Advances and Applications in Mathematical Sciences
Volume 23, Issue 11, September 2024, Pages 901-912

ALLOCATION AND REORGANIZATION OF CENTRALIZED RESOURCES WITH THE AIM OF DEVELOPING NEW EFFECTIVE DECISION-MAKING UNITS WITH THE HIGHEST PRODUCTIVITY USING THE DATA ENVELOPMENT ANALYSIS TECHNIQUE

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Abstract

The data envelopment analysis (DEA) is a technique for evaluating the performance and measuring the efficiency of a collection of decision-making units with multiple inputs and outputs. One of the DEA applications is to gather resources and inputs and organize and reallocate them in order to develop new efficient decision-making units. Using a linear programming model, the current study makes an attempt to present a method to develop efficient decision-making units by regrouping and reallocating the resources.

1. Introduction

To address practical and managerial issues in most cases, groups of comparable and homogeneous decision-making units operate under a central decision-making process [10-18]. In this regard, the decision-maker has enough power and responsibility to control and plan the production process, determine decision-making parameters, and manage resource consumption. In most cases, due to the scarcity of available resources as well as the heavy financial expenses to provide them, managers are always looking for a solution to obtain the maximum possible productivity as a result of

2020 Mathematics Subject Classification: Primary: 91B28; Secondary: 90C90, 90B10.
Keywords: Data Envelopment Analysis, Centralized Resource Reallocation, Efficiency.
Received September 9, 2024; Accepted September 30; 2024

consuming these valuable resources. Therefore, it is necessary to adopt a strategy that, by allocating the resources and organizing the units, avoids the wasteful consumption of resources and inputs, e.g., expensive equipment, and obtains maximum productivity and income. Today, a large number of institutions and economic enterprises are facing the problems associated with reallocating resources and inputs in order to obtain the maximum possible output and income. For example, a hospital hires a number of specialty doctors for different departments. It is obvious that the board of directors is trying to gain achievements such as training of expert staff, providing quality services to clients, earning money, and publishing and printing specialized articles. It is obvious that some goals, e.g., earning money and training specialized human resources, are considered as the main goals, while the others, e.g., publishing specialized books and articles, are among the intermediate goals, or in a country, the central government may decide to allocate a fixed budget to the provinces to mitigate poverty or increase and strengthen their economic and social infrastructure. In such issues, the question that arises is that, considering the large volume of information, with what scientific and practical solution is the central decision-maker able to fairly allocate the fixed cost to the decision-making units (the provinces here). In address this issue, the current study presents a method to make all new units productive by re-allocating centralized resources using the DEA technique.

The DEA technique was first introduced by Charnes, Cooper, and Rhodes in 1987, and then developed by Banker, Charnes and Cooper in 1984. Their models were quickly and widely used by researchers to evaluate the performance and measure the efficiency of decision-making units in different economic, commercial, and similar areas. After presenting the initial models of the DEA technique, the researchers decided to provide methods to improve the efficiency of inefficient units and use the available potential in the inputs. Among these methods, reallocation of resources and reorganization of units in order to develop new efficient units can be mentioned. The problem of reallocation of resources and reorganization of units has become one of the most important application areas of DEA technique, with the potential of providing valuable insights for resource allocation [1-2-13-15].

Various studies have been conducted on the problem of reallocation of resources due to its increasingly important role in the DEA technique. Considering the effect of resource allocation on unit efficiency, Golany et al. [11] used a collective model of the DEA for resource allocation. Cook and Zhu [8] presented a view that maximizes the level of the post-allocation efficiency using a set of shared weights. Amirteimoori and et al. [3-4] proposed a method to reduce a constant amount of resources in all the decision-making units without changing the efficiency. Hosseinzadeh Lotfi et al. [16] proposed a method based on the modified Russell model for re-allocation of centralized resources. Using the reverse DEA technique, Hadi-Vencheh et al. [12] presented a method for re-allocation of resources and estimation of input and output of units. Hosseinzadeh Lotfi et al. [17] presented a method for re-allocation of centralized resources with probabilistic data based on the DEA technique. Yu et al. [20] used data envelopment analysis technique to allocate resources in 18 airports in Taiwan. Lozano et al. [19] presented a method for centralized resource reallocation. Asmild et al. [7] proposed a method based on the BCC model for centralized resource allocation. Amirteimoori and Kordrostami [5] proposed a method to develop the implementation of supply and demand changes in centralized decision-making. Hosseinzadeh Lotfi et al. [15] and Hatami Marbini et al. [14] introduced the common weights method in the DEA technique and ideal planning for resource allocation. Amirteimoori et al. [2] assumed that after allocating the input resources and model set, all the decision-making units have to be efficient under a set of common weights. Wu et al. [21] proposed a multi-objective linear programming model for resource allocation. Zhang et al. [22] proposed a linear programming model with bounded variables for resource allocation.

