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GENERATOR INTERSECTION GRAPH

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Abstract

In this paper, we introduce graphical structure of a group G called as Generator intersection graph, denoted by $\mathbb{I}(G)$. We will discuss some basic properties of this graph such as connectivity, diameter, clique number, regularity, completeness etc. Moreover we discussed in brief results on generator intersection graph of $\mathbb{Q}_8$, $\mathbb{Z}_n$ and $\mathbb{D}_n$. The discussion contains the cyclic properties, regularity, Euler graph, diameter, girth, triangularity etc., of corresponding generator intersection graph of $\mathbb{Q}_8$, $\mathbb{Z}_n$ and $\mathbb{D}_n$.

1. Introduction

The study of zero divisor graph was firstly done by I. Beck [3] in 1988. After this Anderson and Livingston [2] gives in brief the properties of Zero divisor graph over the commutative ring with unit element. Later on many researcher come forward to studied zero divisor graph on various algebraic structures such as semi group, group, ring, commutative ring, non

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commutative ring, field, vector space etc. Presently S. Akabari, M. Habibi [1], studied zero divisor graph on ideals of the ring. Later on Anshuman Das studied the zero divisor graph on various algebraic structures like, Non-Zero Component Union Graph of a Finite Dimensional Vector Space[4], Non-Zero Component Union Graph of a finite Dimensional Vector Space[5], Subspace Inclusion Graph of Vector Space[6], Non-Zero Component Graph of a Finite Fields[7], etc and try to characterized the algebraic structures and their properties with graph and vice versa. R. A. Muneshwar and K. L. Bondar [9], introduced a union graph, inclusion graph and intersection graph of a topological space which can be found in [10], [11]. In previous work [8] we introduced the graphical structure of vector space over a finite field called as "Bases Intersection Graph" and discussed some basic properties of it.

2. Definition and Preliminaries

In this section for the sake of understanding of reader we recall some basic definitions related to generator intersection graph. An ordered pair $G = (V, E)$ is called graph where V is set of vertices and E is set of edges, be the binary relation on V If there is an edge between any two element u, v of V then they are adjacent. $H = (W, F)$ is sub graph of $G = (V, E)$ where $\phi \neq W \subseteq V$ and $F \subseteq E$. If V is finite, the graph G is said to be finite, otherwise it is infinite. If all the vertices of G are pairwise adjacent, then G is said to be complete. A complete sub graph of a graph G is called a clique. A maximal clique is a clique which is maximal with respect to inclusion. The clique number of G, written as $\omega(G)$, is the maximum size of a clique in G. The chromatic number of G, denoted as $\chi(G)$, is the minimum number of colorsneeded to label the vertices so that the adjacent vertices receive different colors. A graph is said to be triangulated if for any vertex u in V, there exist u, w in V, such that (u, v, w) is a triangle. The distance between two vertices $u, v \in V$, $d(u, v)$ is defined as the length of the shortest path joining u and v, if it exists. Otherwise, $d(u, v)$ is defined as ∞. The diameter of a graph is defined as $diam(G) = \max (u, v) \in V\, d(u, v)$, the largest distance between pairs of vertices of the graph, if it exists. Otherwise, $diam\,(G)$ is defined as ∞. The girth of a graph is the length of its shortest cycle, if it exists. Otherwise, it is defined as ∞. If G is a group than G is called abelian group if $\forall\, a, b \in G, a * b = b * a$ otherwise it is called non abelian group. The group G is called cyclic group if $\forall\, g \in G$ and for a fixed $a \neq e \in G, g = a^n$ for some $n \in \mathcal{I}$ this, a is called generator of group G.

3. Generator Intersection Graph

Definition 3.1. Let $(G, *)$ be any finite group. We define a "Generator intersection group", $\mathbb{I}(G) = (\mathcal{G}, E)$ where $\mathcal{G}$ is set of generators of group G, and for $S_1, S_2, \in \mathcal{G}, S_1 \sim S_2$ or $(S_1, S_2) \in E$ if and only if $S_1 \cap S_2 \neq \phi$.

