Advances and Applications in Mathematical Sciences
Volume 23, Issue 3, January 2024, Pages 233-249

NOVEL ALGORITHM FOR SEQUENCING PROBLEM WITH DEADLINES BASED ON *Z*-NUMBERS

K. PARAMESWARI[1] and G. VELAMMAL[2]

[1]Assistant Professor
[2]Associate Professor and Head (Retd.)
Department of Mathematics
Sri Meenakshi Government Arts College for Women(A)
Madurai, Tamilnadu, India
E-mail: paramusps@yahoo.co.in

Abstract

Some circumstances in real life, decisions are often taken based on imprecise information. Ali Askar Zadeh suggested fuzzy numbers to represent this issue. An improvised version of this is Z number which he formulated while dealing with Computing with Words (CWW). Z number describes a real-life quantity, as described in a natural language as fuzzy number with its reliability. Sequencing problem is ordering a collection of jobs to be done on a finite number of service facilities to optimise some efficiency factor. This paper intends to improvise an existing algorithm to solve the Sequencing Problems with Deadlines in CWW environment with suitable adaptations.

1. Introduction

One of the specialized scheduling problems in which ordering of the jobs completely determines schedule is known as pure sequencing problem. Also, there is a single resource, or machine, and all processing times are deterministic.

Order of tasks to be done in a chain [1, 2], in which the next task is started once the previous one is completed is known as sequencing. In addition, with sequencing some places have deadlines for finishing the given job with its profit [3]. Such type of problems is known as "Sequencing

2020 Mathematics Subject Classification: 03E72, 68T20, 30C80, 90B35, 65Yxx.
Keywords: CWW, Z number-ranking of Z number, Lexicographic order, Sequencing problem with deadline in Z parameter, ZSPWD.
Received December 1, 2023; Accepted December 31, 2023

Problem with Deadlines or simply SWD." Also, they need to maximize total profit for the corresponding work within the stipulated time. There were so many greedy algorithms available to solve sequencing problem with deadlines, where the data involving in crisp value.

John Bruno and Peter Downey [4] described about complexity of task sequencing with deadlines, set-up times and changeover costs. M.M.M. Fahmy [5] scheduling non-periodic jobs on soft real-time single processor system by implementing fuzzy logic algorithm.

Rajani Kumari, Dr. Vivek Kumar Sharma, Sandeep Kumar[6] introduced a new job shop scheduling algorithm with same fuzzy rules. Real-time task scheduling with fuzzy deadlines and processing times were given by Marin Litoiu, Roberto Tadei [7].

Pranab K. Muhuria, K. K. Shukla [8, 9] provided real-time scheduling of periodic tasks with processing times and deadlines as parametric fuzzy numbers and fuzzy uncertainty. A two-dimensional fuzzy ranking approach to job scheduling problems by Koun-Tem Sun [10]. Takeshi Itoha, Hiroaki Ishii [11] described fuzzy due-date scheduling problem with fuzzy processing time.

Sometimes in real life, the problems available in CWW (Computing With Words) approach with reliability. Zadeh [12] gave this new idea namely in Z number, which contains two components. Each component is a fuzzy number, where the 2nd component is the reliability of the 1st component.

Job scheduling problems in different categories were dealt in crisp and fuzzy logic but very less of them done in sequencing problem and not in Z number. This paper introduces a new greedy technique to find the optimum solution for Sequencing with Deadline problem, where the parameters are in Z number, denote it as ZSPWD problem.

Here, we take such type of problems, in which parameters are in Z number.

Consider the three types of situations:

(1) Crisp: "Ram completes his work with the profit of Rs.5000 for the deadline 2 hours".

(2) Fuzzy: "Ram completes his work nearly 2 to 3 hours for the profit around Rs.5000 to 7000".

(3) Z number: "Ram completes his work nearly 2 to 3 hours, very surely and earn the profit Rs.5500 to 9000, likely".

Compare with the above two, third one gives more flexible result because of its reliability information.

This paper introduces a new simple ZSPWD algorithm to solve the ZSPWD problem with the processing time for each task is of one or more unit. By using this new technique to solve the problems involving

(i) Profits and deadlines both are in Z number parameters will be given.

