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ON INTUITIONISTIC FUZZY CONTRA γ^* GENERALIZED CONTINUOUS MAPPINGS

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Abstract

In this paper, we have introduced the notion of intuitionistic fuzzy contra γ^* generalized continuous mappings. Furthermore we have provided some properties of intuitionistic fuzzy contra γ^* generalized continuous mappings and discussed some fascinating theorems.

I. Introduction

Atanassov [1] introduced the idea of intuitionistic fuzzy sets using the notion of fuzzy sets. Coker [2] introduced intuitionistic fuzzy topological spaces using the notion of intuitionistic fuzzy sets. Later this was followed by the introduction of intuitionistic fuzzy γ^* generalized closed sets by Riya, V. M and Jayanthi, D [7] in 2017 which was simultaneously followed by the introduction of intuitionistic fuzzy γ^* generalized continuous mappings [8] by

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the same authors. We have now extended our idea towards intuitionistic fuzzy contra γ^* generalized continuous mappings and discussed some of their properties.

2. Preliminaries

Definition 2.1[1]. An intuitionistic fuzzy set (IFS for short) A is an object having the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$$

where the functions $\mu_A : X \to [0, 1]$ and $\nu_A : X \to [0, 1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \le \mu_A(x) + \nu_A(x) \le 1$ for each $x \in X$. Denote by IFS(X), the set of all intuitionistic fuzzy sets in X.

An intuitionistic fuzzy set A in X is simply denoted by $A = \langle x, \mu_A, \nu_A \rangle$ instead of denoting $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$.

Definition 2.2[1]. Let A and B be two IFSs of the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$$

and

$$B = \{\langle x, \mu_B(x), \nu_B(x)\rangle : x \in X\}.$$

Then,

(a) $A \subseteq B$ if and only if $\mu_A(x) \le \mu_B(x)$ and $\nu_A(x) \ge \nu_B(x)$ for all $x \in X$,

(b) $A = B$ if and only if $A \subseteq B$ and $A \supseteq B$,

(c) $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle : x \in X\}$,

(d) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle : x \in X\}$,

(e) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle : x \in X\}$.

The intuitionistic fuzzy sets $0_\sim = \langle x, 0, 1\rangle$ and $1_\sim = \langle x, 1, 0\rangle$ are respectively the empty set and the whole set of X.

Definition 2.3[2]**.** An intuitionistic fuzzy topology (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms:

(i) $0_\sim, 1_\sim \in \tau$,

(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,

(iii) $\cup G_i \in \tau$ for any family $\{G_i : i \in J\} \in \tau$.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X. The complement A^c of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 2.4[12]**.** Two IFSs A and B are said to be q-coincident $(A_q B$ in short) if and only if there exits an element $x \in X$ such that $\mu_A(x) > \nu_B(x)$ or $\nu_A(x) < \mu_B(x)$.

Definition 2.5[12]**.** Two IFSs A and B are said to be not q-coincident (B in short) if and only if $A \subseteq B^c$.

Definition 2.6[3]**.** An intuitionistic fuzzy point (IFP for short), written as $p_{(\alpha, \beta)}$, is defined to be an IFS of X given by

$$p_{(\alpha, \beta)} = \begin{cases} (\alpha, \beta) & \text{if } x = p, \\ (0, 1) & \text{otherwise.} \end{cases}$$

An IFP $p_{(\alpha, \beta)}$ is said to belong to a set A if $\alpha \leq \mu_A$ and $\beta \geq \nu_A$.

Definition 2.7[4]**.** An IFS $A = \langle x, \mu_A, \nu_A \rangle$ in an IFTS (X, τ) is said to be an

(i) intuitionistic fuzzy γ closed set (IFγCS in short) if $cl(\text{int}(A)) \cap \text{int}(cl(A)) \subseteq A$

(ii) intuitionistic fuzzy γ open set (IFγOS in short) if $A \subseteq \text{int}(cl(A)) \cup cl(\text{int}(A))$

Definition 2.8[4]**.** Let A be an IFS in an IFTS (X, τ). Then the γ-interior and γ-closure of A are defined as

$\gamma \operatorname{int}(A) = \bigcup\{G \mid G$ is an IFγOS in X and $G \subseteq A\}$

$\gamma cl(A) = \bigcup\{K \mid K$ is an IFγCS in X and $A \subseteq K\}$

Note that for any IFS A in (X, τ), we have $\gamma cl(A^c) = (\gamma \operatorname{int}(A))^c$ and $\operatorname{int}(A)^c = (\gamma \operatorname{int}(A))^c$.

