) \neq 0$ in (X, T) [10].

Theorem 2.1[12]. *If λ is a fuzzy σ-boundary set in a fuzzy topological space (X, T), then λ is a fuzzy F_σ-set in (X, T).*

Theorem 2.2[18]. *If η is a fuzzy G_δ-set in a weak fuzzy Oz-space (X, T), then $\operatorname{int}(\eta)$ is a fuzzy F_σ-set in (X, T).*

Theorem 2.3[16]. *If $\lambda \leq 1 - \mu$ for any two fuzzy F_σ-sets λ and μ in a fuzzy F'-space (X, T), then*

(i) $\lambda + cl(\mu) \leq 1$, *in* (X, T).

(ii) $\mu + cl(\lambda) \leq 1$, *in* (X, T).

Theorem 2.4[11]. *If* λ *is a fuzzy residual set in a fuzzy submaximal space* (X, T), *then* λ *is a fuzzy* G_δ*-set in* (X, T).

Theorem 2.5[15]. *If* λ *is a fuzzy first category set in a fuzzy globally disconnected space* (X, T), *then* λ *is a fuzzy* F_σ*-set in* (X, T).

Theorem 2.6[17]. *If* λ *is a fuzzy somewhere dense set in a fuzzy topological space* (X, T), *then there exist a fuzzy regular closed set* η *in* (X, T) *such that* $\eta \leq cl(\lambda)$.

Theorem 2.7[13]. *If* λ *is a fuzzy residual set in a fuzzy topological space* (X, T), *then there exists a fuzzy* G_δ*-set* μ *in* (X, T) *such that* $\mu \leq \lambda$.

Theorem 2.8[9]. *Let* (X, T) *be a fuzzy topological space. Then the following are equivalent*:

(1) (X, T) *is a fuzzy Baire space*

(2) $Int(\lambda) = 0$, *for every fuzzy first category set* λ *in* (X, T)

(3) $Cl(\mu) = 1$, *for every fuzzy residual set* μ *in* (X, T).

Theorem 2.9[9]. *If the fuzzy topological space* (X, T) *is a fuzzy Baire space, then* (X, T) *is a fuzzy second category space*.

Theorem 2.10[10]. *If* λ *is a fuzzy* F_σ*-set in a fuzzy almost P-space* (X, T), *then* $cl(\lambda) \neq 1$, *in* (X, T).

3. Fuzzy F_σ-Complemented Spaces

Motivated by the works of *M.* Henriksen and R. G. Woods [5] on cozero complemented spaces in classical topology, the notion of fuzzy F_σ- complemented space is defined as follows:

Definition 3.1. A fuzzy topological space (X, T) is called a fuzzy F_σ- complemented space if for each fuzzy F_σ-set λ in (X, T), there exists a fuzzy F_σ-set μ in (X, T) such that $\lambda \leq 1 - \mu$ and $cl(\lambda \vee \mu) = 1$.

Example 3.1. Let $X = \{a, b, c, d\}$. Let $I = [0, 1]$. The fuzzy sets α, β and γ are defined on X as follows:

$\alpha : X \to I$ is defined by $\alpha(a) = 0.8$, $\alpha(b) = 0.7$, $\alpha(c) = 0.6$, $\alpha(d) = 0.9$,

$\beta : X \to I$ is defined by $\beta(a) = 0.7$, $\beta(b) = 0.8$, $\beta(c) = 0.9$, $\beta(d) = 0.6$,

$\gamma : X \to I$ is defined by $\gamma(a) = 0.6$, $\gamma(b) = 0.9$, $\gamma(c) = 0.8$, $\gamma(d) = 0.7$,

$\delta : X \to I$ is defined by $\delta(a) = 0.4$, $\delta(b) = 0.3$, $\delta(c) = 0.4$, $\delta(d) = 0.4$.

Then, $T = \{0, \alpha, \beta, \gamma, \alpha \vee \beta, \alpha \vee \gamma, \beta \vee \gamma, \alpha \wedge \beta, \alpha \wedge \gamma, \beta \wedge \gamma, \alpha \vee [\beta \wedge \gamma], \beta \vee [\alpha \wedge \gamma], \gamma \vee [\alpha \wedge \beta], \alpha \wedge [\beta \vee \gamma], \beta \wedge [\alpha \vee \gamma], \gamma \wedge [\alpha \vee \beta], \alpha \vee \beta \vee \gamma, 1\}$ is a fuzzy topology on X. By computation one can find that the fuzzy F_σ-sets in (X, T) are $\delta, 1 - (\alpha \wedge \beta), 1 - \alpha$ and $1 - (\beta \wedge [\alpha \vee \gamma])$.

Now $1 - (\alpha \wedge \beta) \leq 1 - \delta; cl\{[1 - (\alpha \wedge \beta)] \vee \delta\} = cl(\delta) = 1$, $(1 - \alpha) \leq 1 - \delta$; $cl\{(1 - \alpha) \vee \delta\} = cl(\delta) = 1$ and $1 - (\beta \wedge [\alpha \vee \gamma]) \leq 1 - \delta; cl\{(\beta \wedge [\alpha \vee \gamma] \vee \delta)\} = cl(\delta) = 1$, in (X, T). Hence (X, T) is a fuzzy F_σ- complemented space.

Example 3.2. Let $X = \{a, b, c\}$. Let $I = [0, 1]$. The fuzzy sets α, β and γ are defined on X as follows:

$\alpha : X \to I$ is defined by $\alpha(a) = 0.8$; $\alpha(b) = 0.7$; $\alpha(c) = 0.5$,

$\beta : X \to I$ is defined by $\beta(a) = 0.6$; $\beta(b) = 0.5$; $\beta(c) = 0.7$

$\gamma : X \to I$ is defined by $\gamma(a) = 0.7$; $\gamma(b) = 0.6$; $\gamma(c) = 0.8$.

