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ANALYTICAL SOLUTIONS OF COVID-19 FRACTIONAL ORDER MATHEMATICAL MODEL BY CONFORMABLE FRACTIONAL DIFFERENTIAL TRANSFORM METHOD

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Abstract

In this paper, we will discuss an analytical solution and numerical simulation of fractional order mathematical model on COVID-19 under conformable sense with the help of conformable fractional differential transform method for different values of order q, where $q \in (0, 1]$. The underlying mathematical model on COVID-19 is consist of four compartments, like, susceptible class, healthy class and infected class, quarantine class. We conclude that use of fractional epidemic model provides better understanding and biologically more insights about the disease dynamics.

1. Introduction

The different models on the CORONA virus disease have been discussed by some authors using the various methods like Adams-Moulton type, Laplace transform coupled with Adomain decomposition method, generalized-Bashforth-Moulton method, q-homotopy analysis transform method which can be found in [1, 2, 3, 4, 5].

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The differential transform method was applied in the different fields by many researchers which can be found in [6, 7]. In this paper, we will study a new application of the conformable fractional differential transform method [8, 9] to obtain an approximate solutions for the system of fractional order mathematical model on COVID-19 (1.1).

Definition 1.1 Conformable definition[8, 9]**.**

Given a function $f : [0, \infty) \to R$. Then the conformable fractional derivative of f order α is defined by

$$(T_\alpha f)(t) = \lim_{\varepsilon \to 0} \frac{f(t + \varepsilon t^{1-\alpha}) - f(t)}{\varepsilon}$$

for all $t > 0$, $\alpha \in (0, 1]$.

Recently, some researchers develop a new mathematical model [12], based on susceptible individuals S, healthy or resistant individuals H, infected and quarantine individuals I, Q respectively. All the parameters involved in the model are assumed to be non-negative. The susceptible individuals initially move to the infectious class with a constant flow rate. The suspected or infected individuals move to the quarantine class and confirmed cases are send back to the infected compartment for further treatment. In this paper, we investigate the COVID-19 new model under conformable fractional derivative as follows:

$$\begin{cases} \dfrac{d^{q_1} S(t)}{dt^{q_1}} = \lambda - \gamma S(t) I(t) - (d + \mu) S(t), \\ \dfrac{d^{q_2} H(t)}{dt^{q_2}} = \alpha - \beta H(t) I(t) + \theta I(t) - (d + \mu) H(t), \\ \dfrac{d^{q_3} I(t)}{dt^{q_3}} = \gamma S(t) I(t) + \beta H(t) I(t) + \delta Q(t) - (d + \mu + \eta + \theta) I(t), \\ \dfrac{d^{q_4} Q(t)}{dt^{q_4}} = \eta I(t) - (d + \mu + \delta) Q(t), \\ S(0) \geq 0,\ H(0) \geq 0,\ I(0) \geq 0,\ Q(0) \geq 0, \end{cases} \tag{1.1}$$

where $0 < q_i \leq 1$ for $i = 1, 2, 3, 4$.

In the above model.

$\lambda \to$ Recruitment rate susceptible,

$\gamma \to$ Disease transmission rate,

$d \to$ Natural death rate,

$\alpha \to$ Recruitment rate of healthy human,

$\beta \to$ Transmission rate of healthy human,

$\mu \to$ Disease related death rate infected or suspected individuals,

$\delta \to$ Rate at which quarantine people get infection,

$\theta \to$ Cure rate of infected people in the quarantine class.

The study of the mathematical models under fractional derivatives instead of usual ordinary derivatives produces more significant results which are more helpful in understanding.

2. Conformable Fractional Differential Transform Method

For further use and better understanding of readers, we will recall some results on conformable fractional differential transform method.