(ii) Profits are in Z number whereas the deadlines are in crisp and Operations and ordering on Z number plays a vital role for solving ZSPWD problems. Arithmetic Operations on Z numbers were discussed by Akif V. Alizadeh [13], Aliev and others [14, 15]. Shahila Bhanu and Velammal [16] also gave operations on Zadeh's Z numbers. Rani and Velammal [17] have dealt with combining Z valuations.

Stephen [18] introduced the novel binary operations on Z numbers in an easiest manner. This approach is tune with Zadeh's Computations With Words philosophy [17, 18] and can be put to use in practical applications easily. This can be taken for the situations, where we need to operate two Z numbers.

In a similar manner, for ordering of Z number, Parameswari[21] introduced simple elementary methods namely, "Lexicographic ordering based on Z number" and "Momentum Ranking Function (MRF)" [22]. This MRF gives more accurate solution for a maximization problem, in case of minimization take the inverse function of MRF, that is, 1/MRF.

This paper describes how to find out maximum total profit earned and the total processing time by executing the tasks within the specified deadline? by using this new algorithm.

For finding a solution of a problem involving CWW approach with reliability is needful part of our life. This algorithm fulfils our completion by using some elementary operations and ordering for the given task. So that, there is no doubts, this paper provides a simple and useful solutions for the users.

2. Preliminaries

Arithmetic Operations and Ordering of Z numbers

Definition 2.1. Z number. For any fuzzy number A and its reliability B, which is also a fuzzy number in [0, 1], an ordered pair of fuzzy number $Z = (A, B)$, is called Zadeh's Z number.

Definition 2.2. MIN Operation. Minimum of two fuzzy numbers is defined by $MIN(X, Y) = \begin{cases} X, \text{ if } r(X) \leq r(Y) \\ Y, \text{ if } r(Y) < r(X) \end{cases}$, by choosing r as any of ranking function for fuzzy number.

Definition 2.3. MIN R Operation. Let $*$ be any one of the basic arithmetic operations addition, subtraction, multiplication or division. Let R_k be any suitably chosen ranking function. Then the MIN R operation is defined by $(A, B)(*, MIN)(C, D) = (A * C, MIN(B, D))$, where $A * C$ is calculated by using the extension principle and

$$MIN(B, D) = B \text{ if } R_k(B) < R_k(D) \text{ or } D \text{ if } R_k(D) < R_k(B)$$

Definition 2.4. Lexicographic Order $L(R_1, R_2)$ for Z numbers.

Let R_1 and R_2 be any two ranking function and let $Z_1 = (A_1, B_1)$ $Z_2 = (A_2, B_2)$ be any two Z numbers. Define $Z_1 \preccurlyeq Z_2$ under the Lexicographic order $L(R_1, R_2)$ if and only if

(i) $R_1(A_1) < R_1(A_2)$ (or) (ii) $R_1(A_1) = R_1(A_2)$ and $R_2(B_1) \geq R_2(B_2)$.

Definition 2.5. Flipped Lexicographic Ordering $FL(R_1, R_2)$ for Z numbers. Let R_1 and R_2 be two ranking functions. Let $Z_1 = (A_1, B_1)$ and $Z_2 = (A_2, B_2)$.

We say, $Z_1 \preccurlyeq Z_2$ under $FL(R_1, R_2)$ if and only if (i) $R_1(B_1) > R_2(B_2)$ or (ii) $R_2(B_1) = R_2(B_2)$ and $R_1(A_1) \geq R_1(A_2)$.

Note: Depending on the area of application, R_1 and R_2 can be suitably chosen. So, this Lexicographic approach is highly flexible.

Again, the decision maker can decide which is critical-the first component or the second component. According he or she can use the $L(R_1, R_2)$ or $FL(R_1, R_2)$ to rank or order.

Definition 2.6. Momentum Ranking Function (MRF).

The Momentum Ranking Function (MRF) of any Z number (X, Y) is defined by $MRF(Z) = r_1(X) \times r_2(Y)$, for any two suitable ranking function r_1 and r_2.

