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DECOMPOSITION OF INTUITIONISTIC FINE CLOSED SETS IN INTUITIONISTIC FINE TOPOLOGICAL SPACES

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Abstract

A collection of new sets called intuitionistic fine generalized locally closed sets and a collection of new functions namely intuitionistic fine generalised locally continuous maps and intuitionistic fine generalized locally irresolute maps are introduced in intuitionistic fine topological space. Further few of their properties are discussed. Also the properties of intuitionistic fine generalized locally closed sets and functions in intuitionistic fine topological spaces are illustrated.

1. Introduction

Fundamental idea of intuitionistic sets, intuitionistic fuzzy sets and ITS were brought forth and developed by D. Coker [3] [4]. P. L. Powar et al. [10] introduced fine sets, that forms the fine topological space. The authors [11] [12] introduced intuitionistic fine space and intuitionistic fine g-closed set. M Gansterand et al. [7] introduced notions of locally closed sets in ITS and generalized locally closed sets in intuitionistic fuzzy topological spaces were

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introduced and their properties were discussed by K. Balachandran [2]. Different types of generalised locally closed sets namely α-weakly generalised locally closed sets by R. K Moorthy [9], feebly generalized locally closed sets in bi-topological spaces by S. V. Vani [13], regular generalized locally closed sets by I. Arokiarani [1] were investigated and studied. Authors in their papers [7] [2] [1] introduced the concept of LC-continuous functions, GLC-continuous functions in different spaces.

This article introduces intuitionistic fine generalized locally closed sets in intuitionistic fine topological spaces briefly $I_f GLCS$, $I_f GLCS^*$, $I_f GLCS^{**}$ and illustrates their properties. Further intuitionistic fine generalised locally continuous and irresolute functions, briefly $I_f GL$ continuous, $I_f GL^*$ continuous, $I_f GL^{**}$ continuous, $I_f GL$ irresolute, $I_f GL^*$ irresolute, $I_f GL^{**}$ irresolute are introduced and their behaviors are investigated.

2. Preliminaries

Definition 2.1[3]**.** Suppose X is a non empty set, an intuitionistic set (IS) C is an object of the form $C = \langle X, C_1, C_2 \rangle$, where C_1 and C_2 are subsets of X satisfying $C_1 \cap C_2 = \phi \cdot C_1$ is said to be the set of members of C, while C_2 is said to be set of non-members of C.

Definition 2.2[4]**.** An intuitionistic topology (IT) on a non empty set X is a family τ of IS's in X satisfying below:

(1) $\underset{\sim}{X}, \underset{\sim}{\phi}$ belongs to τ.

(2) $C_1 \cap C_2 \in \tau$ for every $C_1, C_2 \in \tau$.

(3) $\cup C_j \in \tau$ for any family $\{C_j : j \in K\} \subseteq \tau$.

(X, τ) is known as an intuitionistic topological space (ITS) and $IS \subseteq \tau$ is called an intuitionistic open set $(IOS) \subseteq X$. Intuitionistic closed set (ICS) is complement of IOS.

Definition 2.3[3]**.** Suppose (X, τ) is an ITS, let $C = \langle X, C_1, C_2 \rangle$ an IS in X. The definitions of Interior and closure of C are.

$Icl(C) = \bigcap\{J : J \text{ is an ICS in } X, C \subseteq J\}$.

$I\,\text{int}(C) = \bigcup\{J : J \text{ is an ICS in } X, C \supseteq J\}$.

Definition 2.4[10]**.** Suppose (X, τ) is a TS, we define $\tau(C_\beta) = \tau_\beta(say)$ $= \{Z_\beta(\neq X) : Z_\beta \cap C_\beta \neq \phi$, for $C_\beta \in \tau$ and $C_\beta \neq \phi, X$ for some $\beta \in K$, where K is the index set$\}$. We define $\tau_f = \{\phi, X\} \cup \{\tau_\beta\}$. Collection, τ_f of subsets of X is said to be fine collection of subsets of X and (X, τ, τ_f) is called the fine space X generated by τ on X. A subset U of a fine space X is said to be fine open sets if $U \in \tau_f$. Fine closed sets denoted by F_f^c are complements of fine open sets.

