Advances and Applications in Mathematical Sciences
Volume 22, Issue 12, October 2023, Pages 2255-2267

SEPARATION AXIOMS IN NEUTROSOPHIC FUZZY SUPRA TOPOLOGICAL SPACES

RUNU DHAR

Department of Mathematics
Maharaja Bir Bikram University
Agartala, Tripura, India
E-mail: runu.dhar@gmail.com

Abstract

This paper reveals the investigation of separation axioms in neutrosophic fuzzy supra topological spaces. We define neutrosophic fuzzy supra topological T_i-spaces $(i = 1, 2)$. We call them NFST-T_i-spaces $(i = 1, 2)$. We investigate some of their basic properties and characterization theorems in neutrosophic fuzzy supra topological spaces. We establish the relationship between neutrosophic fuzzy supra T_0-space and neutrosophic fuzzy supra T_1-space. We investigate the necessary and sufficient conditions for two neutrosophic fuzzy supra topological spaces to be NFST-T_i- spaces $(i = 1, 2)$. We study the notion of neutrosophic supra hereditary property and its applications.

1. Introduction

Human beings are dealing with real life problems due to uncertainty in their everyday life. To overcome this uncertainty, Zadeh [28] revealed the notion of fuzzy sets theory. As it was limited to control all types of real life problems due to uncertainty, Atanaosv [2] invented the notion of intuitionistic fuzzy sets theory associating with membership and non-membership values. Still it was not sufficient to resolve all types of situations in decision making problems under uncertainty. Thereafter, Smarandache [23, 24] considered the elements with membership, non-membership and indeterministic values and talked the notion of neutrosophic sets and further

2020 Mathematics Subject Classification: 03E72; 54A05; 54A40; 54J05.
Keywords: neutrosophic set, neutrosophic topological space, neutrosophic supra topological space, separation axioms.
Received October 15, 2023; Revised October 19, 2023; Accepted October 19, 2023

investigated on neutrosophic sets [25]. Salama and Alblowi [20, 21] referred the notion of neutrosophic topological spaces. Salama et al. [22] investigated on new neutrosophic crisp topological concepts.

The notions of supra topological spaces, supra open sets and supra closed sets were introduced by Mashhour et al. [13]. Thereafter, ME Abd El-Monsef et al. [1] introduced the concept of fuzzy supra topological as a natural generalization of the notion of supra topological spaces. Babu and Aswini [3] studied on fuzzy neutrosophic supra topological spaces. Turah [27] studied on intuitionistic fuzzy supra topological spaces.

Further several researchers [5, 6, 7, 8, 9, 10, 11, 14, 15, 16, 17, 18] contributed themselves to investigate neautrosophic topological spaces in different directions. Following their works, we introduce and study separation axioms in neutrosophic fuzzy supra topological spaces. The chapter is subdivided is as follows. The next section briefly focuses on some known definitions and results which are relevant for investigation. In section 3, we introduce and investigate separation axioms in neutrosophic supra topological spaces. Conclusion appears in last section.

2. Preliminaries

Here we procure some basic concepts to reveal the study.

Definition 2.1[28]**.** Let X be a universal set. A fuzzy set (in short, FS) A in X is characterized by a membership function μ_A which associates with each point x in X with a real number in the interval $[0, 1]$, with the value of $\mu_A(x)$ at x representing the "grade of membership" of X in A. Let $X = \{x\}$ be a universal set. A Fuzzy set A in X is a set of order pair $A = \{(x, \mu_A(x)) : x \in X\}$ where $\mu_A : X \to [0, 1]$ and $\mu_A(x)$ is the degree of belongingness of x in A.