Example 2.7. Consider the Z number $((1, 2, 3, 5), (.75, .8, .9, 1))$

- Here $A = (1, 2, 3, 5)$, and $B = (.75, .8, .9, 1)$
- "Choose two-ranking function: r_1 as Center of Gravity method, and r_2 as Median method" with $r_1(C) = \dfrac{(c^2 + d^2 + cd - a^2 - b^2 - ab)}{3(c + d - a - b)}$ and $r_2(C) = \dfrac{a + b + c + d}{4}$ for the trapezoidal number $C(a, b, c, d)$.
- $r_1(A) = \dfrac{3^2 + 5^2 + 3 \times 5 - 1^2 - 2^2 - 1 \times 2}{3(3 + 5 - 1 - 2)} = 2.8$ and $r_2(B) = \dfrac{.75 + .8 + .9 + 1}{4} = .8625.$
- $M(r_1, r_2)(Z) = r_1(A) \times r_2(B) = 2.8 \times .8625 = 2.415.$

Definition 2.8. Ordering of Z numbers using the $MRF - M(r_1, r_2)Z$

Choose any two ranking functions r_1 and r_2. Let $z_1 = (A_1, B_1), z_2 = (A_2, B_2)$ be two Z numbers.

We define $z_1 \leq z_2$ if and only if $M(r_1, r_2)(A_1, B_1) \leq M(r_1, r_2)(A_2, B_2)$ or simply $MRF(r_1) \leq MRF(z_2)$.

Examples 2.9. (1) Let $Z_1 = (A_1, B_1) = ((3, 5, 6, 7), (.8, .85, .9, 95))$ and $Z_2 = (A_2, B_2) = ((3, 4, 5, 6), (.7, .8, .85, 9))$. Here, $M(r_1, r_2)(A_1, B_1)$

$= r_1(A_1) \times r_2(B_1) = 5.2 \times .875 = 4.8125.$ $M(r_1, r_2)(A_2, B_2) = r_1(A_2) \times r_2(B_2)$ $= 4.5 \times .8125 = 3.65625.$ Hence, $Z_2 \leq Z_1$.

(2) Let $Z_1 = (A_1, B_1) = ((3, 5, 6, 7), (.7, .75, .8, 85))$ and $Z_2 = (A_2, B_2)$ $= ((3, 4, 5, 6), (.85, .9, .95, 1))$. $M(r_1, r_2)(Z_1) = M(r_1, r_2)(A_1, B_1) = r_1(A_1) \times r_2(B_1)$ $= 5.2 \times .775 = 4.03.$ $M(r_1, r_2)(Z_2) = M(r_1, r_2)(A_2, B_2) = r_1(A_2) \times r_2(B_2) = 4.5 \times .925$ $= 4.1625.$ Hence, $Z_1 \leq Z_2$.

3. Proposed Method

The objective of a job sequencing with deadline problem is to find a sequence of jobs to be completed within their deadlines and gives maximum profit with the total processing time. Let us consider the set of n given jobs associated with its deadlines. For these jobs, to find the sequence of jobs which maximize the profit.

Denote the i^{th} job or task as ID[i], deadline of i^{th} job is DL[i] and the profit received for the job ID[i] is PJ[i]. Hence, finding the optimal solution of job sequencing with deadlines by using the proposed algorithm gives feasible solution with maximum profit. Also, we need to follow some conditions:

- Each job has deadline and it can process the job within its deadline; only one job can be processed at a time.
- Only one Machine is available for processing all jobs.
- All jobs arrived at the same time.

Definition 3.1. n-ary Z number.

In any Z number both the components are in n-dimensional, then the corresponding Z number is called n-ary Z number.

Example 3.2. $Z_1 = ((a_{11}, a_{12}, \ldots, a_{1n}), (b_{11}, b_{12}, \ldots, b_{1n}))$ and $Z_2 = ((a_{21}, a_{22}, \ldots, a_{2n}), (b_{21}, b_{22}, \ldots, b_{2n}))$ be two n-ary $(n > 1)$ positive Z number $(a_{ij} > 0,\ i = 1$ to 2 and $j = 1$ to $n)$.