Then, $T = \{0, \alpha, \beta, \gamma, \alpha \vee \beta, \beta \vee \gamma, \alpha \vee \gamma, \alpha \wedge \beta, \alpha \wedge \gamma, \gamma \wedge [\alpha \vee \beta], 1\}$ is a fuzzy topology on X. By computation one can find that the fuzzy F_σ-sets in (X, T), are $1 - (\alpha \wedge \beta), 1 - (\alpha \wedge \gamma)$ and $1 - \alpha$.

Now $1 - (\alpha \wedge \gamma) \leq 1 - [1 - (\alpha \wedge \beta)]; [1 - \alpha] \leq 1 - [1 - (\alpha \wedge \beta)]$, in (X, T). But $cl\{[1 - (\alpha \wedge \gamma)] \vee (1 - ((\alpha \wedge \beta)))\} = 1 - (\alpha \wedge \beta) \neq 1$, implies that (X, T) is not a fuzzy F_σ-complemented space.

Proposition 3.1. *If* δ *is a fuzzy* G_δ*-set in a fuzzy* F_σ*-complemented space* (X, T), *then there exists a fuzzy* G_δ*-set* η *in* (X, T) *such that* $1 - \eta \leq \delta$ *and* $\operatorname{int}(\delta \wedge \eta) = 0$.

Proof. Let δ be a fuzzy G_δ-set in (X, T). Then, $1-\delta$ is a fuzzy F_σ-set in (X, T). Since (X, T) is a fuzzy F_σ-complemented space, for the fuzzy F_σ-set $1-\delta$, there exists a fuzzy F_σ-set μ in (X, T) such that $1-\delta \leq 1-\mu$ and $cl([1-\delta] \vee \mu) = 1$. This implies that $\mu \leq \delta$ and $1 - cl([1-\delta] \vee \mu) = 0$. Then, $1-[1-\mu] \leq \delta$ and $\operatorname{int}(1-([1-\delta] \vee \mu)) = 0$. This implies that $\operatorname{int}(\delta \wedge [1-\mu]) = 0$, in (X, T). Let $\eta = 1-\mu$. Then, η is a fuzzy G_δ-set in (X, T). Thus, for the fuzzy G_δ-set δ in (X, T), there exists a fuzzy G_δ-set η in (X, T) such that $1-\eta \leq \delta$ and $\operatorname{int}(\delta \wedge \eta) = 0$.

Corollary 3.1. *If δ is a fuzzy G_δ-set in a fuzzy F_σ-complemented space (X, T), then there exists a fuzzy G_δ-set η in (X, T) such that $1-\eta \leq \delta$ and $\operatorname{int}(\eta) \leq 1-\operatorname{int}(\delta)$, in (X, T).*

Proof. Let δ be a fuzzy G_δ-set in (X, T). Since (X, T) is a fuzzy F_σ-complemented space, by Proposition 3.1, there exists a fuzzy G_δ-set η in (X, T) such that $1-\eta \leq \delta$ and $\operatorname{int}(\delta \wedge \eta) = 0$. Now $\operatorname{int}(\delta \wedge \eta) = 0$, implies that $\operatorname{int}(\delta) \wedge \operatorname{int}(\eta) = 0$ and then $\operatorname{int}(\eta) \leq 1-\operatorname{int}(\delta)$, in (X, T).

Corollary 3.2. *If δ is a fuzzy G_δ-set in a fuzzy F_σ-complemented space (X, T), then there exists a fuzzy F_σ-set θ in (X, T) such that $\theta \leq \delta$ and $\operatorname{int}(\delta) \leq cl(\theta)$.*

Proof. Let δ be a fuzzy G_δ-set in (X, T). Since (X, T) is a fuzzy F_σ-complemented space, by Corollary 3.1, there exists a fuzzy G_δ-set η in (X, T) such that $1-\eta \leq \delta$ and $\operatorname{int}(\eta) \leq 1-\operatorname{int}(\delta)$. This implies that $\operatorname{int}(\delta) \leq 1-\operatorname{int}(\eta) = cl(1-\eta)$. Let $\theta = 1-\eta$ and then θ is a fuzzy F_σ-set in (X, T). Hence, for the fuzzy G_δ-set δ, there exists a fuzzy F_σ-set θ in (X, T) $\theta \leq \delta$ and $\operatorname{int}(\delta) \leq cl(\theta)$.

Proposition 3.2. *If λ is a fuzzy F_σ-set in a fuzzy F_σ-complemented space (X, T), then $cl(\lambda) \vee cl(1-\lambda) = 1$, in (X, T).*

Proof. Let λ be a fuzzy F_σ-set in (X, T). Since (X, T) is a fuzzy F_σ-complemented space, there exists a fuzzy F_σ-set μ in (X, T) such that

$\lambda \leq 1 - \mu$ and $cl(\lambda \vee \mu) = 1$. Now $\lambda \leq 1 - \mu$, implies that $\mu \leq 1 - \lambda$ and $\lambda \vee \mu \leq \lambda \vee (1 - \lambda)$. This implies that $cl(\lambda \vee \mu) \leq cl(\lambda \vee (1 - \lambda))$, in (X, T). Then, $cl(\lambda \vee \mu) \leq cl(\lambda) \vee cl(1 - \lambda)$ and $1 \leq cl(\lambda) \vee cl(1 - \lambda)$. That is, $cl(\lambda) \vee cl(1 - \lambda) = 1$, in (X, T).