Result 2.9[6]. Let A be an IFS in (X, τ), then

$$\gamma cl(A) \supseteq A \cup cl(\operatorname{int}(A)) \cap \operatorname{int}(cl(A))$$

$$\gamma \operatorname{int}(A) \subseteq A \cup cl(\operatorname{int}(A)) \cap \operatorname{int}(cl(A))$$

Corollary 2.10[3]. *Let A, $A_i(I \in J)$ be intuitionistic fuzzy sets in X and B, $B_i(j \in K)$ be intuitionistic fuzzy sets in Y and $f : X \to Y$ be a function. Then*

a) $A_1 \subseteq A_2 \Rightarrow f(A_1) \subseteq f(A_2)$

b) $B_1 \subseteq B_2 \Rightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$

c) $A \subseteq f^{-1}(f(A))$ [*If f is injective, then* $A = f^{-1}(f(A))$]

d) $f(f^{-1}(B)) \subseteq B$ [*If f is surjective, then* $B = f(f^{-1}(B))$]

e) $f^{-1}(\bigcup B_j) = \bigcup f^{-1}(B_j)$

f) $f^{-1}(\bigcap B_j) = \bigcap f^{-1}(B_j)$

g) $f^{-1}(0_\sim) = 0_\sim$

h) $f^{-1}(1_\sim) = 1_\sim$

i) $f^{-1}(B^c) = (f^{-1}(B))^c$.

Definition 2.11[7]. An IFS A of an IFTS (X, τ) is said to be an intuitionistic fuzzy γ^* generalized closed set (briefly IFγ^*GCS) if $cl(\operatorname{int}(A)) \cap \operatorname{int}(cl(A)) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in (X, τ).

Definition 2.12[8]. A mapping $f : (X, \tau) \to (Y, \sigma)$ is called an intuitionistic fuzzy γ^* generalized continuous (IF γ^*G continuous for short) mapping if $f^{-1}(V)$ is an IF γ^*GCS in (X, τ) for every IFCS V of (Y, σ).

3. Intuitionistic Fuzzy Contra γ^* Generalized Continuous Mappings

In this section we have introduced intuitionistic fuzzy contra γ^* generalized continuous mappings and investigated some of their properties.

Definition 3.1. A mapping $f : (X, \tau) \to (Y, \sigma)$ is said to be an intuitionistic fuzzy contra γ^* generalized continuous (IF contra γ^*G continuous for short) mapping if $f^{-1}(A)$ is an IF γ^*GCS in X for every IFOS A in Y.

We use the notation $A = \langle x, (\mu_a, \mu_b), (\nu_a, \nu_b) \rangle$ instead of $A = \langle x, (a/\mu_a, b/\mu_b), (a/\nu\mu_a, b/\nu_b) \rangle$ in the following examples.

Example 3.2. Let $X = \{a, b\}, Y = \{u, v\}$ and $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b) \rangle, G_2 = \langle y, (0.4_u, 0.4_v), (0.6_u, 0.6_v) \rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. The IFS $G_2 = \langle y, (0.4_u, 0.4_v), (0.6_u, 0.6_v) \rangle$ is an IFOS in Y. Then $f^{-1}(G_2) = \langle x, (0.4_u, 0.4_v), (0.6_u, 0.6_v) \rangle$ is an IF γ^*GCS in X as $f^{-1}(G_2) \subseteq G_1$ and $cl(\text{int}(f^{-1}(G_2))) \cap \text{int}(cl(f^{-1}(G_2))) = 0_\sim \subseteq G_1$, where G_1 is an IFOS in X.

Therefore f is an IF contra γ^*G continuous mapping.

Theorem 3.3. *Every IF contra continuous mapping is an IF contra γ^*G continuous mapping but not conversely in general.*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be an IF contra continuous mapping [6]. Let V be an IFOS in Y. Then $f^{-1}(V)$ is an IFCS in X, by hypothesis. Since every IFCS is an IF γ^*GCS [8], $f^{-1}(V)$ is an IF γ^*GCS in X. Hence f is an IF contra γ^*G continuous mapping.

Example 3.4. In Example 3.2, f is an IF contra γ^*G continuous mapping but since $f^{-1}(G_2) = \langle x, (0.4_a, 0.4_b), (0.6_a, 0.6_b)\rangle$ is not an IFCS in X, as $cl(f^{-1}(G_2)) = G_1^c \neq f^{-1}(G_2)$, f is not an IF contra continuous mapping.

Theorem 3.5. *Every IF contra semi continuous mapping is an IF contra γ^*G continuous mapping but not conversely in general.*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be an IF contra semi continuous mapping. Let V be an IFOS in Y. Then $f^{-1}(V)$ is an IFSCS in X, by hypothesis. Since every IFSCS is an IF γ^*GCS [8], $f^{-1}(V)$ is an IF γ^*GCS in X. Hence f is an IF contra γ^*G continuous mapping.