Definition 2.1[8, 9]**.** Assume $f(t)$ is infinitely α-differentiable function for some $\alpha \in (0, 1]$. Conformable fractional differential transform of $f(t)$ is defined as

$$F_{\alpha}(k) = \frac{1}{\alpha^{k} k!} [(T_{\alpha}^{t_0} f)^{(k)}(t)]_{t=t_0}.$$

Definition 2.2[8, 9]**.** Let $F_{\alpha}(k)$ be the conformable fractional differential transform of $f(t)$. Inverse conformable fractional differential transform of $F(k)$ is defined as

$$f(t) = \sum_{k=0}^{\infty} F_{\alpha}(k)(t - t_0)^{\alpha k} = \sum_{k=0}^{\infty} \frac{1}{\alpha^{k} k!} [(T_{\alpha}^{t_0} f)^{(k)}(t)]_{t=t_0} (t - t_0)^{\alpha k}.$$

CFDT of initial conditions for integer order derivatives are defined as

$$F_\alpha(k) = \begin{cases} \text{if } \alpha k \in Z^+, & \frac{1}{(\alpha k)!}\left[\frac{d^{\alpha k f(t)}}{dt^{\alpha k}}\right]_{t-t_0} \text{ for } k = 0, 1, 2, \ldots, \left(\frac{n}{\alpha} - 1\right), \\ \text{if } \alpha k \notin Z^+, & 0 \end{cases} \quad (5.1)$$

where α is the order of conformable fractional ordinary differential equation (CFODE).

Theorem 2.3[8, 9]. *If* $f(t) = u(t) \pm v(t)$, *then* $F_\alpha(k) = U_\alpha(k) \pm V_\alpha(k)$.

Theorem 2.4[8, 9]. *Let c be a constant. If* $f(t) = c\,u(t)$, *then* $F_\alpha(k) = c\,U_\alpha(k)$.

Theorem 2.5[8, 9]. *If* $f(t) = u(t)v(t)$, *then* $F_\alpha(k) = \sum_{l=0}^{k} U_\alpha(l)V_\alpha(k-l)$.

Theorem 2.6[8, 9]. *If* $f(t) = T_\alpha^{t_0}(u(t))$, *then* 0.

Theorem 2.7[8, 9]. *If* $f(t) = (t - t_0)^p$, *then* $F_\alpha(k) = \delta\left(k - \frac{p}{\alpha}\right)$ *where*

$$\delta(k) = \begin{cases} 1, & \textit{if } k = 0 \\ 0, & \textit{if } k \neq 0. \end{cases}$$

3. Analytical Solutions of Model using CFDTM

Apply CFDTM on mathematical model on Corona (1.1), and then by using Theorems 2.3-2.7, the first equation is transformed into

$$q(k+1)S_q(k+1) = \lambda\delta(k) - \gamma\sum_{l=0}^{k} S_q(l)I_q(k-l) - (d+\mu)S_q(k)$$

this implies

$$S_q(k+1) = \frac{\lambda\delta(k) - \gamma\sum_{l=0}^{k} S_q(l)I_q(k-l) - (d+\mu)S_q(k)}{q(k+1)} \quad (1)$$

By using Theorems 2.3-2.7, second equation of model (1.1) is transformed into

$$q(k+1)H_q(k+1) = \alpha\delta(k) - \beta\sum_{l=0}^{k} H_q(l)I_q(k-l) + QI_q(k) - (d+\mu)H_q(k)$$

this implies,

$$H_q(k+1) = \frac{\alpha\delta(k) - \beta\sum_{l=0}^{k} H_q(l)I_q(k-l) + QI_q(k) - (d+\mu)H_q(k)}{q(k+1)} \tag{2}$$

By using Theorems 2.3-2.7, third equation of model (1.1) is transformed into

$$q(k+1)I_q(k+1)$$

$$= \gamma\sum_{l=0}^{k} S_q(l)I_q(k-l) + \beta\sum_{l=0}^{k} H_q(l)I_q(k-l) + \delta Q_q(k) - (d+\mu+\eta+\theta)I_q(k)$$

this implies

$$I_q(k+1)$$

$$= \frac{\gamma\sum_{l=0}^{k} S_q(l)I_q(k-l) + \beta\sum_{l=0}^{k} H_q(l)I_q(k-l) + \delta Q_q(k) - (d+\mu+\eta+\theta)I_q(k)}{q(k+1)} \tag{3}$$

By using Theorems 2.3-2.7, fourth equation of model (1.1) is transformed into

$$q(k+1)Q_q(k+1) = \eta I_q(k) - (d+\mu+\delta)Q_q(k)$$

this implies

$$Q_q(k+1) = \frac{\eta I_q(k) - (d+\mu+\delta)Q_q(k)}{q(k+1)} \tag{4}$$

Subject to initial conditions and using (5.1) which reduces to

$$S(0) = 43994,\ H(0) = 0,\ I(0) = 1,\ Q(0) = 1.$$

Now, we take values for parameters as follows.