Considering the fact that the resource allocation problem is one of the classic research topics in the fields of management and economics, many researchers have focused on its development and management. Managers are always looking for ways to get the most possible income and output by using the available resources in production technology and to prevent the wasteful consumption of these valuable resources, especially when the resources include expensive and rare equipment. In this regard, the current study presents a method to aggregate the resources and inputs and then re-allocate them in such a way that all new units are efficient and with the highest productivity.

2. Problem Description and Mathematical Models

This section addresses the resource allocation problem and determination of the model set. To this end, an introduction to the resource allocation problem is presented in Section 2.1 and the proposed method for re-allocation of centralized resources is presented in Section 2.2.

2.1. Preliminaries

The idea of re-allocation of centralized resources was first proposed by Lozano and Villa [19] and their model was used as the basis for subsequent studies on re-allocation of centralized resources. In order to reallocate centralized resources, they first aggregated the inputs and outputs in the form of $\sum_{j=1}^{n} y_{rj}$ $r = 1, 2, \ldots, s$, $\sum_{j=1}^{n} x_{ij}$ $i = 1, 2, \ldots, m$ and proposed the following radial model.

Model (2.1):

$$\min \theta - \varepsilon\left(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+\right)$$

$$\text{s.t} \quad \sum_{k=1}^{n}\sum_{j=1}^{n} \lambda_{kj} x_{ij} = \theta \sum_{j=1}^{n} x_{ij} - s_i^- \quad i = 1, 2, \ldots, m$$

$$\sum_{k=1}^{n}\sum_{j=1}^{n} \lambda_{kj} y_{rj} = \sum_{j=1}^{n} y_{rj} + s_r^+ \quad r = 1, 2, \ldots, s$$

$$\sum_{j=1}^{n} \lambda_{kj} = 1 \quad k = 1, 2, \ldots, n$$

$$\lambda_{kj} \geq 0,\ s_i^- \geq 0,\ s_r^+ \geq 0 \ \forall\, k, j, i, r$$

where s_i^- is the auxiliary variable of the aggregated inputs and s_r^+ is the covariate of aggregated outputs, ε is a positive non-Archimedean number that is smaller than any positive real number. λ_{kj} is the contribution of the new k-th decision-making unit from the j-th old decision-making unit. Now let $(\theta^*, \lambda^*, s^{-*}, s^{+*})$ is the optimal solution of the model (2.1), then the image of the model (2.1) contains n new efficient decision-making units with the following coordinates:

$$x_k^* = \sum_{j=1}^{n} \lambda_{kj}^* x_j, \; y_k^* = \sum_{j=1}^{n} \lambda_{kj}^* y_j \;\; k = 1, 2, \ldots, n$$

Apa et al. [6] presented a model in order to provide the MPSS image for decision-making units. In their method, it was first necessary to determine the CCR effective units by solving the CCR model. Eslami et al. [9] presented a method to provide the nearest MPSS decision-making unit for decision-making units. To this end, they had to solve four models, one of which was nonlinear, and sometimes more than one MPSS unit was derived as a model for a decision-making unit.

2.2. Proposed Method

Consider an organization consisting of n comparable homogeneous decision-making units with coordinates (x_j, y_j) $j = 1, 2, \ldots, n$ where an m-fold vector with values $x_j = (x_{1j}, \ldots, x_{mj})$ can generate an output s-fold vector $y_j = (y_{1j}, \ldots, y_{sj})$. Now consider a situation where a central decision-maker in such an organization is able to control the resources of the decision-making units.

The current study presents a model for the resource allocation and the organization of decision-making units under the supervision of a central decision-making unit, so that the maximum possible reduction in the total inputs and the maximum possible increase in the total outputs are achieved, and develop decision-making units with the aim of preventing wasteful consumption of resources as well as achieving new efficient units with the highest productivity. Now, in order to re-allocate centralized resources and organize decision-making units with the highest possible efficiency, inputs and outputs are aggregated in the form of $\sum_{j=1}^{n} y_{rj}$ $(r = 1, 2, \ldots, s)$, $\sum_{j=1}^{n} x_{ij}$ $(i = 1, 2, \ldots, m)$, and then model (2.2) in the following form is solved.