Definition 2.5[11]**.** Suppose (X, τ) is an ITS, we define $\tau(C_\beta) = \hat{\tau}_\beta(say)$ $= \{Z_\beta(\neq \underset{\sim}{X}) : Z_\beta \cap C_\beta \neq \underset{\sim}{\phi}$, for $C_\beta \in \tau$ and $C_\beta \neq \underset{\sim}{\phi}, \underset{\sim}{X}$ for some $\beta \in K$ where K is the index set$\}$. We define $\hat{\tau}_f = \{\underset{\sim}{\phi}, \underset{\sim}{X}\} \cup \{\hat{\tau}_\beta\}$. Collection, $\hat{\tau}_f$ of subsets of X is called an intuitionistic fine collection of subsets of X and $(X, \tau, \hat{\tau}_f)$ is called an intuitionistic fine space I_fTS generated by τ on X. A subset U of an I_fTS on X is said to be intuitionistic fine open sets if $U \in \hat{\tau}_f$. Intuitionistic fine closed sets (I_fTS) are complements of intuitionistic fine open sets (I_fOS).

Remark 2.6[11]**.**

(1) Let $(X, \tau, \hat{\tau}_f)$ be a I_fTS then the arbitrary union of I_fOS in X is I_fOS in X.

(2) Let $(X, \tau, \hat{\tau}_f)$ be a I_fTS then intersection of two union of I_fOS in X need not be I_fOS in X.

Example 2.7. Suppose $X = \{p, q, r\}$ and $\tau = \{\underset{\sim}{X}, \underset{\sim}{\phi}, C_1, C_2\}$ where $C_1 = \langle X, \{r\}, \{p, q\}\rangle$ and $C_1 = \langle X, \{r\}, \{p\}\rangle$.

Let $C_\beta = C_1$ and C_2.

$\tau(C_\beta) = \hat{\tau}_\beta = \langle X, \{p\}, \{\phi\}\rangle$,

$\langle X, \{q\}, \{\phi\}\rangle, \langle X, \{r\}, \{\phi\}\rangle, \langle X, \{\phi\}, \{p\}\rangle$

$\langle X, \{\phi\}, \{q\}\rangle, \langle X, \{\phi\}, \{r\}\rangle, \langle X, \{p\}, \{q\}\rangle$

$\langle X, \{q\}, \{r\}\rangle, \langle X, \{r\}, \{p\}\rangle, \langle X, \{q\}, \{p\}\rangle$

$\langle X, \{r\}, \{q\}\rangle, \langle X, \{p\}, \{r\}\rangle, \langle X, \{p, q\}, \{\phi\}\rangle$

$\langle X, \{q, r\}, \{\phi\}\rangle, \langle X, \{p, r\}, \{\phi\}\rangle, \langle X, \{\phi\}, \{p, q\}\rangle$

$\langle X, \{\phi\}, \{p, r\}\rangle, \langle X, \{q\}, \{p, r\}\rangle,$

$\langle X, \{r\}, \{p, q\}\rangle, \langle X, \{q, r\}, \{p\}\rangle$

$\langle X, \{p, r\}, \{q\}\rangle, \langle X, \{p, q\}, \{r\}\rangle, \langle X, \{\phi\}, \{\phi\}\rangle.$

$\hat{\tau}_f = \{\underset{\sim}{X}, \underset{\sim}{\phi}\} \cup \{\hat{\tau}_\beta\}.$

Hence $\hat{\tau}_f = I_f OS = \{X, \phi, \langle X, \{p\}, \{\phi\}\rangle\},$

$\langle X, \{q\}, \{\phi\}\rangle, \langle X, \{r\}, \{\phi\}\rangle, \langle X, \{\phi\}, \{p\}\rangle$

$\langle X, \{\phi\}, \{q\}\rangle, \langle X, \{\phi\}, \{r\}\rangle, \langle X, \{p\}, \{q\}\rangle$