$$\alpha = 0.535,\ \beta = 0.0056,\ \gamma = 0.5944,\ \delta = 0.27,\ \eta = 0.0025,$$

$$d = 0.025,\ \lambda = 0.0043217,\ \mu = 3.5,\ \theta = 0.5.$$

Therefore, the series solutions of the transformed expressions, when $k = 6$ and $q_1 = \frac{1}{2}, q_2 = \frac{1}{2}, q_3 = \frac{1}{2}, q_4 = \frac{1}{2}$ for $S(t)$, $H(t)$, $I(t)$ and $Q(t)$ are obtained as

$$s(t) = 43994 - 362457.7585566t^{\frac{1}{2}} - 1365958888.5691562t$$

$$- 23820587478589.24t^{\frac{3}{2}} - 3.1123008424749856e + 17t^2$$

$$- 3.2521025442105115e + 21t^{\frac{5}{2}} - 2.830404572988943e + 25t^3$$

$$- 2.10964668079509e + 29t^{\frac{7}{2}} + \dots$$

$$h(t) = 0 + 2.0700000000000003t^{\frac{1}{2}} + 26138.967758000003t$$

$$+ 3736439611.8805118t^{\frac{3}{2}} + 5948406376736.038t^2 + 6.223579134700689e$$

$$+ 16t^{\frac{5}{2}} + 5.419209429451661e + 20t^3 + 4.0426262328678816e + 24t^{\frac{7}{2}} + \dots$$

$$i(t) = 1 + 52292.552200000006t^{\frac{1}{2}} + 1367025941.8777246t$$

$$+ 23820056256442.363t^{\frac{3}{2}} + 3.1122410041716326e + 17t^2$$

$$+ 3.2520399972482275e + 21t^{\frac{5}{2}} + 2.8303501099306783e + 25t^3$$

$$+ 2.1096060523942766 2e + 29t^{\frac{7}{2}} + \dots$$

$$q(t) = 1 - 7.585t^{\frac{1}{2}} + 159.51645550000003t + 2277972.993163793t^{\frac{3}{2}}$$

$$+ 29770747866.798428t^{2} + 311178908421901.44t^{\frac{5}{2}}$$

$$+ 2.7096396897210363e + 18t^{3} + 2.021384847589842e + 22t^{\frac{7}{2}} + \ldots.$$

For $q_1 = 0.3, q_2 = 0.4, q_3 = 0.6, q_4 = 0.7$ and hence $S(t), H(t), I(t)$ and $Q(t)$ are obtained as

$$s(t) = 43994 - 604096.2642610001t^{\frac{3}{10}} - 1895091361.2202196t^{\frac{3}{5}}$$

$$- 27552751961158.49t^{\frac{9}{10}} - 2.998238770238271e + 17t^{\frac{6}{5}}$$

$$- 2.608460402856581e + 21t^{\frac{3}{2}} - 1.889233356555197e + 25t^{\frac{9}{5}}$$

$$- 1.1708036160359847e + 29t^{\frac{21}{10}} + \ldots$$

$$h(t) = 0 + 2.5875000000000004t^{\frac{4}{10}} + 27224.284986458337t^{\frac{4}{5}}$$

$$+ 6085168866.184664t^{\frac{12}{10}} + 429214754278.396t^{\frac{8}{5}} + 3.7473829817806136e$$

$$+ 16t^{2} + 2.7167526543688303e + 20t^{\frac{12}{5}} + 1.6864744339479867e + 24t^{\frac{14}{5}} +$$

$$i(t) = 1 + 43577.12683333334t^{\frac{6}{10}} + 949173956.4474993t^{\frac{6}{5}}$$

$$+ 13777881525218.57t^{\frac{18}{10}} + 1.499292856317466e + 17t^{\frac{12}{5}}$$

$$+ 1.3043812150367266e + 21t^{\frac{15}{10}} + 9.447261634298385e + 24t^{\frac{18}{5}}$$

$$+ 5.854697774811895e + 28t^{\frac{21}{5}} + \ldots$$

$$q(t) = 1 - 5.417857142857144t^{\frac{7}{10}} + 92.50256067176872t^{\frac{7}{5}}$$