Example 3.6. In Example 3.2, f is an IF contra γ^*G continuous mapping. We have $\text{int}(cl(f^{-1}(G_2))) = \text{int}(G_1^c) = G_1 \not\subseteq f^{-1}(G_2) = \langle x, (0.4_a, 0.4_b), (0.6_a, 0.6_b)\rangle$. Hence $f^{-1}(G_2)$ is not an IFSCS in X.

Hence f is not an IF contra semi continuous mapping.

Theorem 3.7. *Every IF contra pre continuous mapping is an IF contra γ^*G continuous mapping but not conversely in general.*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be an IF contra pre continuous mapping [6]. Let V be an IFOS in Y. Then $f^{-1}(V)$ is an IFPCS in X, by hypothesis. Since every IFPCS is an IF γ^*GCS [8], $f^{-1}(V)$ is an IF γ^*GCS in X. Hence f is an IF contra γ^*G continuous mapping.

Example 3.8. Let $X = \{a, b\}$ and $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ be IFTs on X and Y respectively, where $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$ and $G_2 = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$ and $G_3 = \langle y, (0.4_u, 0.4_v), (0.6_u, 0.6_v)\rangle$. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. The IFS $G_3 = \langle y, (0.4_u, 0.4_v), (0.6_u, 0.6_v)\rangle$ is an IFOS in Y. Then $f^{-1}(G_3) = \langle x, (0.4_u, 0.4_v), (0.6_u, 0.6_v)\rangle$ is an IF γ^*GCS as $cl(\text{int}(f^{-1}(G_3))) \cap \text{int}(cl(f^{-1}(G_3))) = G_1^c \cap G_2 = G_2 \subseteq G_1$ where $f^{-1}(G_3) \subseteq G_1$

but not an IFPCS as $cl(\text{int}(f^{-1}(G_3))) = G_1^c \not\subseteq f^{-1}(G_3)$. Hence f is not an IF contra pre continuous mapping.

Theorem 3.9. *Every IF contra α continuous mapping is an IF contra $\gamma^* G$ continuous mapping but not conversely in general.*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be an IF contra α continuous mapping [6]. Let V be an IFOS in Y. Then $f^{-1}(V)$ is an IFαCS in X, by hypothesis. Since every IFαCS is an IF γ^*GCS [8], $f^{-1}(V)$ is an IF γ^*GCS in X. Hence f is an IF contra γ^*G continuous mapping.

Example 3.10. Let $X = \{a, b\}, Y = \{u, v\}$ and $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle, G_2 = \langle y, (0.4_u, 0.4_v), (0.6_u, 0.6_v)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. The IFS $G_2 = \langle y, (0.4_u, 0.4_v), (0.6_u, 0.6_v)\rangle$ is an IFOS in Y. Then $f^{-1}(G_2) = \langle x, (0.4_u, 0.4_v), (0.6_u, 0.6_v)\rangle$ is an IF γ^*GCS in X as $cl(\text{int}(f^{-1}(G_2))) \cap \text{int}(cl(f^{-1}(G_2))) = 0_\sim \subseteq G_1 = 0_\sim \subseteq G_1$ where $f^{-1}(G_2) \subseteq G_1$ but not an IFγCS in (X, τ) as $cl(\text{int}(f^{-1}(G_2^c))) = G_1^c \not\subseteq f^{-1}(G_2)$. Hence f is not an IF contra α continuous mapping.

Theorem 3.11. *Every IF contra γ continuous mapping is an IF contra $\gamma^* G$ continuous mapping but not conversely in general.*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be an IF contra γ continuous mapping [4]. Let V be an IFOS in Y. Then $f^{-1}(V)$ is an IFγCS in X, by hypothesis. Since every IFγCS is an IF γ^*GCS [8], $f^{-1}(V)$ is an IF γ^*GCS in X. Hence f is an IF contra γ^*G continuous mapping.

Example 3.12. Let $X = \{a, b\}, Y = \{u, v\}$ and $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle, G_2 = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$ and $G_3 = \langle y, (0.4_u, 0.6_v), (0.6_u, 0.4_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping

$f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. The IFS $G_3 = \langle y, (0.4_u, 0.6_v), (0.6_u, 0.4_v)\rangle$ is an IFOS in Y. Then $f^{-1}(G_3) = \langle x, (0.4_u, 0.6_v), (0.6_u, 0.4_v)\rangle$ is an IFγ^*GCS in X as $cl(\text{int}(f^{-1}(G_3))) \cap \text{int}(cl(f^{-1}(G_3))) = G_1^c \subseteq G_1 = G_1^c \subseteq G_1$ and $f^{-1}(G_3) \subseteq G_1$ but not an IFγCS in (X, τ) as $cl(\text{int}(f^{-1}(G_3))) \cap \text{int}(cl(f^{-1}(G_3)))$ $= G_1^c \subseteq G_1 = G_1^c \not\subseteq f^{-1}(G_3)$. Hence $f^{-1}(G_3)$ is not an IFγCS in X. Hence f is not an IF contra γ continuous mapping.