Corollary 3.3. *If* δ *is a fuzzy* G_δ*-set in a fuzzy* F_σ*-complemented space* (X, T), *then* $cl(\delta) \vee cl(1 - \delta) = 1$, *in* (X, T).

Proof. Let δ be a fuzzy G_δ-set in (X, T). Then, $1 - \delta$ is a fuzzy F_σ-set in (X, T). Since (X, T) is a fuzzy F_σ-complemented space, by Proposition 3.2, then $cl(1 - \delta) \vee cl(1 - [1 - \delta]) = 1$, in (X, T). Thus, $cl(\delta) \vee cl(1 - \delta) = 1$, in (X, T).

Proposition 3.3. *If* λ *is a fuzzy* F_σ*-set in a fuzzy* F_σ*-complemented space* (X, T), *then* $\text{int}(1 - \lambda) \leq 1 - \text{int}(\lambda)$, *in* (X, T).

Proof. Let λ be a fuzzy F_σ-set in (X, T). Since (X, T) is a fuzzy F_σ-complemented space, by Proposition 3.2, $cl(\lambda) \vee cl(1 - \lambda) = 1$, in (X, T). This implies that $1 - [cl(\lambda) \vee cl(1 - \lambda)] = 0$ and $[1 - cl(\lambda)] \wedge [1 - cl(1 - \lambda)] = 0$, in (X, T). Thus $[1 - cl(\lambda)] \wedge [1 - (1 - \text{int}(\lambda))] = 0$ and $\text{int}(1 - \lambda) \wedge \text{int}(\lambda) = 0$. Hence $\text{int}(1 - \lambda) \leq 1 - \text{int}(\lambda)$, in (X, T).

Proposition 3.4. *If* λ *is a fuzzy* σ*-boundary set in a fuzzy* F_σ*-complemented space* (X, T), *then* $cl(\lambda) \vee cl(1 - \lambda) = 1$, *in* (X, T).

Proof. Let λ be a fuzzy σ-boundary set in (X, T). Then, by Theorem 2.1, λ is a fuzzy F_σ-set in (X, T). Since (X, T) is a fuzzy F_σ-complemented space, by Proposition 3.2, for the fuzzy F_σ-set λ, $cl(\lambda) \vee cl(1 - \lambda) = 1$, in (X, T).

Corollary 3.4. *If* λ *is a fuzzy* σ*-boundary set in a fuzzy* F_σ*-complemented space* (X, T), *then* $\text{int}(1 - \lambda) \leq 1 - \text{int}(\lambda)$, *in* (X, T).

Proof. Let λ be a fuzzy σ-boundary set in (X, T). Then, by Proposition 3.4, $cl(\lambda) \vee cl(1 - \lambda) = 1$, in (X, T). This implies that

$1 - [cl(\lambda) \vee cl(1-\lambda)] = 0$ and $[1 - cl(\lambda)] \wedge [1 - cl(1-\lambda)] = 0$, in (X, T). Thus $[1 - cl(\lambda)] \wedge [1 - \text{int}(\lambda)] = 0$ and $\text{int}(1-\lambda) \wedge \text{int}(\lambda) = 0$. Hence $\text{int}(1-\lambda) \leq 1 - \text{int}(\lambda)$, in (X, T).

Corollary 3.5. *If λ is a fuzzy σ-boundary set in a fuzzy F_σ-complemented space (X, T), then $1 - cl(\lambda) \leq cl(1-\lambda)$ in (X, T).*

Proof. Let λ be a fuzzy σ-boundary set in (X, T). Then, by Corollary 3.4, $\text{int}(1-\lambda) \leq 1 - \text{int}(\lambda)$, in (X, T). This implies that $1 - [\text{int}(1-\lambda)] \geq 1 - [1 - \text{int}(\lambda)]$ and $1 - [1 - cl(\lambda)] \geq 1 - [cl(1-\lambda)]$. Then, $1 - [cl(1-\lambda)] \leq cl(\lambda)$, in (X, T). Hence it follows that $1 - cl(\lambda) \leq cl(1-\lambda)$, in (X, T).

Proposition 3.5. *If λ is a fuzzy F_σ-set in a fuzzy F_σ-complemented space (X, T), then there exists a fuzzy F_σ-set μ in (X, T) such that $\lambda \leq 1 - \mu$ and $1 - cl(\mu) \leq cl(\lambda)$, in (X, T).*

Proof. Let λ be a fuzzy F_σ-set in (X, T). Since (X, T) is a fuzzy F_σ-complemented space, there exists a fuzzy F_σ-set μ in (X, T) such that $\lambda \leq 1 - \mu$ and $cl(\lambda \vee \mu) = 1$. Then, $1 - cl(\lambda \vee \mu) = 0$ and by Lemma 2.1, $\text{int}[1 - (\lambda \vee \mu)] = 0$. This implies that $\text{int}[(1-\lambda) \wedge (1-\mu)] = 0$ and $\text{int}(1-\lambda) \wedge \text{int}(1-\mu) = 0$. Then, $\text{int}(1-\lambda) \leq 1 - \text{int}(1-\mu)$ and $\text{int}(1-\lambda) \leq 1 - (1 - cl(\mu))$. It follows that $1 - cl(\lambda) \leq cl(\mu)$. Hence, for the fuzzy F_σ-set λ, there exists a fuzzy F_σ-set μ in (X, T) such that $\lambda \leq 1 - \mu$ and $1 - cl(\mu) \leq cl(\lambda)$.