$$+ 1129801.8304290469t^{\frac{21}{10}} + 12300148648.24998t^{\frac{16}{5}}$$

$$+ 107079010004356.11e + 21t^{\frac{35}{10}} + 7.763206363687738e + 17t^{\frac{21}{5}}$$

$$+ 4.819430193659376e + 21t^{\frac{49}{10}} + \ldots.$$

For $q_1 = 0.8$, $q_2 = 0.8$, $q_3 = 0.8$, $q_4 = 0.8$ and hence $S(t)$, $H(t)$, $I(t)$ and $Q(t)$ are obtained as

$$s(t) = 43994 - 226536.099097875t^{\frac{4}{5}} - 533577690.8473266t^{\frac{8}{5}}$$

$$- 5815573114889.95t^{\frac{12}{5}} - 4.748994205436685e + 16t^{\frac{16}{5}}$$

$$- 3.101446670732986e + 20t^{4} - 1.6870525914364716e + 24t^{\frac{24}{5}}$$

$$- 7.859046313148332e + 27t^{\frac{28}{5}} + \ldots$$

$$h(t) = 0 + 1.2937500000000002t^{\frac{4}{5}} + 10210.53428046875t^{\frac{8}{5}}$$

$$+ 912216702.1192657t^{\frac{12}{5}} + 907654781606.4512t^{\frac{16}{5}}$$

$$+ 5935267576885878.0t^{4} + 3.2301005300591358e + 19t^{\frac{24}{5}}$$

$$+ 1.5059956285610361e + 23t^{\frac{28}{5}} + \ldots$$

$$i(t) = 1 + 32682.845125000003t^{\frac{4}{5}} + 533994508.5459862t^{\frac{8}{5}}$$

$$+ 5815443421982.999t^{\frac{12}{5}} + 4.748902899431812e + 16t^{\frac{16}{5}}$$

$$+ 3.101387021301487e + 20t^{4} + 1.6870201289240594e + 24t^{\frac{24}{5}}$$

$$+ 7.858894960560939e + 27t^{\frac{28}{5}} + \ldots$$

$$q(t) = 1 - 4.740625t^{\frac{4}{5}} + 62.31111542968751t^{\frac{8}{5}} + 556145.750284129t^{\frac{12}{5}}$$

$$+ 4542655619.32349t^{\frac{16}{5}} + 29676333276929.992t^{4}$$

$$+ 1.6150711117512205e + 17t^{\frac{24}{5}} + 7.530245362184356e + 20t^{\frac{28}{5}} + \ldots$$

For $q_1 = q_2 = q_3 = q_4 = 1$ and hence $S(t), H(t), I(t)$ and $Q(t)$ are obtained as

$$s(t) = 43994 - 181228.8792783t - 1365958888.5691562t^{2}$$

$$- 107217992520952.2t^{3} - 1.1212350713573536e + 19t^{4}$$

$$- 1.4656859966044585e + 24t^{5} - 2.2991644515586486e + 29t^{6}$$

$$- 4.207738078551882e + 34t^{7} + \ldots$$

$$h(t) = 0 + 1.0350000000000001t + 26138.967758000003t^{2}$$

$$+ 16575159853.59114t^{3} + 214197553639755.1t^{4} + 2.8026540629606425e$$

$$+ 19t^{5} + 4.396378351973526e + 24t^{6} + 8.045832599916284e + 29t^{7} + \ldots$$

$$q(t) = 1 + 26146.276100000003t + 1367025941.8777246t^{2}$$

$$+ 107215840839691.12t^{3} + 1.1212135240501512e + 19^{4}$$

$$+ 1.4656578299360666e + 24t^{5} + 2.299120267958349e + 29t^{6}$$

$$+ 4.20765721793823e + 34t^{7} + \ldots$$

$$i(t) = 1 - 3.7925t + 159.51645550000003t^{2} + 10250878.469237069t^{3}$$

$$+ 1072002800061.7483t^{4} + 1.40131349253177e + 17t^{5}$$

$$+ 2.1981676658218506e + 22t^{6} + 4.022876525686686e + 27t^{7} + \ldots$$

4. Numerical Simulation and Discussion

Here we use python software to plot the pictorial behaviour of solution of given fractional COVID-19 model. In this section, we provide numerical simulation by using table and graphs. The mathematical analysis of measles epidemic model with nonlinear system of fractional differential equation has been presented. We observe that fractional order COVID-19 measles model has more degree of freedom as compared to ordinary derivatives COVID-19 model. The compression for some different values of order q has been obtained and numerical simulation are shown using CFDTM. All the calculated values of solution of given model for fixed parameters are shown in table 1, table 2, table 3 and table 4. It is observed that the solution of the fractional COVID-19 model obtained by using conformable fractional differential transform method is approximately same with solution obtained for integer order COVID-19 model. Graphical interpretation of solution obtained by using CFDTM of given fractional model for different fractional order are shown in figure 1 and figure 2.