Remark 3.13. Every IF contra generalized continuous mapping is an IF contra γ^*G continuous mapping but not conversely in general.

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be an IF contra generalized continuous mapping [9]. Let V be an IFOS in Y. Then $f^{-1}(V)$ is an IFGCS in X, by hypothesis. Since every IFGCS is an IFγ^*GCS [8], $f^{-1}(V)$ is an IFγ^*GCS in X. Hence f is an IF contra γ^*G continuous mapping.

Example 3.14. In Example 3.2, f is an IFγ^*G continuous mapping but not an IF contra generalized continuous mapping as $cl(f^{-1}(G_2)) = G_1^c \not\subseteq G_1$, where $f^{-1}(G_2) \subseteq G_1$.

Remark 3.15. Every IF contra γ generalized continuous mapping is an IF contra γ^*G continuous mapping but not conversely in general.

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be an IF contra γ generalized continuous mapping [10]. Let V be an IFOS in Y. Then $f^{-1}(V)$ is an IFγGCS in X, by hypothesis. Since every IFγGCS is an IFγ^*GCS [8], $f^{-1}(V)$ is an IFγ^*GCS in X. Hence f is an IF contra γ^*G continuous mapping.

Example 3.16. Let $X = \{a, b\}$, $Y = \{u, v\}$ and $G_1 = \langle x, (0.5_a, 0.36_b), (0.5_a, 0.7_b)\rangle$, $G_2 = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$ and $G_3 = \langle y, (0.3_u, 0.2_v), (0.7_u, 0.8_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping

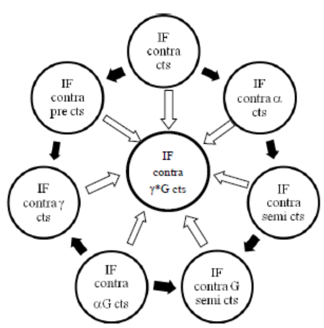
$f:(X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. The IFS $G_3 = \langle y, (0.3_u, 0.2_v), (0.7_u, 0.8_v)\rangle$ is an IFOS in Y. Then $f^{-1}(G_3) = \langle x, (0.3_u, 0.2_v), (0.7_u, 0.8_v)\rangle$ is an IFγ^*GCS in X as $cl(\text{int}(f^{-1}(G_3))) \cap \text{int}(cl(f^{-1}(G_3))) = 0_\sim \cap G_1 = 0_\sim \subseteq G_1, G_2$ and $f^{-1}(G_3) \subseteq G_1, G_2$ but not an IFγCS in (X, τ) as $\alpha cl(\text{int}(f^{-1}(G_3)))$ $= f^{-1}(G_3) \cup cl(\text{int}(cl(f^{-1}(G_3)))) = f^{-1}(G_3) \cup G_1^c = G_1^c \not\subseteq G_1, G_2$. Hence f is not an IF contra γ generalized continuous mapping.

Remark 3.17. Every IF contra generalized semi continuous mapping is an IF contra γ^*G continuous mapping but not conversely in general.

Proof. Let $f:(X, \tau) \to (Y, \sigma)$ be an IF contra generalized semi continuous mapping [11]. Let V be an IFOS in Y. Then $f^{-1}(V)$ is an IFGSCS in X. Since every IFGSCS is an IFγ^*GCS [8], $f^{-1}(V)$ is an IFγ^*GCS in X. Hence f is an IF contra γ^*G continuous mapping.

Example 3.18. In Example 3.16, f is an IF contra γ^*G continuous mapping but not an IF contra generalized semi continuous mapping as G3 is an IFOS in Y, but $f^{-1}(G_3)$ is not an IFGSCS in X, since $scl(f^{-1}(G_3)) = f^{-1}(G_3) \cup \text{int}(cl(f^{-1}(G_3))) = f^{-1}(G_3) \cup G_1 = G_1 \not\subseteq G_2$, but $f^{-1}(G_3) \subseteq G_2$.

The relation between various types of intuitionistic fuzzy continuity is given in the following diagram. In this diagram 'cts.' means continuous.



Theorem 3.19. *A mapping $f : (X, \tau) \to (Y, \sigma)$ is an IF contra $\gamma^* G$ continuous mapping if and only if the inverse image of each IFCS in Y is an IFγ^* GOS in X.*

Proof. Necessity: Let A be an IFCS in Y. This implies A^c is an IFOS in Y. Then $f^{-1}(A^c)$ is an IFγ^*GCS in X, by hypothesis. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is an IFγ^*GOS in X.