Note: If $n = 2, 3, 4, 5, 6, 7, 8$ then the Z-number is called Interval, Triangular, Trapezoidal, Pentagonal, Hexagonal, Septagonal, Octagonal Z-number respectively,

Definition 3.2. R Type Operations on n-ary Z number.

Let $Z_1 = ((a_{11}, a_{12}, \ldots, a_{1n}), (b_{11}, b_{12}, \ldots, b_{1n}))$ and $Z_2 = ((a_{21}, a_{22}, \ldots, a_{2n}), (b_{21}, b_{22}, \ldots, b_{2n}))$ be two n-ary $(n > 1)$ positive Z number $(a_{ij} > 0,\ i = 1$ to 2 and $j = 1$ to $n)$,

Then the arithmetic operations on Z number $(+,\ MIN)$ is defined as:

$$Z_1(+,\ MIN)Z_2 = ((a_{11} + a_{21}, a_{12} + a_{22}, \ldots, a_{1n} + a_{2n}),$$
$$MIN((b_{11}, b_{12}, \ldots, b_{1n}), (b_{21}, b_{22}, \ldots, b_{2n}))$$

Definition 3.2. Generalized Momentum Ranking Function (GMRF).

Introduce new ranking function for the n-ary Z number $((a_1, a_2, \ldots, a_{n-1}, a_n), (b_1, b_2, \ldots, b_{n-1}, b_n))$. For the n-ary Z number $((a_1, a_2, \ldots, a_{n-1}, a_n), (b_1, b_2, \ldots, b_{n-1}, b_n))$ with the ranking function

$$r_1((a_1, a_2, \ldots, a_{n-1}, a_n))$$

$$= \begin{cases} \dfrac{2a_1 + 3a_2 + \ldots + \left(\frac{n+1}{2}+1\right)a_{\frac{n+1}{2}} + \left(\frac{n+1}{2}\right)a_{\left(\frac{n+1}{2}+1\right)} \ldots + 2a_n}{\frac{(n+1)(n+3)}{4} + \frac{n-1}{2}}, & \text{if } n \text{ is odd} \\[2ex] \dfrac{2a_1 + 3a_2 + \ldots + \left(\frac{n}{2}+1\right)a_{\frac{n}{2}} + \left(\frac{n}{2}+1\right)a_{\left(\frac{n}{2}+1\right)} + \left(\frac{n}{2}\right)a_{\left(\frac{n}{2}+1\right)} \ldots + 2a_n}{\frac{(n+1)(n+3)}{4} + \frac{n-1}{2}}, & \text{if } n \text{ is even} \end{cases}$$

and $r_2((b_1, b_2, \ldots, b_{n-1}, b_n)) = \dfrac{b_1 + b_2 + \ldots + b_{n-1} + b_n}{n}$, the momentum ranking function for the n-ary Z number $GMRF(Z) = r_1((a_1, a_2, \ldots, a_{n-1}, a_n)) \times r_2((b_1, b_2, \ldots, b_{n-1}, b_n))$

Consider the following:

	i	:	1	2	3	$\cdots$	n
Job ID,	$ID[i]$	:	$ID[1]$	$ID[2]$	$ID[3]$	$\cdots$	$ID[n]$
Profits for the Job,	$PJ[i]$	:	$DL[1]$	$DL[2]$	$DL[3]$	$\cdots$	$DL[n]$
Deadline for processing the Job	$DL[i]$	:	$PT[1]$	$PT[2]$	$PT[3]$	$\cdots$	$PT[n]$

where profit and deadlines are on Z numbers. In general, this formation is called Sequencing Problem with Deadlines on Z numbers, shortly named as ZSPWD problem.

Here, Consider the ZSPWD in two cases.