Table 1. Table of susceptible class (population) at different values of q using CFDTM.

S.N.	$q = 0.5$	$q = 0.3$	$q = 0.8$	$q = 1$
1	4.39940000e+04	0.00000000e+00	2.29675648e+02	1.00000000e+00
2	-7.54792786e+33	1.44637729e+29	7.54778250e+33	7.23214306e+26
3	-8.53943052e+34	1.63637473e+30	8.53926606e+34	8.18216401e+27
4	-3.52977177e+35	6.76395146e+30	3.52970379e+35	3.38209574e+28
5	-9.66120273e+35	1.85133518e+31	9.66101668e+35	9.25700438e+28
6	-2.10967499e+36	4.04268043e+31	2.10963436e+36	2.02141194e+29
7	-3.99346063e+36	7.65249872e+31	3.99338372e+36	3.82638514e+29

Table 2. Table of resistive class (population) at different values of q using CFDTM.

S.N.	$q = 0.5$	$q = 0.4$	$q = 0.8$	$q = 1$
1	4.39940000e+04	0.00000000e+00	1.00000000e+00	1.00000000e+00
2	-6.31939606e+31	7.41113728e+27	1.70546212e+34	1.14301325e+28

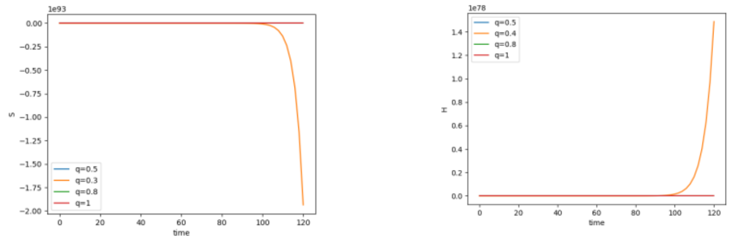
3	-2.70915098e+32	5.16135504e+28	3.13446992e+35	3.41267508e+29
4	-6.34778486e+32	1.60626377e+29	1.72086079e+36	2.48851810e+30
5	-1.16142682e+33	3.59454437e+29	5.76086213e+36	1.01891953e+31
6	-1.85567390e+33	6.71414707e+29	1.47064856e+37	3.04087438e+31
7	-2.72133081e+33	1.11865858e+30	3.16278506e+37	7.42995616e+31

Table 3. Table of infected class (population) at different values of q using CFDTM.

S.N.	$q = 0.5$	$q = 0.6$	$q = 0.8$	$q = 1$
1	4.39940000e+04	0.00000000e+00	1.00000000e+00	1.00000000e+00
2	-1.51756154e+35	2.90803860e+30	1.51753231e+35	1.45407090e+28
3	-7.36055689e+36	1.41047221e+32	7.36041514e+36	7.05261130e+29
4	-7.12886473e+37	1.36607403e+33	7.12872744e+37	6.83061253e+30
5	-3.57006871e+38	6.84117083e+33	3.56999996e+38	3.42070680e+31
6	-1.24558162e+39	2.38685508e+34	1.24555763e+39	1.19346989e+32
7	-3.45769634e+39	6.62583644e+34	3.45762975e+39	3.31303579e+32

Table 4. Table of quarantined class (population) at different values of q using CFDTM.