$\langle X, \{q\}, \{r\}\rangle, \langle X, \{r\}, \{p\}\rangle, \langle X, \{q\}, \{p\}\rangle$

$\langle X, \{r\}, \{q\}\rangle, \langle X, \{p\}, \{r\}\rangle, \langle X, \{p, q\}, \{\phi\}\rangle$

$\langle X, \{q, r\}, \{\phi\}\rangle, \langle X, \{p, r\}, \{\phi\}\rangle, \langle X, \{\phi\}, \{p, q\}\rangle$

$\langle X, \{\phi\}, \{p, r\}\rangle, \langle X, \{q\}, \{p, r\}\rangle,$

$\langle X, \{r\}, \{p, q\}\rangle, \langle X, \{q, r\}, \{p\}\rangle$

$\langle X, \{p, r\}, \{q\}\rangle, \langle X, \{p, q\}, \{r\}\rangle, \langle X, \{\phi\}, \{\phi\}\rangle.$

Let $A = \langle X, \{q\}, \{r\}\rangle$ and $B = \langle X, \{r\}, \{q\}\rangle$

$A \cap B = \langle X, \{\phi\}, \{q, r\}\rangle$ which is not in $I_f OS$.

Definition 2.8[11]**.** Suppose $(X, \tau, \hat{\tau}_f)$ is an $I_f TS$, let $C = \langle X, C_1, C_2\rangle$ is an $I_f S$ in *X*. An intuitionistic fine closure and intuitionistic fine interior of C are:

$I_f cl(C) = \bigcap\{J : J \text{ is an } I_f CS \text{ in } X, C \subseteq J\}$

$I_f \operatorname{int}(C) = \bigcup\{J : J \text{ is an } I_f OS \text{ in } X, C \supseteq J\}$

Definition 2.9[12]. In $(X, \tau, \hat{\tau}_f)$, an intuitionistic fine set $(I_f S)C$ of X is called an intuitionistic fine g-closed set in an intuitionistic fine topological space $(I_f TS)$ if $I_f cl(C) \subseteq w$ whenever $C \subseteq w$ and w is intuitionistic fine open set denoted by $I_f g$ -closed $(I_f GCS)$.

Complement of $I_f GCS$ is $I_f g$ -open set $(I_f GOS)$.

Definition 2.10[12]. Let $(X, \tau, \hat{\tau}_f)$ be an $I_f TS$.

(i) For every $C \subseteq X$, the space union of all $I_f GOS \subseteq C$ is called an $I_f g$ - interior of C denoted as $i^*_{I_f}(C)$. (ii) For every $C \subseteq X$, the space intersection of all $I_f GCS$ containing C is called $I_f g$ -closure of C denoted as $cl^*_{I_f}(C)$.

3. Intuitionistic Fine Generalized Locally Closed Sets

Definition 3.1. Suppose $(X, \tau, \hat{\tau}_f)$ is an $I_f TS$. A subset C in X is called intuitionistic fine generalised locally closed set $(I_f GLCS)$, if $C = M \cap N$, where M is $I_f GOS$ in $(X, \tau, \hat{\tau}_f)$ and N is $I_f GCS$ in $(X, \tau, \hat{\tau}_f)$. Notation for collection of intuitionistic fine generalised locally closed set in X is $I_f GLCS(X)$.

Remark 3.2. Every $I_f GCS$ (resp., $I_f GOS$) is $I_f GLCS$.

Definition 3.3. Suppose C is any subset of an $I_f TSX$, $C \in I_f GLCS^*(X)$, if $\exists$ an intuitionistic fine g-open set M and an intuitionistic fine closed set N of $(X, \tau, \hat{\tau}_f)$, such that $C = M \cap N$.

Definition 3.4. Suppose C is any subset of an $I_f TSX$, $C \in I_f GLCS^{**}(X)$, if $\exists$ an intuitionistic fine open set M and an $I_f GLCS$ N of $(X, \tau, \hat{\tau}_f)$, such that $C = M \cap N$.