Proposition 3.6. *If λ is a fuzzy F_σ-set in a fuzzy F_σ-complemented space (X, T), then $\lambda \vee (1-\lambda)$ is a fuzzy dense set in (X, T).*

Proof. Let λ be a fuzzy F_σ-set in (X, T). Since (X, T) is a fuzzy F_σ-complemented space, by Proposition 3.2, $cl(\lambda) \vee cl(1-\lambda) = 1$, in (X, T). Now $cl[\lambda \vee (1-\lambda)] = cl(\lambda) \vee cl(1-\lambda) = 1$. Hence $\lambda \vee (1-\lambda)$ is a fuzzy dense set in (X, T).

The following propositions give conditions under which fuzzy topological spaces become fuzzy F_σ-complemented spaces.

Proposition 3.7. *If* $cl(\lambda)$ *is a fuzzy* G_δ*-set for each fuzzy* F_σ*-set in a fuzzy topological space* (X, T) *such that* $cl(\lambda) \vee cl[1 - cl(\lambda)] = 1$, *then* (X, T) *is a fuzzy* F_σ*-complemented space.*

Proof. Let λ be a fuzzy F_σ-set in (X, T). By hypothesis, $cl(\lambda)$ is a fuzzy G_δ-set in (X, T) and then $1 - cl(\lambda)$ is a fuzzy F_σ-set in (X, T). Now $\lambda \leq cl(\lambda)$, implies that $\lambda \leq 1 - [1 - cl(\lambda)]$ in (X, T). Also by hypothesis, $cl(\lambda) \vee cl[1 - cl(\lambda)] = 1$ and then $cl(\lambda \vee [1 - cl(\lambda)]) = cl(\lambda) \vee cl[1 - cl(\lambda)] = 1$. Thus, for the fuzzy F_σ-set λ, there exists a fuzzy F_σ-set $1 - cl(\lambda)$ in (X, T) such that $\lambda \leq 1 - [1 - cl(\lambda)]$ and $cl(\lambda \vee [1 - cl(\lambda)]) = 1$. Hence (X, T) is a fuzzy F_σ-complemented space.

Proposition 3.8. *If* $cl(\lambda)$ *is a fuzzy* G_δ*-set such that* $\operatorname{int} cl(\lambda) = 0$, *for each fuzzy* F_σ*-set in a fuzzy topological space* (X, T), *then* (X, T) *is a fuzzy* F_σ*-complemented space.*

Proof. Let λ be a fuzzy F_σ-set in (X, T). By hypothesis, $cl(\lambda)$ is a fuzzy G_δ-set in (X, T) and $\operatorname{int} cl(\lambda) = 0$, in (X, T). Now $1 - \operatorname{int} cl(\lambda) = 1 - 0 = 1$ and then $cl[1 - cl(\lambda)] = 1$, in (X, T). Now $cl(\lambda) \vee cl[1 - cl(\lambda)] = cl(\lambda) \vee 1 = 1$. Thus, $cl(\lambda)$ is a fuzzy G_δ-set for a fuzzy F_σ-set in the fuzzy topological space (X, T) such that $cl(\lambda) \vee cl[1 - cl(\lambda)] = 1$. Hence, by Proposition 3.7, (X, T) is a fuzzy F_σ-complemented space.

Proposition 3.9. *If* (X, T) *is a topological space in which fuzzy* F_σ*-sets are fuzzy dense and fuzzy disjoint, then* (X, T) *is a fuzzy* F_σ*-complemented space.*

Proof. Let λ and μ be any two fuzzy F_σ-sets in (X, T). Then, by hypothesis, $\lambda \wedge \mu = 0$. This implies that $\lambda \leq 1 - \mu$. Now $cl(\lambda \vee \mu)$ $= cl(\lambda) \vee cl(\mu) = 1 \vee 1 = 1$. Hence, for the fuzzy F_σ-set λ, there exists a fuzzy F_σ-set μ in (X, T) such that $\lambda \leq 1 - \mu$ and $cl(\lambda \vee \mu) = 1$, implies that (X, T) is a fuzzy F_σ-complemented space.

Proposition 3.10. *If* λ *is a fuzzy residual set in a fuzzy* F_σ*-complemented space* (X, T), *then there exists a fuzzy* F_σ*-set* μ *in* (X, T) *such that* $\mu \leq \lambda$ *and* $\text{int}(1-\lambda) \leq 1 - \text{int}(\mu)$.

Proof. Let λ be a fuzzy residual set in (X, T). Then, by Theorem 2.7, there exists a fuzzy G_δ-set η in (X, T) such that $\eta \leq \lambda$. This implies that $1 - \lambda \leq 1 - \eta \ldots$ (A) and $1 - \eta$ is a fuzzy F_σ-set in (X, T). Since (X, T) is a fuzzy F_σ-complemented space, for the fuzzy F_σ-set $1 - \eta$, there exists a fuzzy F_σ-set μ in (X, T) such that $1 - \eta \leq 1 - \mu \ldots$ (B) and $cl([1-\eta] \vee \mu) = 1$. From (A) and (B), $1 - \lambda \leq 1 - \eta \leq 1 - \mu$ and then $1 - \lambda \leq 1 - \mu$, in (X, T). This implies that $\mu \leq \lambda$, in (X, T).