Definition 2.2[2]**.** Let X be a non-empty set. An intuitionistic fuzzy set (in short, IFS) A in X is an object having the form $A = \{(x, \mu_A(x), \nu_A(x)) : x \in X\}$, where the functions $\mu_A(x), \nu_A(x)$ $: X \to [0, 1]$ define respectively, the degree of membership and degree of non-membership of the element $x \in X$ to the set A, which is a subset of X

and for every element $x \in X$, $0 \leq \mu_A(x) + \nu_A(x) \leq 1$. Furthermore, we have $\pi(x) = 1 - \mu_A(x) - \nu_A(x)$ called the intuitionistic fuzzy set index or hesitation margin of x in A. $\pi(x)$ is the degree of indeterminacy of $x \in X$ to the IFS A and $\pi_A(x) \in [0, 1]$ i.e., $\pi_A(x) : X \to [0, 1]$ and $0 \leq \pi_A(x) \leq 1$ for every $x \in X$. $\pi_A(x)$ expresses the lack of knowledge of whether x belongs to IFS A or not.

Definition 2.3[20]**.** Let X be a universal set. A neutrosophic set (in short, NS) A in X is a set contains triplet having truthiness, falseness and indeterminacy membership values taken from $[0, 1]$. These can be characterized independently and denoted by T_A, F_A, I_A respectively. The neutrosophic set is denoted as follows:

$$A = \{(x, T_A(x), F_A(x), I_A(x)) : x \in X \text{ and } T_A(x), F_A(x), I_A(x) \in [0, 1]\}.$$

There is no restriction on the sum of $T_A(x)$, $F_A(x)$ and $I_A(x)$, so

$$0 \leq T_A(x), F_A(x), I_A(x) \leq 3.$$

The null and full NSs on a non-empty set X are denoted by 0_N and 1_N, defined as follows:

Definition 2.4[20]**.** The neutrosophic sets 0_N and 1_N in X are represented as follows:

(i) $0_N = \{\langle x, 0, 0, 1\rangle : x \in X\}$.

(ii) $0_N = \{\langle x, 0, 1\ 1\rangle : x \in X\}$.

(iii) $0_N = \{\langle x, 0, 1, 0\rangle : x \in X\}$.

(iv) $0_N = \{\langle x, 0, 0, 0\rangle : x \in X\}$.

(v) $1_N = \{\langle x, 1, 0, 0\rangle : x \in X\}$.

(vi) $1_N = \{\langle x, 1, 0, 1\rangle : x \in X\}$.

(vii) $1_N = \{\langle x, 1, 1, 0\rangle : x \in X\}$.

(viii) $1_N = \{\langle x, 1, 1, 1\rangle : x \in X\}$.

Clearly, $0_N \subseteq 1_N$. We have, for any neutrosophic set A, $0_N \subseteq A \subseteq 1_N$.

Definition 2.5[26]**.** The following statements are true for neutrosophic sets A and B on X:

(i) $T_A(x) \leq T_B(x)$, $F_A(x) \geq F_B(x)$ and $I_A(x) \leq I_B(x)$ for all $x \in X$ if and only if $A \subseteq B$.

(ii) $A \subseteq B$ and $B \subseteq A$ if and only if $A = B$.

(iii) $A \cap B = \{(x, \min\{T_A(x), T_B(x)\}, \min\{F_A(x), F_B(x)\}, \min\{I_A(x), I_B(x)\}\}$.

(iv) $A \cup B = \{(x, \max\{T_A(x), T_B(x)\}, \max\{F_A(x), F_B(x)\}, \max\{I_A(x), I_B(x)\}\}$.

Remark 2.6[26]**.** More generally, the intersection and the union of a collection of neutrosophic sets $\{A_i\}_{i \in \Lambda}$, are defined by

(i) $\bigcap_{i \in \Lambda} A_i = \{(x, \inf_{i \in \Lambda}\{T_{A_i}(x)\}, \sup_{i \in \Lambda}\{F_{A_i}(x)\}, \inf_{i \in \Lambda}\{I_{A_i}(x)\} : x \in X\}$.

(ii) $\bigcup_{i \in \Lambda} A_i = \{(x, \sup_{i \in \Lambda}\{T_{A_i}(x)\}, \inf_{i \in \Lambda}\{F_{A_i}(x)\}, \sup_{i \in \Lambda}\{I_{A_i}(x)\} : x \in X\}$.