Remark 3.5. Every $I_f GCS$ is $I_f CS$ in $I_f TS$ [12] therefore $I_f GLCS$, $I_f GLCS^*$ and $I_f GLCS^{**}$ coincides.

Example 3.6. Suppose $X = \{r, s, t\}$ and $\tau = \{\underset{\sim}{X}, \underset{\sim}{\phi}, A\}$ where $A = \langle X, \{r\}, \{t\}\rangle$.

$\hat{\tau}_f = \{\underset{\sim}{X}, \underset{\sim}{\phi}, \langle X, \{r\}, \{\varphi\}\rangle\}$,

$\langle X, \{s\}, \{\phi\}\rangle, \langle X, \{t\}, \{\phi\}\rangle, \langle X, \{\phi\}, \{r\}\rangle$

$\langle X, \{\phi\}, \{s\}\rangle, \langle X, \{\phi\}, \{t\}\rangle, \langle X, \{r\}, \{s\}\rangle$

$\langle X, \{s\}, \{t\}\rangle, \langle X, \{t\}, \{r\}\rangle, \langle X, \{s\}, \{r\}\rangle$

$\langle X, \{t\}, \{s\}\rangle, \langle X, \{r\}, \{t\}\rangle, \langle X, \{r, s\}, \{\phi\}\rangle$

$\langle X, \{s, t\}, \{\phi\}\rangle, \langle X, \{r, t\}, \{\phi\}\rangle, \langle X, \{\phi\}, \{s, t\}\rangle$

$\langle X, \{\phi\}, \{r, t\}\rangle, \langle X, \{r\}, \{s, t\}\rangle, \langle X, \{s\}, \{r, t\}\rangle$,

$\langle X, \{s, t\}, \{r\}\rangle, \langle X, \{r, t\}, \{s\}\rangle$

$\langle X, \{r, s\}, \{t\}\rangle, \langle X, \{\phi\}, \{\phi\}\rangle$.

$\hat{\tau}_f = \{\underset{\sim}{X}, \underset{\sim}{\phi}, \langle X, \{r\}, \{\varphi\}\rangle\}$,

$\langle X, \{\phi\}, \{s\}\rangle, \langle X, \{\phi\}, \{t\}\rangle, \langle X, \{r\}, \{s\}\rangle$

$\langle X, \{s\}, \{\phi\}\rangle, \langle X, \{t\}, \{\phi\}\rangle, \langle X, \{\phi\}, \{r\}\rangle$

$\langle X, \{t\}, \{s\}\rangle, \langle X, \{r\}, \{t\}\rangle, \langle X, \{r, s\}, \{\phi\}\rangle$

$\langle X, \{s\}, \{t\}\rangle, \langle X, \{t\}, \{r\}\rangle, \langle X, \{s\}, \{r\}\rangle$

$\langle X, \{\phi\}, \{s, t\}\rangle, \langle X, \{\phi\}, \{r, t\}\rangle, \langle X, \{s, t\}, \{\phi\}\rangle$

$\langle X, \{r, t\}, \{\phi\}\rangle, \langle X, \{s, t\}, \{r\}\rangle, \langle X, \{r, t\}, \{s\}\rangle$,

$\langle X, \{r\}, \{s, t\}\rangle, \langle X, \{s\}, \{r, t\}\rangle$

$\langle X, \{t\}, \{r, s\}\rangle, \langle X, \{\phi\}, \{\phi\}\rangle$.

$I_f GLCS = I_f GLCS^* = I_f GLCS^{**} = P(X)$.

Theorem 3.7. *Suppose* C *is any subset of an* $I_f TS\ X$. *If* $C \in I_f GLCS^*(X)$ *or* $C \in I_f GLCS^{**}(X)$, *then* C *is an* $I_f GLCS$.