Example 3.2. Let $K_4 = \{e, a, b, a * b\}$, a Kelvin 4 group having generator sets as, $S_1 = \{a, b\}, S_2 = \{a, a * b\}, S_3 = \{b, a * b\}$ then $\mathbb{I}(K_4)$ is shown in Figure 1.

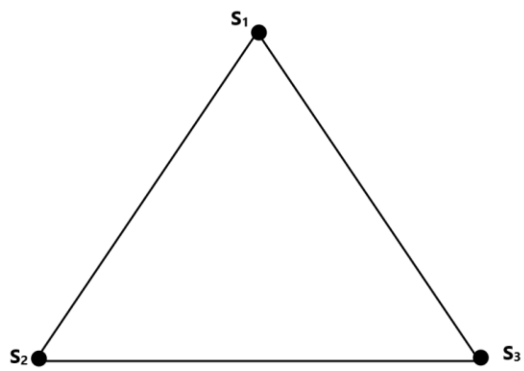

Figure 1. $\mathbb{I}(K_4)$.

Example 3.3. Let $\mathbb{Z}_6 = \{0, 1, 2, 3, 4, 5\}$ under addition mod 6 be a group having generator sets as, $S_1 = \{1\}, S_2 = \{5\}$ then $\mathbb{I}(\mathbb{Z}_6)$ is shown in Figure 2.

Now we generalize some properties of generator intersection graph.

Figure 2. $\mathbb{I}(\mathbb{Z}_6)$.

4. Some Results on Generator Intersection Graph of Group *G*

Theorem 4.1. *Let G be any group then G is cyclic if and only if* $\mathbb{I}(G)$ *is null graph*.

Proof. Firstly suppose G is cyclic group and hence $G = \langle a \rangle$, where $a \neq e \in G$. Therefore there is nothing in common between any two generator set of a group G. By the definition of generator intersection graph there is no edge in $\mathbb{I}(G)$, and hence $\mathbb{I}(G)$ is null graph.

Conversely, let $\mathbb{I}(G)$ is a null graph and therefore there is no element common in two different generating sets of a group G. That is G generated by single element and hence G is cyclic group.

Corollary 4.2. *If G is cyclic group then* $\mathbb{I}(G)$ *is not connected and hence it is not complete*.

Proof. It is obvious from theorem 4.1.

Remark. Since $\mathbb{Z}_n$ is cyclic group then the corresponding generator inter- section graph $\mathbb{I}(\mathbb{Z}_n)$ is null graph, and hence not connected, not complete. Thus here after we work only for non cyclic group.

Theorem 4.3. *If G is not cyclic group then* $\mathbb{I}(G)$ *is connected graph*.

Proof. It is given that G is not cyclic group and therefore G is not generated by single element. Which means generating set of G contains at least two elements. Let $S_1 = \{a, b\}$ be any generating set of G. Then clearly $a \neq b^n$ or $b \neq a^n$, for $n \in I$. As $S_1 = \{a, b\}$, $a, b \in G$, is a generating set, we have $S_2 = \{a^{-1}, b\}$ is also a generating set, (provided $O(a)$ and $O(b) \neq 2$ if it is the case we will take $S_2 = \{a, a * b\}$, such that, $S_1 \cap S_2 \neq \phi$ and hence $S_1 \sim S_2$. Thus $\mathbb{I}(G)$ is connected graph.

5. The Generator Intersection Graph of $\mathbb{Q}_8$

In this section we will briefly discussed about the generator intersection graph of Quaternian group i.e. $\mathbb{I}(\mathbb{Q}_8)$.

Theorem 5.1. *If* $G = \mathbb{Q}_8$ *be the Quaternian group and* $\mathbb{I}(\mathbb{Q}_8)$ *be the corresponding generator intersection graph then following are hold.*

1. $\mathbb{Q}_8$ *is not cyclic.*

2. $\mathbb{I}(\mathbb{Q}_8)$ *is connected.*

3. $\mathbb{I}(\mathbb{Q}_8)$ *is not complete.*

4. $\mathbb{I}(\mathbb{Q}_8)$ *is regular graph.*

5. $\mathbb{I}(\mathbb{Q}_8)$ *is an Euler graph.*

6. $\mathbb{I}(\mathbb{Q}_8)$ *is Hamiltonian graph.*

7. *The diameter of* $\mathbb{I}(\mathbb{Q}_8)$ *is* 2.