Definition 2.7[20]**.** Let $A = \{(x, T_A(x), F_A(x), I_A(x)) : x \in X$ and $T_A(x), F_A(x), I_A(x) \in [0, 1]\}$ be a neutrosophic set on X. Then the complement of A, denoted by $CO(A)$ may be defined in three ways as listed below:

(i) $CO(A) = \{(x, T_A(x), F_A(x), I_A(x)) : x \in X\}$.

(ii) $CO(A) = \{(x, F_A(x), 1 - I_A(x), T_A(x)) : x \in X\}$.

(iii) $CO(A) = \{(x, 1 - T_A(x), 1 - I_A(x), 1 - F_A(x)) : x \in X\}$.

Definition 2.8[12]**.** Let A and B be two neutrosophic sets on X. Then the difference of A and B is also a neutrosophic set on X, defined as

$$A \setminus B = \{(x, |T_A(x) - T_B(x)|, |F_A(x) - F_B(x)|, 1 - |I_A(x) - I_B(x)| : x \in X\}.$$

Definition 2.9[20]**.** Let X be a non-empty set and T be the collection of neutrosophic subsets of X. Then T is said to be a neutrosophic topology (in short, NT) on X if the following properties hold:

(i) $0_N, 1_N \in T$.

(ii) $U_1, U_2 \in T \Rightarrow U_1 \cap U_2 \in T$.

(iii) $\cup_{i \in \Delta} U_i \in T$, for every collection $\{U_i : i \in \Delta\} \subseteq T$.

Then (X, T) is called a neutrosophic topological space (in short, NTS) over X. The members of T are called neutrosophic open sets (in short, NOS). A neutrosophic set D is called neutrosophic closed set (in short, NCS) if and only if D^c is a neutrosophic open set.

Definition 2.10[20]**.** Let (X, T) be a NTS and U be a NS in X. Then the neutrosophic interior (in short, N_{int}) and neutrosophic closure (in short, N_{cl}) of U are defined by

$N_{\text{int}}(U) = \cup \{E : E$ is a NOS in X and $E \subseteq U\}$,

$N_{cl}(U) = \cap \{F : F$ is a NCS in X and $U \subseteq F\}$.

Remark 2.11[20]**.** Clearly $N_{\text{int}}(U)$ is the largest neutrosophic open set over X which is contained in U and $N_{cl}(U)$ is the smallest neutrosophic closed set over X which contains U.

Proposition 2.12[20]**.** For any NS B in (X, T), we have

(i) $N_{\text{int}}(B^c) = (N_{cl}(B))^c$.

(ii) $N_{cl}(B^c) = (N_{\text{int}}(B))^c$.

Definition 2.13[3]**.** A neutrosophic fuzzy supra topology (in short, NFST) of a non-empty set X is a family T^{μ} of neutrosophic fuzzy supra subsets in X satisfying the following axioms:

(i) $0_N, 1_N \in T^{\mu}$.

(ii) $\cup G_i \in T^{\mu}$, $\forall$ all collection $\{G_i : i \in J\} \subseteq T^{\mu}$.

In this case, the pair (X, T^{μ}) is called a neutrosophic fuzzy supra topological space (in short, NFSTS) and any neutrosophic fuzzy supra set in T^{μ} is known as neutrosophic fuzzy supra open set (in short, NFSOS) in X.

The complement of NFSOS in the NFSTS (X, T^{μ}) is called neutrosophic fuzzy supra closed set (NFSCS).

Example 2.14[3]**.** Let $X = \{y\}$ and consider the family $T^{\mu} = \{0_N, 1_N, P, Q, R\}$ where $P = \{y, (0.3, 0.4, 0.7)\}$, $Q = \{y, (0.2, 0.6, 0.8)\}$, $R = \{y, (0.3, 0.6, 0.7)\}$. Then (X, T^{μ}) is NFSTS on X.