Sufficiency. Let A be an IFOS in Y. Then A^c is an IFCS in Y. By hypothesis $f^{-1}(A^c)$ is IFγ^*GOS in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $(f^{-1}(A))^c$ is an IFγ^*GOS in X. Therefore $f^{-1}(A)$ is an IFγ^*GCS in X. Hence f is an IF contra γ^*G continuous mapping.

Theorem 3.20. *Let $f : (X, \tau) \to (Y, \sigma)$ be a bijective mapping. Suppose that one of the following properties hold*:

(i) $f^{-1}(cl(B)) \subseteq \operatorname{int}(\gamma cl(f^{-1}(B)))$ *for each IFS B in Y*

(ii) $cl(\gamma \operatorname{int}(f^{-1}(B))) \subseteq f^{-1}(\operatorname{int}(B))$ *for each IFS B in Y*

(iii) $f(cl(\gamma \operatorname{int}(A))) \subseteq \operatorname{int}(f(A))$ *for each IFS A in X*

(iv) $f(cl(A)) \subseteq \operatorname{int}(f(A))$ *for each IFγOS A in X*

Then f is an IF contra γ^*G continuous mapping.

Proof (i) $\Rightarrow$ (ii) is obvious by taking complement of (i).

(ii) $\Rightarrow$ (iii) Let $A \subseteq X$. Put $B = f(A)$ in Y. This implies $A = f^{-1}(f(A)) = f^{-1}(B)$ in X. Now $cl(\gamma \operatorname{int}(A)) = cl(\gamma \operatorname{int}(f^{-1}(B))))$ $\subseteq f^{-1}(\operatorname{int}(B))$ by (ii). Therefore $f(cl(\gamma \operatorname{int}(A))) \subseteq f(f^{-1}(B))) = \operatorname{int}(B)$ $= \operatorname{int}(f(A))$.

(iii) $\Rightarrow$ (iv) Let $A \subseteq X$ be an IFγOS. Then $\gamma \operatorname{int}(A) = A$. By hypothesis, $f(cl(\gamma \operatorname{int}(A))) \subseteq \operatorname{int}(f(A))$. Therefore $f(cl(A)) = f(cl(\gamma \operatorname{int}(A))) \subseteq \operatorname{int}(f(A))$.

Suppose (iv) holds. Let A be an IFOS in Y. Then $f^{-1}(A)$ is an IFS in X

and $\gamma \operatorname{int}(f^{-1}(A))$ is an IFγOS in X. Hence by hypothesis, $f(cl(\gamma \operatorname{int}(f^{-1}(A))))$ $\subseteq \operatorname{int}(f(\gamma \operatorname{int}(f^{-1}(A)))) \subseteq \operatorname{int}(f(f^{-1}(A))) = \operatorname{int}(A) \subseteq A$. Therefore $cl(\gamma \operatorname{int}(f^{-1}(A))) = f^{-1}(f(cl(\gamma \operatorname{int}(f^{-1}(A))))) \subseteq f^{-1}(A)$. Now $cl(\operatorname{int}(f^{-1}(A)))$ $\subseteq cl(\gamma \operatorname{int}(f^{-1}(A))) \subseteq f^{-1}(A)$. This implies $f^{-1}(A)$ is an IFPCS in X and hence an IF γ^* GCS in X [7]. Thus f is an IF contra γ^* G continuous mapping.

Theorem 3.21. *Let $f : (X, \tau) \to (Y, \sigma)$ be a mapping. Suppose that one of the following properties hold*:

(i) $f(\gamma cl(A)) \subseteq \operatorname{int}(f(A))$ *for each IFS A in X*

(ii) $\gamma cl(f^{-1}(B)) \subseteq f^{-1}(\operatorname{int}(B))$ *for each IFS B in Y*

(iii) $f^{-1}(cl(B)) \subseteq \gamma \operatorname{int}(f^{-1}(B))$ *for each IFS B in Y*

Then f is an IF contra γ^* G continuous mapping.

Proof. (i) $\Rightarrow$ (ii) Let $B \subseteq Y$. Then $f^{-1}(B)$ is an IFS in X. By hypothesis, $f(\gamma cl(f^{-1}(B))) \subseteq \operatorname{int}(f(f^{-1}(B))) \subseteq \operatorname{int}(B)$. Now $\gamma cl(f^{-1}(B))$ $\subseteq f^{-1}(f(\gamma cl(f^{-1}(B)))) \subseteq f^{-1}(\operatorname{int}(B))$.