8. *Girth of* $\mathbb{I}(\mathbb{Q}_8)$ *is* 3.

9. $\mathbb{I}(\mathbb{Q}_8)$ *is triangulated graph.*

10. *Clique number of* $\mathbb{I}(\mathbb{Q}_8)$ *is* 4.

11. *The chromatic number of* $\mathbb{I}(\mathbb{Q}_8)$ *is* 4.

12. *Domination number of* $\mathbb{I}(\mathbb{Q}_8)$ *is* 3.

Proof. It is given that $G = \mathbb{Q}_8$ is the Quaternian group and $\mathbb{I}(\mathbb{Q}_8)$ be the corresponding generator intersection graph. We have $\mathbb{Q}_8 = \pm 1, \pm i, \pm j, \pm k$, such that $i^2 = j^2 = k^2 = -1$. Then generating sets of $\mathbb{Q}_8$ are given by,

$S_1 = \{i, j\}, S_2 = \{i, -j\}, S_3 = \{i, k\}, S_4 = \{i, -k\},$

$S_5 = \{-i, j\}, S_6 = \{-i, -j\}, S_7 = \{-i, k\}, S_8 = \{-i, -k\}$

$S_9 = \{j, k\}, S_{10} = \{j, -k\}, S_{11} = \{-j, k\}, S_{12} = \{-j, -k\}.$

The corresponding generator intersection graph of $\mathbb{Q}_8$ i.e. $\mathbb{I}(\mathbb{Q}_8)$ is shown in Figure 3.

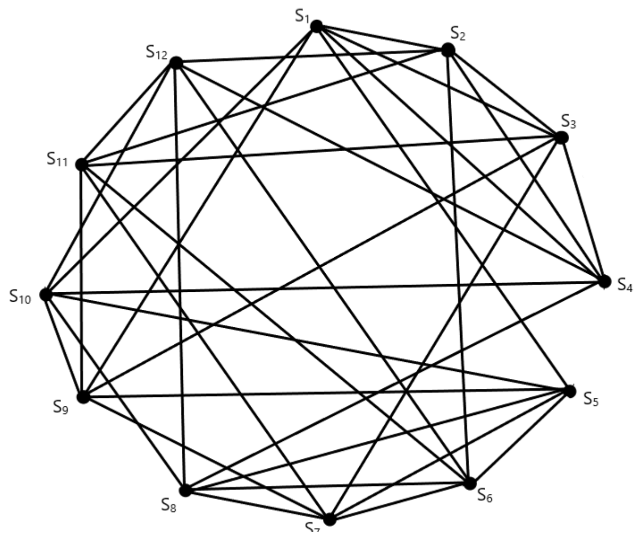

Figure 3. $\mathbb{I}(\mathbb{Q}_8)$.

1. Since $S_1 \cap S_2 = \{i\} \neq \phi$ therefore by definition $S_1 \sim S_2$ and hence $\mathbb{I}(\mathbb{Q}_8)$ is not null graph and hence from theorem 4.1, $\mathbb{Q}_8$ is not cyclic.

2. Let $A_1 = \{a_1, b_1\}$ and $A_2 = \{a_2, b_2\}$ be any two generating sets of $\mathbb{Q}_8$.

Case 1. If $A_1 \cap A_2 \neq \phi$ then clearly by definition of $\mathbb{I}(\mathbb{Q}_8)$, $A_1 \sim A_2$.

Case 2. If $A_1 \cap A_2 \neq \phi$, now as $A_1 = \{a_1, b_1\}$ is a generating set than either $A_1' = \{a_1^{-1}, b_1\}$ or $A_1' = \{a_1, b_1^{-1}\}$ is a generating set such that $A_1 \sim A'$ and $A_2 \sim A'$. Thus $A_1 \sim A' \sim A_2$ be the required path between A_1 and A_2. Thus we see there is a path between any two vertices of $\mathbb{I}(\mathbb{Q}_8)$, hence the graph is connected.