Definition 2.15[3]**.** Let (X, T^{μ}) be NFSTS and $P = \{y, (T_p, I_p, F_p)\}$ be a NFSS in X. Then the neutrosophic fuzzy supra interior and neutrosophic fuzzy supra closure of P are denoted by $NFScl(P)$ and $NFS\,\text{int}(P)$ respectively and are defined as below:

(i) $NFScl(P) = \cap \{Q : Q$ is a NFSCS in X and $P \subseteq Q\}$.

(ii) $NFS\,\text{int}(P) = \cup \{R : R$ is a NFSOS in X and $R \subseteq P\}$.

Remark 2.16[3]**.** (i) Note that $NFScl(P)$ is a NFSCS and $NFS\,\text{int}(P)$ is a NFSOS in X.

(ii) P is a NFSCS in X if and only if $NFScl(P) = P$.

(iii) P is a NFSOS in X if and only if $NFS\,\text{int}(P) = P$.

Example 2.17. Let $X = \{y\}$ and consider the family $T^{\mu} = \{0_N, 1_N, P, R, G\}$ is a NFSTS where $P = \{y, (0.4, 0.3, 0.7)\}$ and $R = \{y, (0.3, 0.3, 0.7)\}$. Let $G = \{y, (0.5, 0.3, 0.4)\}$ be a NFSOS in X.

Then $NFS\,\text{int}(G) = \cup\{P, R : P, R$ are NFSOSs in X and $P \subset G, R \subset G\}$.

$NFS\,\text{int}(G) = \{y, (0.4, 0.3, 0.7)\} = P$.

Thus P is $NFS\,\text{int}(G)$.

Here NFSCSs are complement of NFSOSs.

Thus $CO(P) = \{y, (0.6, 0.7, 0.3)\}$ and $CO(R) = \{y, (0.7, 0.7, 0.3)\}$ are NFSCSs.

Then $NFScl(G) = \cap\{CO(P), CO(R) : CO(P), CO(R)$ are NFSCSs in X and $G \subset CO(P), G \subset CO(R)\}$.

Now $NFScl(G) = \{y, (0.6, 0.7, 0.3)\} = CO(P)$. Thus $CO(P)$ is $NFScl(G)$.

Proposition 2.18[3]. *Let* (X, T^{μ}) *be a NFSTS over X. Then the following properties hold*:

(i) $NFScl(CO(P)) = CO(NFS\ \mathrm{int}(P))$.

(ii) $NFS\ \mathrm{int}(CO(P)) = CO(NFScl(P))$.

3. Separation Axioms Induced by NFSTS

Definition 3.1. A NFSTS (X, T^{μ}) over X is called neutrosophic fuzzy supra topological T_0-space, denoted by NFST-T_0-space if for any distinct pair of points x, y of X, there exists one NFSOS A in T^{μ} such that any one of the following two conditions holds:

(i) $x \in A$ but $y \notin A$ or

(ii) $x \notin A, y \in A$.

Theorem 3.2. *A NFSTS* (X, T^{μ}) *over X is NFST-*T_0*-space if and only if for each pair of distinct points* x, y *of* X, $NFScl\{x\} \neq NFScl\{y\}$.

Proof. Sufficiency. Suppose that $x, y \in X, x \neq y$ and $NFScl\{x\} \neq NFScl\{y\}$.

Let $u \in X$ such that $u \in NFScl\{x\}$ but $u \notin NFScl\{y\}$.

We claim that $x \notin NFScl\{y\}$, for $x \in NFScl\{y\}$, then $NFScl\{x\} \subset NFScl\{y\}$. This contradicts the fact that $u \notin NFScl\{y\}$. Consequently, $x \in \{NFScl\{y\}\}^c$ to which y does not belong. Thus (X, T^{μ}) is NFST-T_0-space.

Sufficiency. Let a NFSTS (X, T^{μ}) over X be NFST-T_0-space and

$x, y \in X, x \neq y$. Then there exists NFSOS A for which $x \in A$ or $y \in A$. Then A^c is a NFSCS for which $x \in A$ and $y \in A^c$.