(ii) $\Rightarrow$ (iii) is obvious by taking complement in (ii).

Suppose (iii) holds. Let A be an IFCS in Y. Then $cl(A) = A$ and $f^{-1}(A)$ is an IFS in X. Now $f^{-1}(A) = f^{-1}(cl(A)) \subseteq \gamma \operatorname{int}(f^{-1}(A)) \subseteq f^{-1}(A)$, by hypothesis. This implies $f^{-1}(A)$ is an IFγOS in X and hence an IF γ^* GOS in X [7]. Therefore f is an IF contra γ^* G continuous mapping.

Theorem 3.22. *Let $f : (X, \tau) \to (Y, \sigma)$ be a bijective mapping. Then f is an IF contra γ^* G continuous mapping if $cl(f(A)) \subseteq f(\gamma \operatorname{int}(A))$ for every IFS A in X.*

Proof. Let A be an IFCS in Y. Then $cl(A) = A$ and $f^{-1}(A)$ is an IFS in X. By hypothesis $cl(f(f^{-1}(B))) \subseteq f(\gamma \operatorname{int}(f^{-1}(A)))$. Since f is an onto,

$f(f^{-1}(A)) = A$. Therefore $A = cl(A) = cl(f(f^{-1}(A))) \subseteq f(\gamma \operatorname{int}(f^{-1}(A)))$. Now $f^{-1}(A) \subseteq f^{-1}(f(\gamma \operatorname{int}(f^{-1}(A)))) = \gamma \operatorname{int}(f^{-1}(A)) \subseteq f^{-1}(A)$. Hence $f^{-1}(A)$ is an IFγOS in X and hence an IFγ^*GOS in X. Thus f is an IF contra γ^*G continuous mapping.

Theorem 3.23. *If $f : (X, \tau) \to (Y, \sigma)$ is an IF contra γ^*G continuous mapping, where X is an IF$\gamma^*T_{1/2}$ space, then the following conditions hold*:

(i) $\gamma cl(f^{-1}(B)) \subseteq f^{-1}(\gamma \operatorname{int}(B))$ *for every IFOS in Y*

(ii) $f^{-1}(cl(\gamma \operatorname{int}(B))) \subseteq \gamma \operatorname{int}(f^{-1}(B))$ *for every IFCS B in Y*

Proof. (i) Let $B \subseteq Y$ be an IFOS. By hypothesis $f^{-1}(B)$ is an IFγ^*GCS in X. Since X is an IF$\gamma^*T_{1/2}$ space, $f^{-1}(B)$ is an IFγCS in X. This implies $\gamma cl(f^{-1}(B)) = f^{-1}(B) \subseteq f^{-1}(\gamma \operatorname{int}(B))$.

(ii) can be proved easily by taking the complement of (i).

Theorem 3.24. *If $f : (X, \tau) \to (Y, \sigma)$ is an IF contra γ^*G continuous mapping and $g : (Y, \sigma) \to (Z, \delta)$ is an IF continuous mapping then $g \circ f : (X, \tau) \to (Z, \delta)$ is an IF contra γ^*G continuous mapping.*

Proof. Let V be an IFOS in Z. Then $g^{-1}(V)$ is an IFOS in Y, since g is an IF continuous mapping. Since f is an IF contra γ^*G continuous mapping, $f^{-1}(g^{-1}(V))$ is an IFγ^*GCS in X. Therefore $g \circ f$ is an IF contra γ^*G continuous mapping.

Theorem 3.25. *If $f : (X, \tau) \to (Y, \sigma)$ is an IF contra γ^*G continuous mapping and $g : (Y, \sigma) \to (Z, \delta)$ is an IF contra continuous mapping then $g \circ f : (X, \tau) \to (Z, \delta)$ is an IFγ^*G continuous mapping.*

Proof. Let V be an IFOS in Z. Then $g^{-1}(V)$ is an IFCS in Y, since g is an IF contra continuous mapping. Since f is an IF contra γ^*G continuous mapping, $f^{-1}(g^{-1}(V))$ is an IFγ^*GOS in X. Therefore $g \circ f$ is an IFγ^*G continuous mapping.

Theorem 3.26. *If* $f : (X, \tau) \to (Y, \sigma)$ *is an IF*γ^**G irresolute mapping and* $g : (Y, \sigma) \to (Z, \delta)$ *is an IF contra continuous mapping then* $g \circ f : (X, \tau) \to (Z, \delta)$ *is an IF*γ^**G continuous mapping.*

Proof. Let V be an IFOS in Z. Then $g^{-1}(V)$ is an IFCS in Y, since g is an IFC continuous mapping. As every IFCS is an IFγ^*GCS, $g^{-1}(V)$ is an IFγ^*GCS in Y. Since f is an IFγ^*G irresolute mapping, $f^{-1}(g^{-1}(V))$ is an IFγ^*GCS in X. Therefore $g \circ f$ is an IF contra γ^*G continuous mapping.