Now $1 - \eta \leq 1 - \mu$ implies that $[1-\eta] \vee \mu \leq [1-\mu] \vee \mu$ and then $cl([1-\eta] \vee \mu) \leq cl([1-\mu] \vee \mu)$. Thus, $1 \leq cl([1-\mu] \vee \mu) \leq cl([1-\mu] \vee \lambda)$. That is, $cl([1-\mu] \vee \lambda) = 1$, in (X, T). Then, $1 - cl([1-\mu] \vee \lambda) = 0$ and it follows that $\text{int}[1 - ([1-\mu]) \vee \lambda] = 0$ and $\text{int}[(1 - ([1-\mu]) \wedge (1-\lambda))] = 0$, in (X, T). This implies that $\text{int}[\mu \wedge (1-\lambda)] = 0$ and then $\text{int}(\mu) \wedge \text{int}(1-\lambda) = 0$. This implies that $\text{int}(1-\lambda) \leq 1 - \text{int}(\mu)$, in (X, T).

Proposition 3.11. *If* λ *is a fuzzy first category set in a fuzzy* F_σ*-complemented space* (X, T), *then there exists a fuzzy* F_σ*-set* δ *in* (X, T) *such that* $\lambda \leq 1 - \delta$ *and* $\text{int}(\lambda) \leq 1 - \text{int}(\delta)$.

Proof. Let λ be a fuzzy first category set in (X, T). Then, $1 - \lambda$ is a fuzzy residual set in a fuzzy F_σ-complemented space (X, T) and by Proposition 3.10, there exists a fuzzy F_σ-set δ in (X, T) such that $\delta \leq 1 - \lambda$ and $\text{int}(1 - [1-\lambda]) \leq 1 - \text{int}(\delta)$. This implies that $\lambda \leq 1 - \delta$ and $\text{int}(\lambda) \leq 1 - \text{int}(\delta)$.

4. Fuzzy F_σ-Complemented Spaces and Other Fuzzy Topological Spaces

The following proposition gives a condition under which weak fuzzy Oz-spaces become fuzzy F_σ-complemented spaces.

Proposition 4.1. *If* $cl(\theta) \vee cl[1 - cl(\theta)] = 1$, *for each fuzzy* F_σ*-set* θ *in a weak fuzzy Oz-space* (X, T), *then* (X, T) *is a fuzzy* F_σ*-complemented space.*

Proof. Let θ be a fuzzy F_σ-set in (X, T). Since (X, T) is a weak fuzzy Oz-space, $cl(\theta)$ is a fuzzy G_δ-set in (X, T). Thus $cl(\theta)$ is a fuzzy G_δ-set for the fuzzy F_σ-set θ in (X, T) such that $cl(\theta) \vee cl[1 - cl(\theta)] = 1$. Then, by proposition 3.7, (X, T) is a fuzzy F_σ-complemented space.

The following proposition gives a condition under which weak fuzzy Oz and fuzzy hyperconnected spaces become fuzzy F_σ-complemented spaces.

Proposition 4.2. *If each fuzzy* F_σ*-set is a fuzzy open set in a weak fuzzy Oz and fuzzy hyperconnected space* (X, T), *then* (X, T) *is a fuzzy* F_σ*-complemented space.*

Proof. Let λ be a fuzzy F_σ-set in (X, T). Then, $\lambda \le cl(\lambda)$ implies that $\lambda \le 1 - (1 - cl(\lambda))$. Since (X, T) is a weak fuzzy Oz-space, $cl(\lambda)$ is a fuzzy G_δ-set and then $1 - cl(\lambda)$ is a fuzzy F_σ-set in (X, T). Let $\mu = 1 - cl(\lambda)$. Thus, for the fuzzy F_σ-set λ, there exists a fuzzy F_σ-set μ in (X, T) such that $\lambda \le 1 - \mu$. By hypothesis, λ and μ are fuzzy open sets in (X, T). Since (X, T) is a fuzzy hyperconnected space, $cl(\lambda) = 1$ and $cl(\mu) = 1$ and then $cl(\lambda \vee \mu) = cl(\lambda) \vee cl(\mu) = 1 \vee 1 = 1$, in (X, T). Thus, for the fuzzy F_σ-set λ, there exists a fuzzy F_σ-set μ in (X, T) such that $\lambda \le 1 - \mu$ and $cl(\lambda \vee \mu) = 1$. Hence (X, T) is a fuzzy F_σ-complemented space.

The following proposition gives a condition for fuzzy hyperconnected spaces to become fuzzy F_σ-complemented spaces.

Proposition 4.3. *If fuzzy* F_σ*-sets are fuzzy open and fuzzy disjoint in a fuzzy hyperconnected space* (X, T), *then* (X, T) *is a fuzzy* F_σ*-complemented space.*

Proof. Let λ and μ be any two fuzzy F_σ-sets in (X, T). By hypothesis, $\lambda \wedge \mu = 0$, and then $\lambda \le 1 - \mu$. Also by hypothesis, λ and μ are fuzzy open sets, in (X, T). Since (X, T) is a fuzzy hyperconnected space, $cl(\lambda) = 1$ and $cl(\mu) = 1$ and then $cl(\lambda \vee \mu) = cl(\lambda) \vee cl(\mu) = 1 \vee 1 = 1$, in (X, T). Thus, for the fuzzy F_σ-sets λ and μ in (X, T) with $\lambda \leq 1 - \mu, cl(\lambda \vee \mu) = 1,$ implies that (X, T) is a fuzzy F_σ-complemented space.

The following proposition shows that fuzzy F'-spaces are not fuzzy F_σ- complemented spaces.