Proof. Given C is a subset of X. Assume that $C \in I_f GLCS^*(X)$ or $C \in I_f GLCS^{**}(X)$. W.K.T every $I_f GCS$ (resp., $I_f GOS$) is $I_f GLCS$ set. Hence C is $I_f GLCS$. □

Theorem 3.8. *Suppose* C *is any subset of an* $I_f TS\ X$, *below are equivalent*:

(1) $C \in I_f GLCS^*(X, \hat{\tau}_f)$ *or* $I_f GLCS$ *or* $I_f GLCS^{**}$.

(2) $C = M \cap I_f cl(A)$, *for some* $I_f GOS\ M$.

(3) $I_f cl(C) - C$ *is* $I_f GCS$.

(4) $C \cup [X - I_f cl(C)]$ *is* $I_f GOS$.

Proof. (1) $\Rightarrow$ (2). Let $C \in I_f GLCS^*(X)$. $\exists$ an intuitionistic fine g-open subset N so that $C = M \cap N$. Since $C \subset M$ and $M \subset I_f cl(C)$, $C \subset M \cap I_f cl(C)$. Conversely, $I_f cl(C) \subset N$. Hence $C = M \cap N \supset M \cap I_f cl(C)$. This implies $C = M \cap I_f cl(C)$.

(2) $\Rightarrow$ (1). Suppose $C = M \cap I_f cl(C)$ for some $I_f GOS\ M$. Then $I_f cl(C)$ is $I_f CS$ and therefore $C = M \cap I_f cl(C) \in I_f GLCS^*(X)$.

(2) $\Rightarrow$ (3). Suppose $C = M \cap I_f cl(C)$, for few $I_f GOS\ M$. Hence $C \in I_f cl(C)$, for some $I_f GOS$ Then $C \in I_f GLCS^*(X, \tau, \hat{\tau}_f)$.

This implies M is $I_f GOS$ and $I_f CS$. Therefore $I_f cl(C) - C$ is $I_f GOS$.

(3) $\Rightarrow$ (2). Suppose $M = X - (I_f cl(C) - C)$. By (iii), M is $I_f GOS$ in X.

Then $C = M \cap cl(C)$ in X holds.

(3) $\Rightarrow$ (4). Suppose $R = I_f cl(C) - C$ be $I_f GOS$ then $X - R = X$

$-[I_f cl(C) - C] = C \cup [X - I_f cl(C)]$. Since $X - R$ is $I_f GOS$, $C \cup (X - I_f cl(C))$ is $I_f GOS$.

(4) $\Rightarrow$ (3). Suppose $M = C \cup (X - I_f cl(C))$. Since $X - M$ is $I_f GOS$ and $X - M = I_f cl(C) - C$ is $I_f GCS$. □

Theorem 3.9. *Suppose C is any subset of an $IS_f TS$ X, below are equivalent*:

(1) $C \in I_f GLCS(X, \tau, \hat{\tau}_f)$.

(2) $C = M \cap cl^*_{I_f}(C)$, *for some* $I_f GOS$.

(3) $cl^*_{I_f}(C) - C$ *is* $I_f GCS$.

(4) $C \cup (X - I_f cl(C))$ *is* $I_f GOS$.

Proof. (1) $\Rightarrow$ (2) Given $C \in I_f GLCS(X, \tau, \hat{\tau}_f)$. Then $\exists$ an $I_f GOS$ subset M and $I_f GCS$ subset N such that $C = M \cap N$. Since $C \subset M$ and $C \subset cl^*_{I_f}(C)$. $C \subset M \cap cl^*_{I_f}(C)$.

Conversely, W.K.T $cl^*_{I_f}(C)$ is $I_f GCS$, $cl^*_{I_f}(C) \subseteq N$ and hence $C = M \cap N \supset M \cap cl^*_{I_f}(C)$. Hence, $C = M \cap cl^*_{I_f}(C)$.

(2) $\Rightarrow$ (1) Suppose $C = M \cap cl^*_{I_f}(C)$, for few $I_f GOS$ M. W.K.T $cl^*_{I_f}(C)$ is $I_f GCS$, $cl^*_{I_f}(C)$ is $I_f GCS$ and hence $C = M \cap I_f(C) \in I_f GLCS^*(X)$.