3. Now if $A_1 = \{a_1, b_1\}$ is a generating set of $\mathbb{Q}_8$ then clearly $A_1' = \{a_1^{-1}, b_1^{-1}\}$ is also a generating set such that $A_1 \cap A_2 \neq \phi$ which means $A_1 \nsim A'$, hence $\mathbb{I}(\mathbb{Q}_8)$ is not complete.

4. Let us consider any vertex of $\mathbb{I}(\mathbb{Q}_8)$. Without loss of generality let $S_1 = \{i, j\}$, then there are 3 vertices of $\mathbb{I}(\mathbb{Q}_8)$ having i in common. Similarly there are 3 vertices of $\mathbb{I}(\mathbb{Q}_8)$ having j in common. Thus in general any vertex

of $\mathbb{I}(\mathbb{Q}_8)$ is connected with 6 vertices and hence degree of each vertex of $\mathbb{I}(\mathbb{Q}_8)$ is 6. Hence the graph is 6-Regular Graph.

5. Since from the above discussion degree of each vertex is 6 which is even hence the graph is Eulerian Graph.

6. We have from 2, $\mathbb{I}(\mathbb{Q}_8)$ is connected graph and degree of each vertex is 6 which is half of the number of vertices . Hence $\mathbb{I}(\mathbb{Q}_8)$ is a hamiltonian graph.

7. Let $A_1 = \{a_1, b_1\}$ and $A_2 = \{a_2, b_2\}$ be any two generating sets of $\mathbb{Q}_8$.

Case 1. If $A_1 \cap A_2 \neq \phi$ then clearly by definition of $\mathbb{I}(\mathbb{Q}_8)$, $A_1 \sim A_2$, and $dist(A_1, A_2) = 1$.

Case 2. If $A_1 \cap A_2 \neq \phi$, now as $A_1 = \{a_1, b_1\}$ is a generating set than either $A_1' = \{a_1^{-1}, b_1\}$ or $A_1' = \{a_1, b_1^{-1}\}$ is a generating set such that $A_1 \sim A'$ and $A_2 \sim A'$. Thus $A_1 \sim A' \sim A_2$ be the required path between A_1 and A_2. Therefore $dist(A_1, A_2) = 2$. Thus the maximum possible distance between any two vertices of $\mathbb{I}(\mathbb{Q}_8)$ is 2, hence the $dist(\mathbb{I}(\mathbb{Q}_8)) = 2$.

8. Clearly the graph $\mathbb{I}(\mathbb{Q}_8)$ has no cycle of length 1 or 2 since it is simple. So we have too many 3-cycles in the graph which is the smallest one, i.e. we have for $S_1 = \{i, j\}, S_2 = \{i, -j\}$ and $S_3 = \{i, k\}$ which implies $S_1 \sim S_2 \sim S_3$. Hence $girth(\mathbb{I}(\mathbb{Q}_8)) = 3$.

9. Let $S_1 = \{a, b\}$ be any vertex then $S_2 = \{a, b^{-1}\}$ is also a generating set. Similarly $S_3 = \{a, a * b\}$ is also a generating set such that $S_1 \sim S_2 \sim S_3 \sim S_1$. Since S_1 was chosen arbitrary and is the vertex of triangle. Hence the graph $\mathbb{I}(\mathbb{Q}_8)$ is triangulated graph.

10. Let $K = \{$ Collection of all generating sets having one element in common$\}$. i.e. without loss of generality let it be i. i.e. $\cap K = \{i\}$. Then clearly this K will be a clique having clique number 4 since we have only four possibilities $\{j, -j, k, -k\}$ along with i to be a basis.

11. From the above discussion we have clique number of $\mathbb{I}(\mathbb{Q}_8)$ is 4, then the chromatic number, $\chi(\mathbb{I}(\mathbb{Q}_8)) \geqslant 4$. To established the equality, we demonstrate 4 coloring of $\mathbb{I}(\mathbb{Q}_8)$, therefor for any vertex of $\mathbb{I}(\mathbb{Q}_8)$ we assure that there are 3 distinct vertices which are adjacent as discussed above, hence the chromatic number is 4.