Since $NFScl\{y\}$ is the smallest NFSCS containing y, so $NFScl\{y\} \subset A^c$ and therefore $x \notin NFScl\{y\}$.

Hence $NFScl\{x\} \neq NFScl\{y\}$.

Definition 3.3. Let (X, T^{μ}) be a NFSTS over X and E be a subset of X. Then the classes T^E of all intersections of E with T^{μ}-NFSOSs of X belong to T^{μ} is a topology on E. This topology is called relative neutrosophic fuzzy supra topological space or neutrosophic fuzzy supra subspace.

Example 3.4. Let $X = \{y\}$ and consider the family $T^{\mu} = \{0_N, 1_N, P, Q, R\}$ where $P = \{y, (0.3, 0.4, 0.3)\}$, $Q = \{y, (0.4, 0.5, 0.3)\}$, $R = \{y, (0.5, 0.6, 0.3)\}$. Then (X, T^{μ}) is NFSTS on X. Again $T^E = \{0_N, 1_N, P\}$ and (E, T^E) is a relative neutrosophic fuzzy supra topological space.

Definition 3.5. Let (X, T^{μ}) be a NFSTS over X and p be any property in X. Then we call p as neutrosophic supra hereditary (in short, NSH) if it appears in a relative neutrosophic fuzzy supra topological space.

Theorem 3.6. *Let NFSTS (X, T^{μ}) over X be NFST-T_0-space. Then the relative neutrosophic fuzzy supra topological space (E, T^E) is NFST-T_0-space.*

Proof. Given (X, T^{μ}) is NFSTS and $E \subseteq X$. Let $e_1, e_2 \in E$ and $e_1 \neq e_2$. We have to show that (E, T^E) is NFST-T_0-space. As $e_1, e_2 \in E$, so $e_1, e_2 \in X$. There exists NFSOS $A \subseteq X$ such that A containing one of e_1, e_2, but not both. Now we have $e_1 \in E$ and $e_1 \in A$. Then $e_1 \in E \cap A$ or $e_2 \in E$ and $e_2 \in A$. Then $e_2 \in E \cap A$.

Hence (E, T^E) is NFST-T_0-space.

Definition 3.7. Let $f : (X, T_1^{\mu}) \to (Y, T_2^{\mu})$ be a neutrosophic fuzzy supra homeomorphism (in short, NFSH). Let *p* be any property in *X*. We say that *p* is neutrosophic fuzzy topological property if *p* appears in *Y*.

Theorem 3.8. *The property $p =$ NFST-T_0-space is neutrosophic fuzzy topological property.*

Proof. Let (X, T_1^{μ}) and (Y, T_2^{μ}) be two NFSTSs and $f : (X, T_1^{\mu}) \to (Y, T_2^{\mu})$ be a NFSH. Let (X, T_1^{μ}) be obey the property $p =$ NFST-T_0-space. We have to show that (Y, T_2^{μ}) is NFST-T_0-space. Let $y_1, y_2 \in Y$ and $y_1 \neq y_2$. Since *f* is bijective, there exists $x_1 \neq x_2$ such that $y_1 = f(x_1), y_2 = f(x_2)$. Since (Y, T_2^{μ}) is NFST-T_0-space, there exists a NFSOS A such that $x_1 \in A, x_2 \notin A$ or $x_1 \notin A, x_2 \in A$. Since *f* is bijective, $f(A)$ is also a NFSOS in Y and for $y_1, y_2 \in Y, y_1 \neq y_2$, we have $f(x_1) \in f(A), f(x_2) \notin f(A)$ or $f(x_1) \notin f(A), f(x_2) \in f(A)$. Hence (Y, T_2^{μ}) is NFST-T_0-space.