(2) $\Rightarrow$ (3). Let $C = M \cap cl^*_{I_f}(C)$, for few $I_f GOS$ M. Then $C \in I_f GLC(X)$. This implies M is $I_f GOS$ and $cl^*_{I_f}(C)$ is $I_f GCS$. Hence, $cl^*_{I_f}(C) - C$ is $I_f GCS$.

(3) $\Rightarrow$ (2) Suppose $M = X = [cl^*_{I_f}(C) - C]$. Since (3), M is $I_f GOS$ in X. Therefore, $C = M \cap cl^*_{I_f}(C)$.

(3) $\Rightarrow$ (4) Suppose $R + cl^*_{I_f}(C) - C$ is $I_f GCS$. $X - R = X - [cl^*_{I_f}(C) - C]$ $= C \cup [X - cl^*_{I_f}(C)]$. W.K.T $X - R$ is $I_f GOS$, hence $C \cup [X - cl^*_{I_f}(C)]$ is $I_f GOS$.

(4) $\Rightarrow$ (3). Suppose $C \cup (X - I_f Cl(C))$. W.K.T $X - M$ is $I_f GCS$ therefore $X - M = cl^*_{I_f}(C) - C$ is $I_f GCS$. □

Theorem 3.10. *Suppose C is any subset of an* $I_f TS$ X, *if* $C \in I_f GLCS$ *or* $I_f GLCS^*$ *or* $I_f GLCS^{**}(X)$, *then* $\exists$ *an* $I_f GOS\ H$ *such that* $C = H \cap I_f cl(C)$.

Proof. Let $C \in I_f GLCS^{**}(X)$. Then $C = H \cap N$ where H is $I_f GOS$ and N is $I_f GCS$. Then $C = H \cap N \Rightarrow C \subset H$. Obviously, $C \subset I_f cl(C)$. Hence

$C \subset H \cap I_f cl(C)$ (1).

W.K.T $I_f cl(C) \subset N$ implies $C = H \cap N \supset H \cap I_f cl(C)$ implies

$C \supset H \cap I_f cl(C)$ (2). (1) and (2) $\Rightarrow C = H \cap cl^*_{I_f}(C)$. □

Theorem 3.11. *Suppose D be a subset of* $(X, \tau, \hat{\tau}_f)$, *if* $C \in I_f GLCS^{**}(X)$, *then* $cl^*_{I_f}(D) - D$ *an* $I_f GCS$ $D \cup [X - cl^*_{I_f}(D)]$ *is an* $I_f GOS$.

Proof. By the definition the proof is clear that $cl^*_{I_f}(D) - D$ is an $I_f GCS$ $D \cup [X - cl^*_{I_f}(D)]$ is an $I_f GOS$. □

4. Intuitionistic Fine Generalized Locally Continuous Maps

In the section below different types of maps called intuitionistic fine generalised locally continuous maps and intuitionistic generalised locally irresolute maps are introduced and few of the properties of them are discussed.

Definition 4.1. Suppose $(Y, \tau, \hat{\tau}_f)$ and $(Z, \delta, \hat{\delta}_f)$ are two I_fTS, $\hat{\tau}_f \subseteq \hat{\delta}_f$. A map $g : (Y, \hat{\tau}_f) \to (Z, \hat{\delta}_f)$ is called I_fGL-continuous (resp., I_fGL^*-continuous, resp., I_fGL^{**}-continuous), if $g^{-1}(C) \in I_fGLCS(X)$ (resp., $g^{-1}(C) \in I_fGLCS^*(Y)$ (resp., $g^{-1}(C)) \in I_fGLCS^{**}(Y)$), for every $C \in I_fCS$ of Z.

Definition 4.2. Suppose $(Y, \tau, \hat{\tau}_f)$ and $(Z, \delta, \hat{\delta}_f)$ are two I_fTS, $\hat{\tau}_f \subseteq \hat{\delta}_f$. A map $g : (Y, \hat{\tau}_f) \to (Z, \hat{\delta}_f)$ is called I_fGL-irresolute (resp., I_fGL^*-irresolute, resp., I_fGL^{**}-irresolute), if $g^{-1}(C) \in I_fGLCS(Y)$ (resp., $g^{-1}(C) \in I_fGLCS^*(Y)$ resp., $g^{-1}(C) \in I_fGLCS^{**}(Y)$), for every $C \in I_fCS$ of Z.