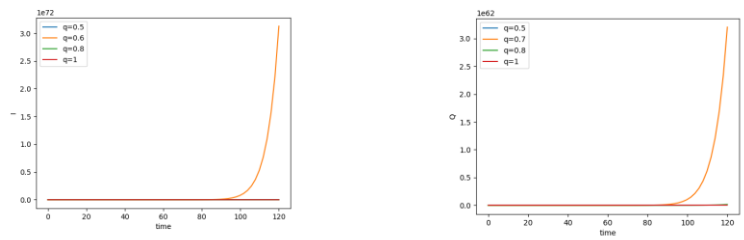
S.N.	$q = 0.5$	$q = 0.3$	$q = 0.8$	$q = 1$
1	4.39940000e+04	0.00000000e+00	1.00000000e+00	1.00000000e+00
2	-5.38590621e+43	1.02986685e+39	5.38580271e+43	5.14928336e+36
3	-6.89395901e+45	1.31822939e+41	6.89382653e+45	6.59108180e+38
4	-1.17789747e+47	2.25231840e+42	1.17787484e+47	1.12614807e+40
5	-8.82426693e+47	1.68733351e+43	8.82409735e+47	8.43658413e+40
6	-4.20773831e+48	8.04583304e+43	4.20765745e+48	4.02287675e+41
7	-1.50770870e+49	2.88296742e+44	1.50767972e+49	1.44146946e+42



(a) Dynamical behaviour of the susceptible class (population)

(b) Dynamical behaviour of the resistive class (population)

Figure 1. Dynamical behaviour of the susceptible and resistive class (population) using FDTM for different values of q.



(a) Dynamical behaviour of the infected class (population)

(b) Dynamical behaviour of the quarantine class (population)

Figure 2. Dynamical behaviour of the infected and quarantine class (population) using FDTM for different values of q.

5. Conclusion

In this work, a fractional order corona virus model under conformable sense has been investigated and solved by using conformable fractional differential transform method. The analytical solutions have been obtained in terms of convergent series with easily computable components in a direct way without using linearisation or perturbation or restrictive assumptions. Solution obtained for integer order COVID-19 model resemble with the that solution of the fractional COVID-19 model obtained by using conformable fractional differential transform method. Graphical interpretation of solution obtained by using CFDTM of given model for different fractional order are shown in figure 1 and figure 2. The compression for some different values of order q has been obtained and numerical simulation are shown.

References

[1] Musibau A. Omoloye, Sunday O. Adewale, Aliyu M. Umar and Asimiyu O. Oladapo, Analysis of coronavirus disease model by differential transformation method (DTM), IJARIIE 7(3) (2021).

[2] Kamal Shah, Thabet Abdeljawad, Ibrahim Mahariq and Fahd Jarad, Qualitative analysis of a mathematical model in the time of COVID-19, BioMed Research International (2020), pp11.

[3] V. Padmavathi, A. Prakash, K. Alagesan and N. Magesh, Analysis and numerical simulation of novel coronavirus (COVID-19) model with Mittag-Leffler Kernel, Mathematical Methods in the Applied Sciences 44(2) (2021), 1863-1877.

[4] Mohammed A. Aba Oud, Aatif Ali, Hussam Alrabaiah, Saif Ullah, Muhammad Altaf Khan and Saeed Islam, A fractional order mathematical model for COVID-19 dynamics with quarantine, isolation, and environmental viral load, Advances in Difference Equations (2021), 1-19.

[5] Nguyen Huy Tuan, Hakimeh Mohammadi and Shahram Rezapour, A mathematical model for COVID-19 transmission by using the Caputo fractional derivative, Chaos, Solitons and Fractals 14 (2020), 110107.

[6] J. K. Zhou, Differential transformation and its applications for electrical circuits, Huazhong University Press, Wuhan, (1986).

[7] Aytac Arikoglu and Ibrahim Ozkol, Solution of fractional differential equations by using differential transform method, Chaos, Solitons and Fractals 34(5) (2007), 1473-1481.

[8] Mostafa Eslami and Hadi Rezazadeh, Multi-step conformable fractional differential transform method for solving and stability of the conformable fractional differential systems, Caspian Journal of Mathematical Sciences (CJMS) 9(2) (2020), 340-348.

[9] Emrah Unal and Ahmet Gokdogan, Solution of conformable fractional ordinary differential equations via differential transform method, OPTIK 128 (2017), 264-273.

[10] Shantanu Das, Functional Fractional Calculus, Springer, (2011).

[11] I. Podlubny, Fractional Differential Equations, Academic Press USA, (1999).

[12] Abdullah, Saeed Ahmad, Saud Owyed, Abdel-Haleem Abdel-Aty, Emad E. Mahmoud, Kamal Shah and Hussam Alrabaiah, Mathematical analysis of COVID-19 via new mathematical model, Chaos, Solitons and Fractals 143 (2021), 110585.