Model (2.2):

$$\max \sum_{i=1}^{m} q_i + \sum_{r=1}^{s} f_r$$

$$\text{s.t} \quad \sum_{k=1}^{n} \sum_{j=E} \lambda_{kj} x_{ij} \leq (d - q_i) \sum_{j=1}^{n} x_{ij}$$

$$\sum_{k=1}^{n} \sum_{j=E} \lambda_{kj} y_{rj} \geq (d + f_r) \sum_{j=1}^{n} y_{rj}$$

$$\sum_{j=E}^{n} \lambda_{kj} = 1 \;\; k = 1, 2, \ldots, n$$

$$\lambda_{kj} \geq 0, \, d \geq 0, \, q_i \geq 0, \, f_r \geq 0, \, \forall \, k, \, j, \, i, \, r$$

Where E is defined as follows. Let the optimal solution is the model (2.3):

Model (2.3):

$$\min \theta_p - \varepsilon(1s^- + 1s^+)$$

$$\text{s.t} \quad X\lambda + s^- = \theta_p x_p$$

$$Y\lambda - s^+ = y_p$$

$$\lambda \geq 0, \, s^- \geq 0, \, s^+ \geq 0$$

In each optimal solution, we have

$$E = \{j \mid \theta_j^* = 1, \, (s^{-*}, \, s^{+*}) = (0, \, 0)\}$$

and ε is a positive non-Archimedean number that is smaller than any positive real number. Also, the variable λ_{kj} is the contribution of the new k-th decision-making unit from the j-th old decision-making unit. Now let $(\lambda^*, d^*, q^*, f^*, s^{-*}, s^{+*})$ is the optimal solution of the model (2.2), then the image of the model (2.2) contains n new decision-making units with the highest productivity and with the following coordinates:

$$x_k^* = \sum_{j=E} \lambda_{kj}^* x_j, \; y_k^* = \sum_{j=E} \lambda_{kj}^* y_j \;\; k = 1, 2, \ldots, n.$$

The model (2.2) acts such that, in case of high market demand, the decision-making units can be imaged to the points (x_k^*, y_k^*) $k = 1, 2, \ldots, n.$

3. Numerical Examples

In this section, a numerical example from previous studies and a real case are used to demonstrate the proposed approach. First, in section 3.1, our proposed method is applied to the data set from Zhang et al. [22] and the results are compared with those of the other methods. Then, in section 3.2, the proposed approach is applied to the data from 12 hospitals, which includes two input and two output indicators and can be under the supervision of a central unit.

3.1. Example 1

The data from Zhang et al. [22] in the single-input and single-output form are listed in Table 3.1, where θ_{CCR} and θ_{BCC} represent the efficiency values derived for the CCR and BCC models in terms of inputs.

Table 3.1. optimal solutions for CCR and BCC models.

DMU	A	B	C	D	E	F	G	H	I	J
x	3	3.5	4	4.5	4.5	5	5.5	6	8	9
y	1	2.5	2.4	2.7	3.6	5	4.4	6	7	7
θ_{CCR}	0.333	0.714	0.6	0.6	0.8	1	0.8	1	0.875	0.778
θ_{BCC}	1	1	0.867	0.804	0.924	1	0.844	1	1	0.889

The units A, B, F, H, I and J are efficient for the CCR model and F and H are efficient for the BCC model. Thus, the units F and H as well as all the points falling between the line FH are MPSS.

Table 3.2. Images of decision-making units with Lozano's and Villa's model and Zhang's et al. model as well as the proposed model.

	Lozano's and Villa's model		Zhang's et al. model		Proposed model	
DMU	x'	y'	x''	y''	x^*	y^*
A	3.5	2.5	5.33	5.33	6	6
B	3.5	2.5	5.25	5.25	6	6
C	3.5	2.5	5.41	3.90	6	6
D	3.5	2.5	5.43	3.52	6	6
E	3.5	2.5	5.32	4.60	6	6
F	3.5	2.5	5.24	5.24	6	6
G	3.5	2.5	5.31	4.25	6	6
H	3.5	2.5	5.22	5.22	6	6
I	3.5	2.5	5.25	5.25	6	6
J	3.5	2.5	5.37	5.37	6	6

According to Table 3.2, in the method presented in the study by Lozano and Villa [19], all the decision-making units are imaged to DMU_B. DMU_B is the efficient unit for the BCC but not the efficient unit for the CCR, which shows that the model presented in Lozano and Villa [19] is not able to provide the MPSS image for the decision-making units. Also, with Zhang's et al. model [22], the efficient units A, B, F, H and I, as well as the weakly efficient unit J are imaged to the MPSS, but the ineffective units C, D, E and G are not imaged to MPSS.