Theorem 4.3. *Suppose* $(Y, \tau, \hat{\tau}_f)$ *and* $(Z, \delta, \hat{\delta}_f)$ *are two* I_fTS. *Let* $g : (Y, \hat{\tau}_f) \to (Z, \hat{\delta}_f)$. *If g is* I_fGL^* *continuous or* I_fGL^{**} *continuous, then it is* I_fGL *continuous.*

Proof. Assume that $(Y, \tau, \hat{\tau}_f)$ and $(Z, \delta, \hat{\delta}_f)$ be two I_fTS and let $g : (Y, \hat{\tau}_f) \to (Z, \hat{\delta}_f)$ be a function. By the definitions of I_fGL^*-continuous and I_fGL^{**}-continuous it is true that g is I_fGL-continuous. □

Theorem 4.4. *If* $i : X_1 \to Y_1$ *is* I_fGL *continuous and* $j : Y_1 \to Z_1$ *is if continuous, then* $i \circ j : X_1 \to Z_1$ *is* I_fGL *continuous.*

Proof. Given $i : X_1 \to Y_1$ is I_fGL continuous and $j : Y_1 \to Z_1$ is if continuous. $i^{-1}(N) \in I_fGLCS(X_1), N \in Y_1$ and $j^{-1}O \in Y_1, O \in Z_1$. Let $O \in Z_1$ (By def.). Then $(i \circ j)^{-1}(O) = (i^{-1}j^{-1})(O) = i^{-1}(j^{-1}(O)) = i^{-1}(N)$ for $N \in Y_1$. From this, $(i \circ j)^{-1}(O) = i^{-1}(N) \in I_fGLCS(X_1), O \in Z_1$. Hence, $i \circ j$ is I_fGL continuous. □

Theorem 4.5. *If* $i : X_1 \to Y_1$ *is* $I_f GL$ *irresolute and* $j : Y_1 \to Z_1$ *is* $I_f GL$ *continuous, then* $j \circ i : X_1 \to Z_1$ *is* $I_f GL$ *continuous.*

Proof. Given $i : X_1 \to Y_1$ is $I_f GL$ irresolute and $j : Y_1 \to Z_1$ is $I_f GL$ continuous, $i^{-1}(N) \in I_f GLCS(X_1)$, for $N \in I_f GLCS(Y_1)$ and $j^{-1}(O) \in I_f GLCS(Y_1)$ for $O \in Z_1$ (By def.). Let $O \in Z_1$. Then $(j \circ i)^{-1}(O) = (i^{-1} j^{-1})(O) = i^{-1}(N) \in I_f GLCS(X_1), O \in Z_1$. Hence $j \circ i$ is $I_f GL$ continuous. □

Theorem 4.6. *If* $i : X_1 \to Y_1$ *is* $I_f GL$ *irresolute,* $j : Y_1 \to Z_1$ *is* $I_f GL$ *irresolute, then* $j \circ i : X_1 \to Z_1$ *is* $I_f GL$ *irresolute.*

Proof. By definition and hypothesis we have $i^{-1}(N) \in I_f GLCS(X_1)$ for $N \in I_f GLCS(Y_1)$ and $j^{-1}(O) \in I_f GLCS(Z_1)$. Let $O \in I_f GLCS(Z_1)$, hence $(j \circ i)^{-1}(O) = (i^{-1} j^{-1}(O)) = i^{-1}(j^{-1}(O)) = i^{-1}(N)$ for $N \in I_f GLCS(Y_1)$. Hence $(i \circ j)^{-1}(O) = i^{-1}(N) \in I_f GLCS(X_1), O \in I_f GLCS(Z_1)$. Therefore $j \circ i$ is $I_f GL$ irresolute. □

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