Proposition 4.4. *If a fuzzy topological space* (X, T) *is a fuzzy* F'*-space, then* (X, T) *is not a fuzzy* F_σ*-complemented space.*

Proof. Suppose that $\lambda \leq 1 - \mu,$ where λ and μ are fuzzy F_σ-sets in (X, T). Since (X, T) is a fuzzy F'-space, by Theorem 2.3, $\lambda + cl(\mu) \leq 1$ and $\mu + cl(\lambda) \leq 1,$ in (X, T). Then, $cl(\mu) \leq 1 - \lambda$ and $cl(\lambda) \leq 1 - \mu,$ and $cl[cl(\mu)] \leq cl[1 - \lambda]$ and $cl[cl(\lambda)] \leq cl[1 - \mu],$ in (X, T). This implies that $cl(\mu) \leq 1 - \text{int}(\lambda)$ and $cl(\lambda) \leq 1 - \text{int}(\mu),$ in (X, T). Now $cl(\lambda \vee \mu)$ $= cl(\lambda) \vee cl(\mu) \leq [1 - \text{int}(\mu)] \vee [1 - \text{int}(\lambda)] = 1 - [\text{int}(\lambda) \wedge \text{int}(\mu)] = 1 - \text{int}(\lambda \wedge \mu).$ That is, $cl(\lambda \vee \mu) \leq 1 - \text{int}(\lambda \wedge \mu),$ in (X, T). This implies that $cl(\lambda \vee \mu) \neq 1,$ in (X, T). Thus, for the fuzzy F_σ-sets with $\lambda \leq 1 - \mu, cl(\lambda \vee \mu) \neq 1,$ in (X, T) shows that (X, T) is not a fuzzy F_σ-complemented space.

Proposition 4.5. *If* λ *is a fuzzy first category set in a fuzzy submaximal and fuzzy* F_σ*-complemented space* (X, T), *then there exists a fuzzy* F_σ*-set* μ *in* (X, T) *such that* $\lambda \leq 1 - \mu$ *and* $\text{int}(1 - \mu) \leq 1 - \text{int}(\mu)$, *in* (X, T).

Proof. Let λ be a fuzzy first category set in (X, T). Then, $1 - \lambda$ is a fuzzy residual set in (X, T). Since (X, T) is a fuzzy submaximal space, by Theorem 2.4, $1 - \lambda$ is a fuzzy G_δ-set in (X, T) and thus λ is a fuzzy F_σ-set in (X, T). Also since (X, T) is a fuzzy F_σ-complemented space, for the fuzzy F_σ-set λ, there exists a fuzzy F_σ-set μ in (X, T) such that $\lambda \leq 1 - \mu$ and $cl(\lambda \vee \mu) = 1.$ Now $cl(\lambda \vee \mu) = cl(\lambda) \vee cl(\mu) \leq cl(1 - \mu) \vee cl(\mu).$ Thus $1 \leq cl(1 - \mu)$ $\vee cl(\mu).$ That is, $cl(1 - \mu) \vee cl(\mu) = 1.$ This implies that $1 - [cl(\mu) \vee cl(1 - \mu)] = 0$ and $[1 - cl(\mu)] \wedge [1 - cl(1 - \mu)] = 0,$ in (X, T). Thus $[1 - cl(\mu)]$ $\wedge [1 - (1 - \text{int}(\mu))] = 0$ and $\text{int}(1 - \mu) \wedge \text{int}(\mu) = 0.$ Hence $\text{int}(1 - \mu)$ $\leq 1 - \text{int}(\mu),$ in (X, T). Hence, for the fuzzy first category set λ, there exists a fuzzy F_σ-set μ in (X, T) such that $\lambda \leq 1 - \mu$ and $\text{int}(1 - \mu) \leq 1 - \text{int}(\mu).$

Corollary 4.1. *If* λ *is a fuzzy residual set in a fuzzy submaximal and fuzzy* F_σ*-complemented space* (X, T), *then there exists a fuzzy* F_σ*-set* μ *in* (X, T) *such that* $\text{int}(\mu) \leq cl(\lambda)$.

Proof. Let λ be a fuzzy residual set in (X, T). Then, $1 - \lambda$ is a fuzzy first category set in (X, T). By Proposition 4.5, there exists a fuzzy F_σ-set μ in (X, T) such that $1 - \lambda \leq 1 - \mu$ and $\text{int}(1 - \mu) \leq 1 - \text{int}(\mu)$, in (X, T). Then, $\mu \leq \lambda$ and $\text{int}(1 - \lambda) \leq \text{int}(1 - \mu) \leq 1 - \text{int}(\mu)$. This implies that $1 - cl(\lambda) \leq 1 - \text{int}(\mu)$ and then $\text{int}(\mu) \leq cl(\lambda)$. Hence, for the fuzzy residual set λ, there exists a fuzzy F_σ-set μ in (X, T) such that $\text{int}(\mu) \leq cl(\lambda)$.

The following corollary shows that fuzzy residual sets in fuzzy submaximal and fuzzy F_σ-complemented spaces are not fuzzy nowhere dense sets.

Corollary 4.2. *If* λ *is a fuzzy residual set in a fuzzy submaximal and fuzzy* F_σ*-complemented space* (X, T), *then* λ *is not a fuzzy nowhere dense set in* (X, T).

Proof. Let λ be a fuzzy residual set in (X, T). Then, by Corollary 4.1, there exists a fuzzy F_σ-set μ in (X, T) such that $\text{int}(\mu) \leq cl(\lambda)$. Then, $\text{int}\, cl(\lambda) \neq 0$, in (X, T) and thus λ is not a fuzzy nowhere dense set in (X, T).

Proposition 4.6. *If* λ *is a fuzzy first category set in a fuzzy globally disconnected and fuzzy* F_σ*-complemented space* (X, T), *then there exists a fuzzy* F_σ*-set* μ *in* (X, T) *such that* $\lambda \leq 1 - \mu$ *and* $\text{int}(1 - \mu) \leq 1 - \text{int}(\mu)$, *in* (X, T).