12. Let us define $D = \{A = \{a, b\}, A' = \{-a, -b\},$ where $a, b \in \pm i, \pm j, \pm k$ and $a \neq \pm b\}$. If S be any vertex of the graph $\mathbb{I}(\mathbb{Q}_8)$ then $\pm i \in S$ or $\pm j \in S$ or $\pm k \in S$ and hence $S \sim A$ or $S \sim A'$. Thus D is domination set. Clearly, D is minimal dominating set and hence domination number of the graph $\mathbb{I}(\mathbb{Q}_8)$ is 2.

6. The Generator Intersection Graph of $\mathbb{D}_n$

Let $\mathbb{D}_n$ is the Dihedral group of n rotations and n reflections, which is a non abelian and hence obviously non cyclic group, generated by one rotation and one reflection. The n number of rotations in $\mathbb{D}_n$ forms a cyclic subgroup of order n in $\mathbb{D}_n$. Therefore there are $\phi(n)$ numbers of elements which generates the subgroup of rotations and there are n reflections. Thus the total number of generating sets which generates $\mathbb{D}_n$ are $n * \phi(n)$. And hence the generator intersection graph of $\mathbb{D}_n$ i.e. $\mathbb{I}(\mathbb{D}_n)$ has $n * \phi(n)$ vertices (For $n \geqslant 3$). In this section we prove all results for $n \geqslant 3$, because for $n = 2$, $\mathbb{D}_n$ is isomorphic to $\mathbb{K}_4$ and hence from Figure 2 graph is complete graph.

Example 6.1. $\mathbb{D}_3 = \{\rho_0, \rho_{120}, \rho_{240}, r_1, r_2, r_3\}$. Then the generating sets of $\mathbb{D}_3$ are given by, $S_1 = \{\rho_{120}, r_1\}, S_2 = \{\rho_{240}, r_1\}, S_3 = \{\rho_{120}, r_2\}, S_4 = \{\rho_{240}, r_2\}, S_5 = \{\rho_{120}, r_3\}, S_6 = \{\rho_{240}, r_3\}$ and hence the corresponding generating intersection graph of $\mathbb{D}_3$ i.e. $\mathbb{I}(\mathbb{D}_3)$ is given by Figure 4.

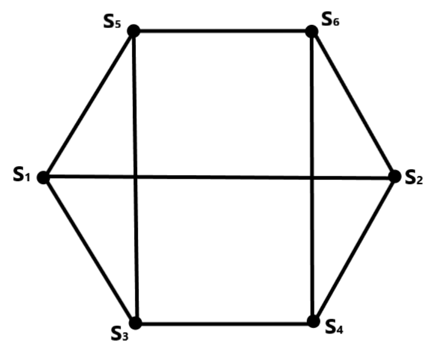

Figure 4. $\mathbb{I}(\mathbb{D}_3)$.

Now we generalize some properties of generator intersection graph of $\mathbb{D}_n$ i.e. $\mathbb{I}(\mathbb{D}_n)$.

Theorem 6.2. *If* $\mathbb{D}_n$ *is a Dihedral Group then degree of every vertex of the graph* $\mathbb{I}(\mathbb{D}_n)$ *is given by* $n + \phi(n) - 2$.

Proof. Let $\mathbb{D}_n$ is a Dihedral Group of order $2n$ then by above discussion the graph $\mathbb{I}(\mathbb{D}_n)$ has $n * \phi(n)$ vertices and each vertex is set of one rotation and one reection viz. $\{\rho, r\}$. Therefore each vertex is connected with those n vertices having the same rotation r in common and $\phi(n)$ vertices having that reection in common with the repetition of 2. Hence degree of vertex is $n + \phi(n) - 2$.

Theorem 6.3. *If* $\mathbb{D}_n$ *is a Dihedral Group then* $\mathbb{I}(\mathbb{D}_n)$ *is regular graph*.

Proof. Proof of the theorem is directly follows from Theorem 6.2.

Remark. As $\mathbb{I}(\mathbb{D}_n)$ is a regular graph then minimum degree δ, edge connectivity, vertex connectivity are same as degree of vertex i.e. $n + \phi(n) - 2$.