Consider the problem in the following array of Job Sequencing Problem with Deadline:

Case 1. Each task takes one unit of time to execute is assumed.

i	:	1	2	3	…	n
$ID[i]$ (Job ID)	:	a	b	c	…	n
$DL[i]$ (Profits for the Job)	:	Z_1	Z_2	Z_3	…	Z_n
$PT[i]$ (Deadline for processing the Job)	:	d_1	d_2	d_3	…	d_n

where the Profits $Z_1, Z_2, Z_3, \dots, Z_n$ and the deadlines $d_1, d_2, d_3, \dots, d_n$ are Z numbers or deadlines may be taken in crisp values.

Case 2. Each task takes more than one unit of processing time is assumed. 7

	i	:	1	2	3	…	n
Job ID,	$ID[i]$	:	a	b	c	…	n
Profits for the Job,	$DL[i]$	:	Z_1	Z_2	Z_3	…	Z_n
Deadline for processing the Job	$PT[i]$	:	d_1	d_2	d_3	…	d_n

Processing time $PT[i] : PT[1]\ PT[2]\ PT[3] \dots PT[n]$, where each Zi's Zi's and PT[i]'F is in Z number parameter. Here Profit values and Processing must be taken in Z number, whereas deadline may be in crisp. Either all items given in the above table are in Z number parameter, or suppose except the profit values other values like DL and PT available in crisp, then also these types of problems can be solved by using the ZSPWD algorithm.

Algorithm 3.1. ZSPWD Algorithm

Step 1. Find out the descending order chain (DOC). This will get by arranging the profit values $PJ[i]$ in descending order by using any suitable ranking function. DOC will be: $PJ[r], PJ[s], PJ[t], \dots$.

Step 2. Find the maximum value of all $DL[i]$, by using any method of ranking of Z numbers. Suppose it will be for di. Take rank of 1st component of di, let it be d. Then create the Optimal Schedule, with the notation $OS[j] = 0$ or empty cell, for $j = 1, 2, \dots, d$.

j =	1	2	3	…..		d	
	0	0	0	…		0	OS[j]

Step 3. Choose the 1st element PJ[r] of the DOC, which is derived in Step 1.

Step 4. If rank of 1st component of $DL[r] = p \leq d$, and $OS[p] = 0$, then assign $OS[p] = ID[r]$ in the pth cell. Otherwise, if $OS[p] \neq 0$, then there are two cases arise to fix the cell. (i) if there is only one zero with $OS[j] = 0, j < p$ then fix that cell (ii) Suppose there are more $OS[j] = 0 (j < p)$, then give the preference to fix the cell having the position of next smallest value from right to left of p, let it be in the position j. And assign $OS[j] = ID[r]$.

Step 5. Take all the other $n - 1$ elements of DOC one by one and proceed the step 4 for the each element.

Step 6. Finally evaluate the Total Profit of Optimal Schedule (TPOS) = Sum of profit values involved in the Optimal Schedule.

Problem 3.1. Find the optimal solution for the following ZSPWD:

Task	Deadline in Hours	Profit in Rupees
T1	(Very High, Sure)	(Near about 15, Somewhat Sure))
T2	(Approximately 2,Very Sure)	(Around 2, Very Sure).
T3	(Average, at least Sure)	(Atleast 18, Less Sure)
T4	(Below High, Somewhat Sure)	(Below 2, Sure)
T5	(Medium, Below Sure)	(Above 25, Near about Sure)
T6	(Very Low, almost Sure)	(Atmost 30, Less Sure)
T7	(Below Average, Sure)	(approximately 8, Near about Sure)
T8	(High, Very Sure)	(Mostly 10, Sure)
T9	(Above Average, Badly Sure)	(almost 12, Highly Sure)
T10	(Low, Extremely Sure)	(Nearly 5, almost Sure)

Take all the above deadline and profit in the form of pentagonal Z number.