Proof. Let λ be a fuzzy first category set in (X, T). Since (X, T) is a fuzzy globally disconnected space, by Theorem 2.5, λ is a fuzzy F_σ-set in (X, T). Also since (X, T) is a fuzzy F_σ-complemented space, for the fuzzy F_σ-set λ, there exists a fuzzy F_σ-set μ in (X, T) such that $\lambda \leq 1 - \mu$ and $cl(\lambda \vee \mu) = 1$. Now $cl(\lambda \vee \mu) = cl(\lambda) \vee cl(\mu) \leq cl(1 - \mu) \vee cl(\mu)$. Thus $1 \leq cl(1 - \mu) \vee cl(\mu)$. That is, $cl(1 - \mu) \vee cl(\mu) = 1$. This implies that

$1-[cl(\mu)\vee cl(1-\mu)]=0$ and $[1-cl(\mu)]\wedge[1-cl(1-\mu)]=0$, in (X, T). Thus $[1-cl(\mu)]\wedge[1-(1-\text{int}(\mu))]=0$ and $\text{int}(1-\mu)\wedge\text{int}(\mu)=0$. Hence $\text{int}(1-\mu)\le 1-\text{int}(\mu)$, in (X, T). Hence, for the fuzzy first category set λ in (X, T), there exists a fuzzy F_σ-set μ in (X, T) such that $\lambda\le 1-\mu$ and $\text{int}(1-\mu)\le 1-\text{int}(\mu)$.

The following corollary shows that fuzzy residual sets in fuzzy globally disconnected and fuzzy F_σ-complemented spaces are not fuzzy nowhere dense sets.

Corollary 4.3. *If λ is a fuzzy residual set in a fuzzy globally disconnected and fuzzy F_σ-complemented space (X, T) then λ is not a fuzzy nowhere dense set in (X, T).*

Proof. Let λ be a fuzzy residual set in (X, T). Then, $1-\lambda$ is a fuzzy first category set in (X, T). By Proposition 4.6, there exists a fuzzy F_σ-set μ in (X, T) such that $1-\lambda\le 1-\mu$ and $\text{int}(1-\mu)\le 1-\text{int}(\mu)$, in (X, T). Then, $\mu\le\lambda$ and $\text{int}(1-\lambda)\le\text{int}(1-\mu)\le 1-\text{int}(\mu)$. This implies that $1-cl(\lambda)\le 1-\text{int}(\mu)$ and then $\text{int}(\mu)\le cl(\lambda)$. Then, $\text{int}\,cl(\lambda)\ne 0$, in (X, T). Hence, λ is not a fuzzy nowhere dense set in (X, T).

Corollary 4.4. *If λ is a fuzzy residual set in a fuzzy globally disconnected and fuzzy F_σ-complemented space (X, T) then there exist a fuzzy regular closed set η in (X, T) such that $\eta\le cl(\lambda)$.*

Proof. Let λ be a fuzzy residual set in (X, T). Then, by Corollary 4.3, λ is not a fuzzy nowhere dense set in (X, T) and thus $\text{int}\,cl(\lambda)\ne 0$, in (X, T). This implies that λ is a fuzzy somewhere dense set in (X, T). Hence, by Theorem 2.6, there exist a fuzzy regular closed set η in (X, T) such that $\eta\le cl(\lambda)$.

Corollary 4.5. *If λ is a fuzzy residual set in a fuzzy submaximal and fuzzy F_σ-complemented space (X, T), then there exists a fuzzy regular closed set η in (X, T) such that $\eta\le cl(\lambda)$.*

Proof. Let λ be a fuzzy residual set in (X, T). By Corollary 4.2, λ is not a fuzzy nowhere dense set in (X, T) and then $\operatorname{int} cl(\lambda) \neq 0$, in (X, T). This implies that λ is a fuzzy somewhere dense set in (X, T). Hence, by Theorem 2.6, there exists a fuzzy regular closed set η in (X, T) such that $\eta \leq cl(\lambda)$.

Proposition 4.7. *If a fuzzy topological space (X, T) is a fuzzy F_σ-complemented space, then (X, T) is not a fuzzy perfectly disconnected space.*

Proof. Let λ be a fuzzy F_σ-set in (X, T). Since the fuzzy topological space (X, T) is a fuzzy F_σ-complemented space, by Proposition 3.5, there exists a fuzzy F_σ-set μ in (X, T) such that $\lambda \leq 1 - \mu$ and $1 - cl(\mu) \leq cl(\lambda)$, in (X, T). This implies that $cl(\lambda) \nleq 1 - cl(\mu)$, in (X, T) Thus, for the fuzzy sets λ and μ with $\lambda \leq 1 - \mu$, $cl(\lambda) \nleq 1 - cl(\mu)$ shows that (X, T) is not a fuzzy perfectly disconnected space.

Proposition 4.8. *If a fuzzy topological space (X, T) is a fuzzy F_σ-complemented space, then (X, T) is not a fuzzy F'-space.*

Proof. Let λ be a fuzzy F_σ-set in (X, T). Since the fuzzy topological space (X, T) is a fuzzy F_σ-complemented space, by Proposition 3.5, there exists a fuzzy F_σ-set μ in (X, T) such that $\lambda \leq 1 - \mu$ and $1 - cl(\mu) \leq cl(\lambda)$, in (X, T). This implies that $cl(\lambda) \nleq 1 - cl(\mu)$, in (X, T) Thus, for the fuzzy F_σ-set sets λ and μ with $\lambda \leq 1 - \mu$, $cl(\lambda) \nleq 1 - cl(\mu)$ shows that (X, T) is not a fuzzy F'-space.