Remark 3.27. The composition of two IF contra γ^*G continuous mappings need not be an IF contra γ^*G continuous mapping. This can be seen from the following example.

Example 3.28. Let $X = \{a, b\}, Y = \{u, v\}$ and $Z = \{p, q\}$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}, \sigma = \{0_\sim, G_3, 1_\sim\}$ and $\delta = \{0_\sim, G_4, 1_\sim\}$ $G_1 = \langle x, (0.5_a, 0.7_b), (0.2_a, 0.2_b)\rangle, G_2 = \langle x, (0.6_a, 0.8_b), (0.2_a, 0.2_b)\rangle$, $G_3 = \langle y, (0.5_u, 0.6_v), (0.5_u, 0.4_v)\rangle$ and $G_4 = \langle z, (0.5_p, 0.8_q), (0.2_p, 0.2_q)\rangle$. Then $(X, \tau), (Y, \sigma)$ and (Z, δ) are IFTSs. Now define a mapping $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$ and $g : (Y, \sigma) \to (Z, \delta)$ by $g(u) = p$ and $g(v) = q$. Here f and g are IFCγ^*G continuous mappings but their composition $g \circ f : (X, \tau) \to (Z, \delta)$ defined by $g(f(a)) = p$ and $g(f(b)) = q$ is not an IF contra γ^*G continuous mapping since $G_4 = \langle z, (0.5_p, 0.8_q), (0.2_p, 0.2_q)\rangle$ is an IFOS in Z but $f^{-1}(g^{-1}(G_4)) = \langle x, (0.5_a, 0.8_b), (0.2_a, 0.2_b)\rangle$ is not an IFγ^*GCS in X as $f^{-1}(g^{-1}(G_4)) \subseteq G_2$ but $cl(\text{int}(cl(f^{-1}(G_4)))) \cap \text{int}(cl(f^{-1}(g^{-1}(G_4))))$ $= 1_\sim \not\subseteq G_2$.

Theorem 3.29. *For a mapping* $f : (X, \tau) \to (Y, \sigma)$, *where* X *is an*

IF $\gamma^{*}T_{1/2}$ *space, the following are equivalent*:

(i) *f is an IF contra* γ^{*} *G continuous mapping*

(ii) *For every IFCS A in Y and for every IFP* $p_{(\alpha, \beta)} \in X$, *if* $f(p_{(\alpha,\beta)})_q A$ *then* $p_{(\alpha,\beta)_q} \gamma\, \mathrm{int}(f^{-1}(A))$

(iii) *For every IFCS in Y and for any IFP* $p_{(\alpha,\beta)} \in X$, *if* $f(p_{(\alpha,\beta)})_q A$ *then there exists an IF* γ^{*} *GOS B such that* $f(p_{(\alpha,\beta)})_q B$ *and* $f(B) \subseteq A$.

Proof. (i) $\Rightarrow$ (ii) Let f be an IF contra γ^{*} continuous mapping. Let $A \subseteq Y$ be an IFCS and let $p_{(\alpha,\beta)} \in X$. Also let $f(p_{(\alpha,\beta)})_q A$ then $p_{(\alpha,\beta)q} f^{-1}(A)$. By hypothesis $f^{-1}(A)$ is an IF γ^{*} GOS in *X*. Since *X* is an IF $\gamma^{*}T_{1/2}$ space, $f^{-1}(A)$ is an IFγOS in X. Hence $\gamma\, \mathrm{int}(f^{-1}(A)) = f^{-1}(A)$. This implies $p_{(\alpha,\beta)q} \gamma\, \mathrm{int}(f^{-1}(A))$.

(ii) $\Rightarrow$ (i) Let $A \subseteq Y$ be an IFCS then $f^{-1}(A)$ is an IFS in *X*. Let $p_{(\alpha,\beta)} \in X$ and let $f(p_{(\alpha,\beta)})_q A$ then $p_{(\alpha,\beta)q} f^{-1}(A)$. By hypothesis this implies $p_{(\alpha,\beta)_q} \gamma\, \mathrm{int}(f^{-1}(A))$. That is $f^{-1}(A) \subseteq \gamma\, \mathrm{int}(f^{-1}(A))$. But $\gamma\, \mathrm{int}(f^{-1}(A)) \subseteq f^{-1}(A)$. Therefore $\gamma\, \mathrm{int}(f^{-1}(A)) \subseteq f^{-1}(A)$. Thus $f^{-1}(A)$ is an IFγOS in *X* and hence an IF γ^{*} GOS in *X* [7]. This implies *f* is an IF contra γ^{*} G continuous mapping.