Theorem 3.9. *The two NFSTSs (X, T_1^{μ}) and (Y, T_2^{μ}) are NFST-T_0-spaces if and only if $X \times Y$ is a NFST-T_0-space.*

Proof. Sufficiency. Let (X, T_1^{μ}) and (Y, T_2^{μ}) be two NFST-T_0-spaces. If $(x_1, y_1) \neq (x_2, y_2)$, then either $x_1 \neq x_2$ or $y_1 \neq y_2$. We claim $x_1 \neq x_2$. Since X is NFST-T_0-space, there exists a NFSOS A such that $x_1 \in A, x_2 \notin A$ or $x_1 \notin A, x_2 \in A$. Now NFSOS $A \times Y \in X \times Y$. So $(x_1, y_1) \in A \times Y$ or $(x_2, y_2) \in A \times Y$. Thus $X \times Y$ is a NFST-T_0-space.

Necessity. Let $X \times Y$ be a NFST-T_0-space. To show that X is NFST-T_0-space. We take $x_1, x_2 \in X, x_1 \neq x_2$. Then there exists two points $(x_1, y), (x_2, y)$ where $(x_1, y) \neq (x_2, y)$. Since $X \times Y$ is a NFST-T_0-space, there exists a NFSOS $A \in X \times Y$ such that $(x_1, y) \in A, (x_2, y) \notin A$ or $(x_1, y) \notin A, (x_2, y) \in A$. Since $(x_1, y) \in A, (x_2, y) \notin A$, therefore $x_1 \in A, x_2 \notin A$ or $x_1 \notin A, x_2 \in A$. Thus (X, T_1^{μ}) is NFST-T_0-space. Similarly, we can show that (Y, T_2^{μ}) is NFST-T_0-space.

Definition 3.10. A NFSTS (X, T^{μ}) over X is called fuzzy neutrosophic supra topological T_1-space, denoted by NFST-T_1 if for any pair of distinct points x, y of X, there exist two NFSOSs A, B in T^{μ} such that the following two conditions hold:

(i) $x \in A, x \notin B$ and

(ii) $y \in B, y \notin A$.

Remark 3.11. Every NFST-T_1-space is NFST-T_0-space. But the converse is not true.

Example 3.12. Let $X = \{x, y\}$ and consider the family $T^{\mu} = \{0_N, 1_N, P\}$ where $P = \{y, (0.4, 0.3, 0.5)\}$. Then (X, T^{μ}) is a NFST-T_0-space on X but not NFST-T_1-space.

Theorem 3.13. *Let NFSTS (X, T^{μ}) over X be NFST-T_1-space. Then the relative neutrosophic fuzzy supra topological space (E, T^E) is NFST-T_1-space.*

Proof. Given (X, T^{μ}) is NFST-T_1-space. Let $e_1, e_2 \in E$ and $e_1 \neq e_2$. We have to show that (E, T^E) is NFST-T_1-space. As $e_1, e_2 \in E$, so $e_1, e_2 \in X$. There exists two NFSOSs $A, B \subseteq X$ such that $e_1 \in A, e_2 \notin B$ and $e_1 \in B, e_2 \notin A$. Now we have $e_1 \in E \cap A, e_2 \notin E \cap B$ and $e_1 \in E \cap B, e_2 \notin E \cap A$. Hence (E, T^E) is NFST-T_1-space.

Theorem 3.14. *The property $p =$NFST-T_1-space is neutrosophic fuzzy topological property.*

Proof. Let (X, T_1^{μ}) and (Y, T_2^{μ}) be two NFSTSs and $f : (X, T_1^{\mu}) \to (Y, T_2^{\mu})$ be a NFSH. Let (X, T_1^{μ}) be obey the property $p =$NFST-T_1-space. We have to show that (Y, T_2^{μ}) is NFST-T_1-space. Let $y_1, y_2 \in Y$ and $y_1 \neq y_2$. Since f is bijective, there exists $x_1 \neq x_2$ such that

$y_1 = f(x_1), y_2 = f(x_2)$. Since (Y, T_2^{μ}) is NFST-T_1-space, there exist two NFSOSs A and B such that $x_1 \in A, x_1 \notin B$ and $x_2 \in B, x_2 \notin A$. Since f is bijective, $f(A)$ and $f(B)$ are also two NFSOSs in Y and for $y_1, y_2 \in Y, y_1 \neq y_2$, we have f $f(x_1) \in f(A), f(x_1) \notin f(B)$ and $f(x_2) \in f(B), f(x_2) \notin f(A)$. Hence (Y, T_2^{μ}) is NFST-T_1-space.