Theorem 6.4. *If* $\mathbb{D}_n$ *is a Dihedral Group then degree of graph* $\mathbb{I}(\mathbb{D}_n)$ *is* $n * \phi(n)[n + \phi(n) - 2]$.

Proof. Let $\mathbb{D}_n$ is a Dihedral Group then by Theorem 5.3, $\mathbb{I}(\mathbb{D}_n)$ is a regular graph and by Theorem 5.2, degree of every vertex of the graph $\mathbb{I}(\mathbb{D}_n)$ is $n + \phi(n) - 2$. Hence degree of the graph $\mathbb{I}(\mathbb{D}_n)$ is $n * \phi(n)[n + \phi(n) - 2]$.

Theorem 6.5. *If $\mathbb{D}_n$ is a Dihedral Group then $\mathbb{I}(\mathbb{D}_n)$ is connected graph, for all $n \geq 2$.*

Proof. Let $\mathbb{D}_n$ is a Dihedral Group of order $2n$. The proof of the theorem is organized in following three cases.

Case 1. If $n = 2$ then the group $\mathbb{D}_n$ is isomorphic to $\mathbb{K}_4$ and hence the graph $\mathbb{I}(\mathbb{D}_n)$ is complete, and we are through.

Case 2. If $n \geqslant 3$, let $S_1 = \{\rho_k, r_l\}$ and $S_2 = \{\rho_{k'}, r_{l'}\}$ be any two generating sets of $\mathbb{D}_n$.

Sub Case 2.1. If $k = k'$ or $l = l'$ then $S_1 \cap S_2 \neq \phi$, i.e. $S_1 \sim S_2$ be a required path from S_1 to S_2.

Sub Case 2.2. If $k \neq k'$ and $l \neq l'$ then we construct $S_3 = \{\rho_{k'}, r_l\}$, such that, $S_1 \cap S_3 \neq \phi \Rightarrow S_1 \sim S_3$. Similarly, $S_1 \cap S_3 \neq \phi \Rightarrow S_2 \sim S_3$. Thus we have $S_1 \sim S_3 \sim S_2$ be a required path from S_1 to S_2. Thus, the graph $\mathbb{I}(\mathbb{D}_n)$ is connected.

Theorem 6.6. *If $\mathbb{D}_n$ is a given Dihedral Group and $\mathbb{I}(\mathbb{D}_n)$ be the corresponding generator intersection graph, then $\mathbb{I}(\mathbb{D}_n)$ is not complete for $n > 3$.*

Proof. It is given that $\mathbb{D}_n$ is a Dihedral Group of order n and $\mathbb{I}(\mathbb{D}_n)$ be the corresponding generator intersection graph. Now for $n = 2, \mathbb{D}_n$ is isomorphic to $\mathbb{K}_4$ and hence by default it is complete.

Now for $n \geqslant 3$, if $S_1 = \{\rho_k, r_l\}$, is a generating set then $S_2 = \{\rho_n - 1, r_2\}$, is also a generating set such that $r_1 \neq r_2$ and $S_1 \cap S_2 \neq \phi$ therefore by definition $S_1 \nsim S_2$ and hence $\mathbb{D}_n$ is not complete.

Theorem 6.7. *If $\mathbb{D}_n$ is a given Dihedral Group and $\mathbb{I}(\mathbb{D}_n)$ be the* *corresponding generator intersection graph, then*

$$diam\,[\mathbb{I}(\mathbb{D}_n)] = \begin{cases} 1 & if\ n = 2 \\ 2 & if\ n \geqslant 3. \end{cases}$$

Proof. It is given that $\mathbb{D}_n$ is a Dihedral Group of order n and $\mathbb{I}(\mathbb{D}_n)$ be the corresponding generator intersection graph.

Case 1. If $n = 2$ then the group $\mathbb{D}_n$ is isomorphic to $\mathbb{K}_4$ and hence the graph $\mathbb{I}(\mathbb{D}_n)$ is complete. So $diam\,[\mathbb{I}(\mathbb{D}_n)] = 2$.