For any Pentagonal Z number $((a_1, a_2, a_3, a_4, a_5), (b_1, b_2, b_3, b_4, b_5))$, use the ranking function $r_1((a_1, a_2, a_3, a_4, a_5))$

$$= \frac{2a_1 + 3a_2 + \ldots + \left(\frac{5+1}{2} + 1\right)a_{\frac{5+1}{2}} + \left(\frac{5+1}{2}\right)a_{\left(\frac{n+1}{2}+1\right)} \ldots + 2a_5}{\frac{(5+1)(5+3)}{4} + \frac{5-1}{2}}$$

$$= \frac{2a_1 + 3a_2 + 4a_3 + 3a_4 + 2a_5}{14},$$

Numerical Values for the above table is:

Task ID[i]	Deadline DL[i]	Profit PJ[i]
T1	((7,8,9,10,11),(.8,.825,.85,.9,.925))	((5,10,15,20,25),(.725,.75,.8,.825,.85))
T2	((.75,1,2,3,4),(.7,.75,.8,.85,.9))	((1,1.5,2,2.5,3),(.8,.825,.85,.9,.95))
T3	((3,4,5,6,7),(.8,.825,.85,.9,1))	((14,16,18,20,22),(.65,.7,.75,.8,.85))
T4	((5,6,7,8,9),(.7,.75,.85,.9,.95))	((.2,.5,1,1.5,1.7),(.8,.825,.85,.9,.925))
T5	((2,3,4,5,6),(.725,.75,.8,.825,.85))	((15,20,25,30,35),(.7,.75,.8,.85,.9))
T6	((.5,1,2,3,4),(.825,.85,.9,.95,1))	((5,10,20,25,30),(.625,.65,.7,.725,.8))
T7	((2,4,5,6,8),(.725,.75,.8,.825,.85))	((5,7,8,10,11),(.7,.75,.85,.9,.95))
T8	((5.5,6,7,8,8.5),(.8,.825,.85,.9,.95))	((7,8,10,11,12),(.725,.75,.8,.825,.85))
T9	((3,3.5,4,4.5,5),(.65,.7,.75,.8,.85))	((10,11,12,13,14),(.8,.825,.85,.9,1))
T10	((1,1.5,3,3.5,4),(.625,.65,.7,.725,.8))	((3.5,4,5,5.5,6),(.825,85,.9,.95,1))

Step 1. To arrange the descending order of the profit values by using $MRF(r_1, r_2)$

First evaluate the rank of above profit and deadline values for each job by using

$$r_1(a_1, a_2, a_3, a_4, a_5) = \frac{2a_1 + 3a_2 + 4a_3 + 4a_4 + 2a_5}{14}$$

$$r_2(b_1, b_2, b_3, b_4, b_5) = \frac{b_1 + b_2 + b_3 + b_4 + b_5}{5}$$

Task ID[i]	Profit PJ[i]			Deadline DL[i]		
	r_1	r_2	$MRF(PJ[i]) = r_1 \times r_2$	r_1	r_2	$MRF(DL[i]) = r_1 \times r_2$
T1	15	0.79	11.85	9	0.86	7.74
T2	2	0.865	1.73	2.1429	0.8	1.7143
T3	18	0.75	13.5	5	0.875	4.375
T4	0.9857	0.86	0.8477	7	0.83	5.81
T5	25	0.8	20	4	0.79	3.16
T6	18.214	0.7	12.75	2.0714	0.905	1.8746
T7	8.2143	0.83	6.8179	5	0.79	3.95
T8	9.6429	0.79	7.6179	7	0.865	6.055
T9	12	0.875	10.5	4	0.75	3
T10	4.8214	0.905	4.3634	2.6429	0.7	1.85

Descending order for the Profit by using above $MRF(PJ[j])$ values is:

$T5, T3, T6, T1, T9, T8, T7, T10, T2, T4.$

Step 2. Maximum of DL[i] is $d = ((7, 8, 9, 10, 11), (.8, .825, .85, .9, .925))$, so generate $r((7, 8, 9, 10, 11)) = 9$ cell with entries $OS[j] = 0$, $j = 1$ to 9 for finding the optimal schedule.