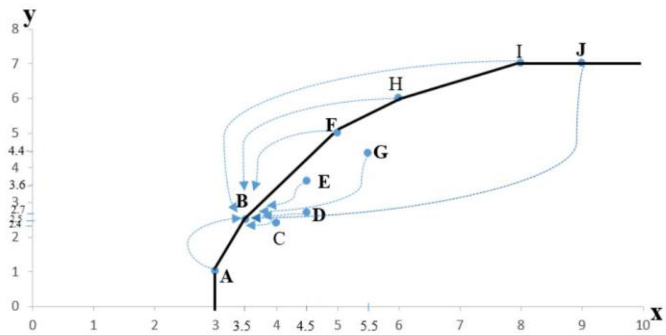

Figure 3.1. Image of all decision-making units with the model proposed by Lozano and Villa.

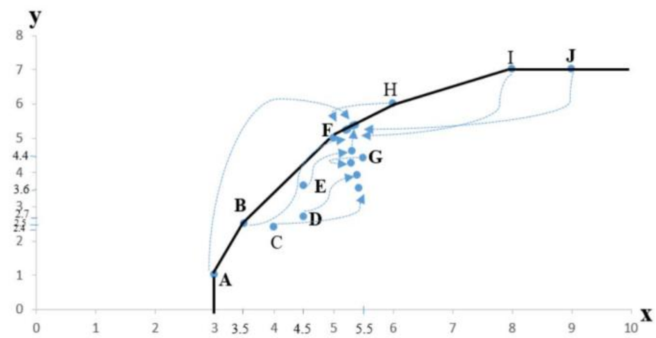

Figure 3.2. Image of all decision-making units with the model proposed by Zhang et al.

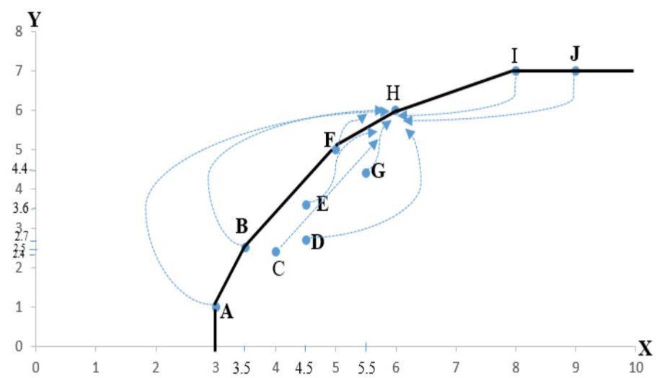

Figure 3.3. Image of all decision-making units with the proposed model.

As shown in Figure 3.2, in the image derived with the proposed model presented in this research model (2.2), all the decision-making units are imaged to point H. As seen in Figure 3.3, DMU_H has the highest efficiency at a fixed scale.

3.2. Examples 2

In this section, the proposed method is used to reallocate the resources of 12 hospitals. Each hospital includes two input indicators, i.e., I_1 doctors and I_2 nurses, and two output indicators, i.e., o_1 treated outpatient patients and o_2 treated hospitalized patients. The values of the input and output indicators of each hospital are listed in Table 3.3.

Table 3.3. Inputs and outputs of the decision-making unit.

DMU	1	2	3	4	5	6	7	8	9	10	11	12	Total
I_1	20	19	25	27	22	55	33	31	30	50	53	38	403
I_2	151	131	160	168	158	275	235	206	244	268	306	284	2566
o_1	100	80	90	120	70	80	100	85	76	75	80	70	1026
o_2	500	350	450	600	300	450	500	450	380	410	440	400	5230

Table 3.3 presents the results from the implementation of the proposed model for re-allocation of centralized resources in order to achieve the highest point of MPSS.

Table 3.4. Efficiency values of units with the BCC and CCR models and the proposed model.

DMU	1	2	3	4	5	6	7	8	9	10	11	12	total
θ_{CCR}	1	0.88	0.8	1	0.65	0.49	0.62	0.62	0.5	0.42	0.4	0.42	
θ_{BCC}	1	2	0.9	1	0.85	0.56	0.64	0.7	0.63	0.51	0.46	0.5	
x_1^*	27	27	27	27	27	27	27	27	27	27	27	27	324
x_2^*	168	168	168	168	168	168	168	168	168	168	168	168	2016
y_1^*	120	120	120	120	120	120	120	120	120	120	120	120	1440
y_2^*	600	600	600	600	600	600	600	600	600	600	600	600	7200

According to Table 3.4, the decision-making units 1 and 4 are efficient for the BCC and CCR. Thus, DMU1 and DMU4 are MPSS. By solving the proposed model of this study, all the units are imaged to DMU4, which has the highest productivity.

5. Conclusion

Using the DEA technique, the current study presents a model for the reallocation of centralized resources and the organization of decision-making units in order to develop new units with the highest MPSS. The highest image of MPSS has the feature that the decision-making units can be imaged to these points if required by the market demand.

Unlike other common methods of re-allocation of concentrated resources, the model proposed in this study is linear, and new units with the highest point of the MPSS procedure are derived only by solving a model.

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