Case 2. If $n \geqslant 3$, let $S_1 = \{\rho_k, r_l\}$, and $S_2 = \{\rho_{k'}, r_{l'}\}$, be any two generating sets of $\mathbb{D}_n$.

Sub Case 2.1. If $k = k'$ or $l = l'$ then $S_1 \cap S_2 \neq \phi$, i.e. $S_1 \sim S_2$ be a required path from S_1 to S_2. Hence distance $(S_1, S_2) = 1$.

Sub Case 2.2. If $k \neq k'$ and $l \neq l'$ then we construct $S_3 = \{\rho_{k'}, r_l\}$, such that, $S_1 \cap S_3 \neq \phi \Rightarrow S_1 \sim S_3$. Similarly, $S_1 \cap S_3 \neq \phi \Rightarrow S_2 \sim S_3$. Thus we have $S_1 \sim S_3 \sim S_2$ be a required path from S_1 to S_2. Hence distance $(S_1, S_2) = 2$. Since, S_1 and S_2 are chosen arbitrary, we conclude $diam\,[\mathbb{I}(\mathbb{D}_n)] = 2\ \forall\, n \geqslant 3$.

Theorem 6.8. *If C is a collection of those generating sets having one fixed rotation in common then C be the maximal clique of $\mathbb{I}(\mathbb{D}_n)$ and clique number is $\omega\,[\mathbb{I}(\mathbb{D}_n)] = n$.*

Proof. Let $\mathbb{D}_n$ is a Dihedral Group of order $2n$. Now for any fixed rotation ρ_k, consider a collection C of generating sets of $\mathbb{D}_n$ having ρ_k in common i.e. $C = \{S_1, S_2, \ldots, S_n\}$, such that $S_i = \{\rho_k, r_i\}$ for all $i = 1, 2, \ldots, n$ and hence $\cap S_i = \{\rho_k\}, \forall\, i = 1, 2, \ldots, n$. Then clearly the collection C is a clique in the graph $\mathbb{I}(\mathbb{D}_n)$.

Claim: C is a maximal clique.

On the contrary, assume that $C' = C \cup S'$ be any clique such that $S' \notin C$, which means $\rho_k \notin S'$. Therefore there exist another rotation ρ'_k

such that S' contains ρ'_k and any one reflection, let it be r_i. Therefore $S' = \{\rho'_K, r_i\}$ be a generating set. Now in C' we have $S' \cap S_i = \{r_i\} \Rightarrow S' \sim S_i$ and $\forall\, k \neq i,\, S' \cap S_k = \{\phi\} \Rightarrow S' \nsim S_k$. For $i = k = 1, 2, \ldots, n,$ a contradiction. Hence C is the required maximal clique of the graph $\mathbb{I}(\mathbb{D}_n)$ and hence clique number of $\mathbb{D}_n$ is $\omega[\mathbb{I}(\mathbb{D}_n)] = n$.

Theorem 6.9. *If $\mathbb{D}_n$ is a given Dihedral Group and $\mathbb{I}(\mathbb{D}_n)$ be the corresponding generator intersection graph, then it is an Euler graph.*

Proof. Let $\mathbb{D}_n$ be a given Dihedral Group and $\mathbb{I}(\mathbb{D}_n)$ be the corresponding generator intersection graph, then there are $n * \phi(n)$ number of vertices and degree of each vertex is $n + \phi(n) - 2.$ Also $\mathbb{I}(\mathbb{D}_n)$ is connected graph.

Case 1. If n is even than degree of each vertex is even. Hence the graph is Euler graph.

Case 2. If n is odd then we have either zero or two vertices of odd degree. Hence in both the cases graph is Eulerian graph.

7. Conclusion

In this paper, we introduced a generator intersection graph $\mathbb{I}(G)$ of a group G. We give some basic results regarding $\mathbb{I}(G)$ and briefly discussed the properties of non abelian group like $\mathbb{K}_4, \mathbb{Q}_8, \mathbb{D}_n$. Discussion contains the basic properties of graph such as connectedness, completeness, girth, diameter, clique number, chromatic number, degree of vertex, degree of graph etc.

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