i	1	2	3	4	5	6	7	8	9
OS[j]	0	0	0	0	0	0	0	0	0

Step 3. (a) Job T5 has 1st maximum profit with the deadline $((2, 3, 4, 5, 6), (.725, .75, .8, .825, .85))$ and $r(2, 3, 4, 5, 6) = 3 <= 9$, and there is a zero value in $OS[4]$. Assign $OS[4] = T5$. Then the optimal schedule becomes:

i	1	2	3	4	5	6	7	8	9
OS[i]	0	0	0	T5	0	0	0	0	0

(b) The next maximum profit is for the job T3 and the corresponding deadline is $((3, 4, 5, 6, 7), (.8, .825, .85, .9, 1))$ with the rank as 5. Also, $OS[5] = 0$, So assign $OS[5] = T3$ in the above Optimal Schedule, we get,

i	1	2	3	4	5	6	7	8	9
OS[i]	0	0	0	T5	T3	0	0	0	0

(c) Next element in the DOC is T6, $d = ((.5, 1, 2, 3, 4), (.825, .85, .9, .95, 1))$ and $r((.5, 1, 2, 3, 4)) \simeq 2$.

In the above optimal schedule, $OS[2] = 0$, choose $OS[2] = T6$.

Then the optimal schedule will be

i	1	2	3	4	5	6	7	8	9
OS[i]	0	T6	0	T5	T3	0	0	0	0

After that, we have the job T1 in DOC with $d = ((7, 8, 9, 10, 11), (.8, .825, .85, .9, .925))$ and $r((7, 8, 9, 10, 11)) = 9$.

and $OS[9] = 0$. Assign $OS[9] = T1$

i	1`	2	3	4	5	6	7	8	9
OS[i]	0	T6	0	T5	T3	0	0	0	T1

Next one is T9 in DOC, deadline for this T9 is $d = ((3, 3.5, 4, 4.5, 5), (.65, .7, .75, .8, 85))$ and $r((3, 3.5, 4, 4.5, 5)) = 4$, $OS[4] \neq 0$, but there is some of $OS[4] = 0$, $p < 4$. So, choose next position 3, with $OS[3] = 0$. Assign $OS[3] = T9$

i	1`	2	3	4	5	6	7	8	9
OS[i]	0	T6	T9	T5	T3	0	0	0	T1

For the next job T8 in DOC, deadline is $d = ((5.5, 6, 7, 8, 8.5), (.8, .825, .85, .9, .95))$ with $r((5.5, 6, 7, 8, 8.5)) = 7$, and $OS[7] = 0$. So assign, $OS[7] = T8$

i	1`	2	3	4	5	6	7	8	9
OS[i]	0	T6	T9	T5	T3	0	T8	0	T1

Next one in the DOC is T7, here deadline for the job T7 is $d = ((2, 4, 5, 6, 8), (.725, .75, .8, .825, .85))$

Also, $r((2, 4, 5, 6, 8)) = 5$, $OS[5] \neq 0$ and there is only possible $OS[p] = 0$, $p < 5$, is $OS[1] = 0$.

i	1`	2	3	4	5	6	7	8	9
OS[i]	T7	T6	T9	T5	T3	0	T8	0	T1

For T10, deadline is $d = ((1, 1.5, 3, 3.5, 4), (.625, .65, .7, .725, .8))$ with $r((1, 1.5, 3, 3.5, 4)) = 2.64 \simeq 3$, none of $OS[p] \neq 0$, $p <= 3$. So take the next element in the descending order is T2 with the deadline $d = ((.75, 1, 2, 3, 4), (.7, .75, .8, .85, .9))$ and $r((.75, 1, 2, 3, 4)) \simeq 2$, here also no entry of $OS[p] \neq 0$, $p <= 2$.

Again select the final element in the DOC is T4, having the deadline is $d = ((5, 6, 7, 8, 9), (.7, .75, .85, .9, .95))$, $r((5, 6, 7, 8.9)) = 7$.

In the optimal schedule $OS[7] \neq 0$, but there is one value which is less than 7, that is 6th entry having zero. So assign $OS[6] = T4$. In DOC there were no more entry. The final optimal schedule is

i	1`	2	3	4	5	6	7	8	9
OS[i]	T7	T6	T9	T5	T3	T4	T8	0	T1

Optimal Maximum Profit = Sum of profit attained for the Jobs 7, 6, 9, 5, 3, 4, 8 and 1 in the optimal schedule.