Remark. From Propositions 4.4 and 4.8, one shall have the following result:

"Fuzzy F_σ-complemented spaces and fuzzy F'-spaces are independent notions."

The following proposition gives a condition for fuzzy F_σ-complemented spaces to become fuzzy Baire spaces.

Proposition 4.9. *If each fuzzy F_σ-set is a fuzzy dense set in a fuzzy F_σ-complemented space (X, T), then (X, T) is a fuzzy Baire space.*

Proof. Let λ be a fuzzy residual set in (X, T). Since the fuzzy topological space (X, T) is a fuzzy F_σ-complemented space, by Proposition 3.10, there exists a fuzzy F_σ-set μ in (X, T) such that $\mu \leq \lambda$. Then, $cl(\mu) \leq cl(\lambda)$. By hypothesis, $cl(\mu) = 1$, in (X, T). This implies that $1 \leq cl(\lambda)$. That is, $cl(\lambda) = 1$. Thus, for a fuzzy residual set λ, $cl(\lambda) = 1$, in (X, T). Hence, by Theorem 2.8, (X, T) is a fuzzy Baire space.

Corollary 4.6. *If each fuzzy F_σ-set is a fuzzy dense set in a fuzzy F_σ-complemented space (X, T), then (X, T) is a fuzzy second category space.*

Proof. The proof follows from Proposition 4.9 and Theorem 2.9.

Proposition 4. 10. *If a fuzzy topological space (X, T) is a fuzzy almost P-space, then (X, T) is not a fuzzy F_σ-complemented space.*

Proof. Suppose that (X, T) is a fuzzy F_σ-complemented space. Then, for each fuzzy F_σ-set λ in (X, T), there exists a fuzzy F_σ-set μ in (X, T) such that $\lambda \leq 1 - \mu$ and $cl(\lambda \vee \mu) = 1$. Now λ and μ are fuzzy F_σ-sets in (X, T) implies that $\lambda \vee \mu$ is a fuzzy F_σ-set in (X, T). Thus, there exists a fuzzy F_σ-set $\lambda \vee \mu$ in the fuzzy almost P-space (X, T) such that $cl(\lambda \vee \mu) = 1$. But this is a contradiction to Theorem 2.10. Hence the fuzzy almost *P*-space (X, T) is not a fuzzy F_σ-complemented space.

References

[1] F. Azarpanah and M. Karavan, On nonregular ideals and z^0-ideals in $C(X)$, Czechoslovak Mathematical Journal 55(2) (2005), 397-407.

[2] K. K. Azad, On fuzzy semi continuity, fuzzy almost continuity and fuzzy weakly continuity, J. Math. Anal. Appl. 82(1) (1981), 14-32.

[3] G. Balasubramanian, Maximal fuzzy topologies, Kybernetika 31(5) (1995), 459-464.

[4] C. L. Chang, Fuzzy topological spaces, J. Math. Anal. Appl. 24 (1968), 182-190.

[5] M. Henriksen and R. G. Woods, Cozero complemented spaces; when the space of minimal prime ideals of a $C(X)$ is compact, Top. Appl. 141(1-3) (2004), 147-170.

[6] R. Levy and J. Shapiro, Rings of quotients of rings of functions, Top. Appl. 146-147 (2005), 253-265.

[7] Miguel Caldas, Govindappa Navalagi and Ratnesh Saraf, On fuzzy weakly semi-open functions, Proyecciones, Universidad Catolica del Norte, Antofagasta-Chile 21(1) (2002), 51-63.

[8] G. Thangaraj and G. Balasubramanian, On somewhat fuzzy continuous functions, J. Fuzzy Math. 11(2) (2003), 725-736.

[9] G. Thangaraj and S. Anjalmose, On fuzzy baire spaces, J. Fuzzy Math. 21(3) (2013), 667-676.

[10] G. Thangaraj and C. Anbazhagan, On fuzzy almost P-spaces, Inter. J. Innov. Sci., Engin. and Tech. 2(4) (2015), 389-407.

[11] G. Thangaraj and C. Anbazhagan, Some remarks on fuzzy P-spaces, Gen. Math. Notes 26(1) (2015), 8-16.

[12] G. Thangaraj and R. Palani, On fuzzy weakly Baire spaces, Bull. Inter. Math. Virt. Inst. 7 (2017), 479-489.

[13] G. Thangaraj and R. Palani, Somewhat fuzzy continuity and fuzzy Baire spaces, Ann. Fuzzy Math. Inform. 12(1) (2016), 75-82.

[14] G. Thangaraj and S. Muruganantham, On fuzzy perfectly disconnected spaces, Int. J. Adv. Math. 2017(5) (2017), 12-21.

[15] G. Thangaraj and S. Muruganantham, On fuzzy globally disconnected spaces, Jour. Tri. Math. Soc. 21 (2019), 37-46.

[16] G. Thangaraj and S. Muruganantham, On fuzzy F'-spaces, J. Phys. Sci. 24 (2019), 97-106.

[17] G. Thangaraj and S. Senthil, On somewhere fuzzy continuous functions, Ann. Fuzzy Math. Inform. 15(2) (2018), 181-198.

[18] G. Thangaraj and M. Ponnusamy, On fuzzy Oz-spaces, Advances and Applications in Mathematical Sciences 22(5) (2023), 939-954.

[19] L. A. Zadeh, Fuzzy sets, Inform. and Control. 8 (1965), 338-353.