Theorem 3.15. *The two NFSTSs (X, T_1^{μ}) and (Y, T_2^{μ}) are NFST-T_1-spaces if and only if $X \times Y$ is a NFST-T_1-space.*

Proof. Sufficiency. Let (X, T_1^{μ}) and (Y, T_2^{μ}) be two NFST-T_0-spaces. If $(x_1, y_1) \neq (x_2, y_2)$, then either $x_1 \neq x_2$ or $y_1 \neq y_2$. We claim $x_1 \neq x_2$. Since X is NFST-T_1-space, there exists a NFSOS A and B such that $x_1 \in A, x_1 \notin B$ or $x_1 \in B, x_2 \notin A$. Now $A \times Y$ and $B \times Y$ are two NFSOSs $X \times Y$. So $(x_1, y_1) \in A \times Y$ but $(x_1, y_1) \notin B$ and $(x_2, y_2) \in B \times Y$ but $(x_2, y_2) \notin A \times Y$. Thus $X \times Y$ is a NFST-T_1-space.

Necessity. Let $X \times Y$ be a NFST-T_1-space. To show that X is NFST-T_1-space. We take $x_1, x_2 \in X, x_1 \neq x_2$. Then there exists two points $(x_1, y), (x_2, y) \in X \times Y$. Since $X \times Y$ is a NFST-T_1-space, there exists a NFSOSs $A, B \in X \times Y$ such that $(x_1, y) \in A, (x_2, y) \notin A$ and $(x_2, y) \in A, (x_1, y) \notin A$. Now there exist two NFSOSs A_1, A_2 such that $A_1 \times A_2 \subset A$. Since $(x_1, y) \in A, (x_2, y) \notin A$ and $(x_2, y) \in A, (x_1, y) \notin B$, therefore $x_1 \in A_1, x_2 \notin A_1$ or $x_2 \notin A_2, x_1 \in A_1$. Thus (X, T_1^{μ}) is NFST-T_1-space. Similarly, we can show that (Y, T_2^{μ}) is NFST-T_1-space.

4. Conclusion

In this paper, we have studied some new separation axioms in neutrosophic fuzzy supra topological spaces. We have introduced and investigated neutrosophic fuzzy supra $T_i (i = 1, 2)$ spaces. We have shown that every neutrosophic fuzzy supra T_1-space is neutrosophic fuzzy supra T_0-space. But the converse is not true. We have established some other relationship between these two spaces. We have studied some characterization theorems of these two newly defined spaces. It is expected that the work done will help in further investigation of the separation axioms in neutrosophic fuzzy supra topological spaces. In the future, it is hoped that the notion of separation which have been studied here can also be extended in neutrosophic crisp supra bi-topological spaces [19], neutrosophic supra tri-topological spaces [4] etc. The author will extend the notion of separation axioms in refined neutrosophic crisp supra topological spaces and neutrosophic crisp supra bi-topological spaces as new possible research works.

References

[1] M. F. Abd. El-Monsef and A. E. Ramadan, On fuzzy supra topological spaces, Indian J. Pure Appl. Math. 18(4) (1987), 322-329.

[2] K. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems 20 (1986), 87-96.

[3] V. A. Babu and J. Aswini, Fuzzy neutrosophic supra topological spaces, Advances and Applications in Mathematical Sciences 20(8) (2021), 1339-1347.

[4] V. A. Babu and P. Rajasekhar, Neutrosophic crisp supra bi and tri-topological spaces, Advances and Applications in Mathematical Sciences 20(5) (2021), 917-929.