(ii) $\Rightarrow$ (iii) Let $A \subseteq Y$ be an IFCS then $f^{-1}(A)$ is an IFS in *X*. Let $p_{(\alpha,\beta)} \in X$. Also let $f(p_{(\alpha,\beta)})_q A$ then $p_{(\alpha,\beta)q} f^{-1}(A)$. By hypothesis this implies $p_{(\alpha,\beta)_q} \gamma\, \mathrm{int}(f^{-1}(A))$. That is $f^{-1}(A) \subseteq \gamma\, \mathrm{int}(f^{-1}(A))$. But $\gamma\, \mathrm{int}(f^{-1}(A)) \subseteq f^{-1}(A)$. Therefore $\gamma\, \mathrm{int}(f^{-1}(A)) \subseteq f^{-1}(A)$. Thus $f^{-1}(A)$ is an IFγOS in *X* and hence an IF γ^{*} GOS in *X* [7]. Let $f^{-1}(A) = B$. Therefore $p_{(\alpha,\beta)q} B$ and $f(B) = f(f^{-1}(A)) \subseteq A$.

(iii) $\Rightarrow$ (ii) Let $A \subseteq Y$ be an IFCS then $f^{-1}(A)$ is an IFS in X. Let $p_{(\alpha,\beta)} \in X$. Also let $f(p_{(\alpha,\beta)})_q A$ then $p_{(\alpha,\beta)q} f^{-1}(A)$. By hypothesis there exists an IFγ^*GOS B in X such that $f(p_{(\alpha,\beta)})_q B$ and $f(B) \subseteq A$. Let $B = f^{-1}(A)$. Since X is an IF$\gamma^* T_{1/2}$ space, $f^{-1}(A)$ is an IFγOS in X and $\gamma\,\mathrm{int}(f^{-1}(A)) \subseteq f^{-1}(A)$. Therefore $p_{(\alpha,\beta)_q} \gamma\,\mathrm{int}(f^{-1}(A))$.

Theorem 3.30. *A mapping $f : (X, \tau) \to (Y, \sigma)$ is an IF contra $\gamma^* G$ continuous mapping if $f^{-1}(\gamma cl(B)) \subseteq \mathrm{int}(f^{-1}(B))$ for every IFS B in Y.*

Proof. Let $B \subseteq Y$ be an IFCS. Then $cl(B) = B$. Since every IFCS is an IFγCS, $\gamma cl(B) = B.$. Now by hypothesis, $f^{-1}(B) = f^{-1}(\gamma cl(B)) \subseteq \mathrm{int}(f^{-1}(B))$ $\subseteq f^{-1}(B)$. This implies $f^{-1}(B) = \mathrm{int}(f^{-1}(B))$. Therefore $f^{-1}(B)$ is an IFOS in X. Hence f is an an IF contra continuous mapping. Then by Theorem 3.3, f is an IF contra γ^*G continuous mapping.

Theorem 3.31. *A mapping $f : (X, \tau) \to (Y, \sigma)$ is an IF contra* γ^*G *continuous mapping, where X is an* IF$\gamma^* T_{1/2}$ *space if and only if $f^{-1}(\gamma cl(B)) \subseteq \gamma\,\mathrm{int}(f^{-1}(cl(B)))$ for every IFS B in Y.*

Proof. Necessity: Let $B \subseteq Y$ be an IFS. Then $cl(B)$ is an IFCS in Y. By hypothesis, $f^{-1}(cl(B))$ is an IFγ^*GOS in X. Since X is an IF$\gamma^* T_{1/2}$ space, $f^{-1}(cl(B))$ is an IFγOS in X. Therefore $f^{-1}(\gamma cl(B)) \subseteq \gamma\,\mathrm{int}(f^{-1}(cl(B)))$ $= \gamma\,\mathrm{int}(f^{-1}(cl(B)))$.

Sufficiency. Let $B \subseteq Y$ be an IFCS. Then $cl(B) = B$. By hypothesis, $f^{-1}(\gamma cl(B)) \subseteq \gamma\,\mathrm{int}(f^{-1}(cl(B))) = \gamma\,\mathrm{int}(f^{-1}(cl(B)))$. Since every IFCS is an IFγCS, B is an IFγCS. So $\gamma cl(B) = B$. Therefore $f^{-1}(B) = f^{-1}(\gamma cl(B))$ $\subseteq \gamma\,\mathrm{int}(f^{-1}(B)) = f^{-1}(B)$. This implies $f^{-1}(B)$ is an IFγOS in X and hence an IFγ^*GOS in X. Hence f is an IF contra γ^*G continuous mapping.

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