[5] R. Das and B. C. Tripathy, Neutrosophic multiset topological space, Neutrosophic Sets and Systems 35 (2020), 142-152.

[6] S. Das and B. C. Tripathy, Pairwise neutrosophic b - open set in neutrosophic bitopological spaces, Neutrosophic Sets and Systems 38 (2020), 135-144.

[7] S. Das, Neutrosophic supra simply open set and neutrosophic supra simply compact space, Neutrosophic Sets and Systems 43 (2021), 105-113.

[8] S. Das and S. Pramanik, Neutrosophic tritopological space, Neutrosophic Sets and Systems 45 (2021), 366-377.

[9] S. Das and R. Das, Neutrosophic separation axioms, Neutrosophic Sets and Systems 49 (2022), 103-110.

[10] S. Das and B. C. Tripathy, Pairwise neutrosophic *b*-open set in neutrosophic bitopological spaces, Neutrosophic Sets and Systems 38 (2020), 135-144.

[11] R. Dhar, S. Paul, S. Paul and G. Pal, Some neutrosophic compact spaces via grills, Electronic Journal of Mathematical Analysis and Applications 11(1) (2023), 231-240.

[12] G. Jayaparthasarathy, V. F. Little Flower and M. Arockia Dasan, Neutrosophic supra topological applications in data mining process, Neutrosophic Sets and System 27 (2019), 80-97.

[13] S. Mashhour, A. A. Allan, F.S. Mahmoud and F. H. Khedr, On supra topological space, Indian J. Pure Appl. Math. 14(4) (1983), 502-510.

[14] T. Y. Ozturk and A. Ozkan, Neutrosophic bitopological spaces, Neutrosophic Sets and Systems 30 (2019), 88-97.

[15] G. Pal and R. Dhar, Compactness in neutrosophic minimal spaces, Journal of Tripura Mathematical Society 22 (2020), 68-74.

[16] G. Pal, R. Dhar and B. C. Tripathy, Minimal structures and grill in neutrosophic topological Spaces, Neutrosophic Sets and Systems 51 (2022), 134-145.

[17] G. Pal, B. C. Tripathy and R. Dhar, On continuity in minimal structure neutrosophic topological space, Neutrosophic Sets and Systems 51 (2022), 360-370.

[18] K. Riad and AI-Hamido, A new approach of neutrosophic topological space, International Journal of Neutrosophic Science 7(1) (2020), 55-61.

[19] K. Riad and AI-Hamido, On neutrosophic crisp supra bi-topological spaces, International Journal of Neutrosophic Science 4(1) (2020), 36-46.

[20] A. A. Salama and S. A. Alblowi, Neutrosophic set and neutrosophic topological space, ISOR J. Math. 3(4) (2012), 31-35.

[21] A. A. Salama and S. A. Alblowi, Generalized neutrosophic set and generalized neutrosophic topological space, Comp. Sci. Engg. 21 (2012), 129-132.

[22] A. A. Salama, F. Smarandache and S. A. Alblowi, New neutrosophic crisp topological concepts, Neutrosophic Sets and Systems 2 (2014), 50-54.

[23] F. Smarandache, Neutrosophy. Neutrosophic Probability, Set and Logic, ProQuest Information and Learning. Ann Arbor, Michigan, USA, 105, 1998.

[24] F. Smarandache, A Unifying Field in logics, Neutrosophy: Neutrosophic Probability, Set and Logic. American Research Press, 1999.

[25] F. Smarandache, Neutrosophic set: a generalization of the intuitionistic fuzzy sets, International Journal of Pure and Applied Mathematics 24 (2005), 287-297.

[26] F. Smarandache and S. Pramanik, New trends in neutrosophic theory and applications. Brussels, Belgium, EU: Pons Editions, 2016.

[27] N. Turah, An overview of intuitionistic fuzzy supratopological spaces, Hacettepe Journal of Mathematics and Statistics 32 (2003), 17-26.

[28] L. A. Zadeh, Fuzzy sets, Information and Control 8 (1965), 338-353.