Step 6. $TPOS = PJ[7](+, MIN)PJ[6](+, MIN)PJ[9](+, MIN)PJ[5](+, MIN)PJ[3](+, MIN)PJ[8](+, MIN)PJ[1] =$ ((5, 7, 8, 10, 11), (.7, .75, .85, .9, .95)) $(+, MIN)$ ((5, 10, 20, 25, 30), (.625, .65, .7, .725, .8)) $(+, MIN)$ ((10, 11, 12, 13, 14), (.8, .825, .85, .9, 1)) $(+, MIN)$ ((15, 20, 25, 30, 35), (.7, .75, .8, .85, .9)) $(+, MIN)$ ((14, 16, 18, 20, 22), (.65, .7, .75, .8, .85)) $(+, MIN)$ ((.2, .5, 1, 1.5, 1.7), (.8, .825, .85, .9, .925)) $(+, MIN)$ ((7, 8, 10, 11, 12), (.725, .75, .8, .825, .85)) $(+, MIN)$ ((5, 10, 15, 20, 25), (.725, .75, .8, .825, .85)) = ((61.2, 82.5, 109, 130.5, 150.7), (.65, .7, .75, .8, .85))

Problem 3.1. Consider the following ZSPWD, with deadline as crisp:

i	Job ID[i]	Profit for the Job I PJ[i] (in Rs.)	Dead Lines for finishing the Job I DL[i] (in Hours)	Processing Time PT[i] (in minutes)
1	a	((50,60,70), (.75,.8,.85))	((1,2,3),(.75,.8,.85))	((30,45,50),(.8,.9,1))
2	b	((80,100,120), (.8,.85,.9))	((0.5,1,2),(.9,.95,1))	((20,30,40),(.9,.95,1))
3	c	((10,20,30), (.9,.925,.95))	((2,3,4),(.8,.85,.95))	((45,50,55),(.7,.8,.9))
4	d	((30,40,50), (.7,.8,.9))	((1,2,3),(.9,.95,1))	((15,20,25),(.85,.9,1))
5	e	((10,20,30), (.7,.85,.9))	((0.75,1,2),(.85,.9,.95))	((35,40,45),(.75,.8,.85))

Step 1. Find DOC for the profit values: Choose $r_1(x, y, z) = \frac{(2x + 3y + 2z)}{7}$ (by definition 3.2), and

$$r_2(x, y, z) = \frac{(x + y + z)}{3}$$

i	MRF for Profit Values			$L(r_1 \times r_2)$	
	r_1	r_2	$MRF(PJ[i]) = r_1 \times r_2$	r_1	r_2
1	60	.8	48	2	.8
2	100	.85	85	1.143	.95
3	20	.925	18.5	3	.867
4	40	.8	32	2	.95
5	20	.817	16.34	1.214	.9

By $MRF(r_1, r_2)$, DOC of the profit becomes

Job ID,	$ID[i]$	:	b	a	d	c	e
Profits for the Job,	$PJ[i]$	:	$PJ[2]$	$PJ[1]$	$PJ[4]$	$PJ[3]$	$PJ[5]$
Deadline for processing the Job	$DL[i]$	:	$DL[2]$	$DL[1]$	$DL[4]$	$DL[3]$	$DL[5]$

Step 2. In the first component of deadline, Choose the maximum value of all deadline is 3, take $d = 3$ and Initialize the Optimal Schedule

$j =$ 1	2	3	
0	0	0	OS[j]

Step 3. First Maximum profit is for Z_2, with deadline as $p = 1$, and $OS[1] = 0$. Fix the 1st cell and allocate it by b. So the optimal schedule becomes:

$j =$ 1	2	3	
b	0	0	OS[j]

Step 4. Next Maximum profit is for Z_1 in DOC, with deadline as $p = 2$, and $OS[2] = 0$, So optimal schedule becomes

$j =$ 1	2	3	
b	a	0	OS[j]

Continuing, next maximum profit is for Z_4, with deadline as $p = 2$, and $OS[p] \neq 0$, for all $p <= 2$. So no changes in the optimal schedule.

The next maximum profit in DOC is Z_3, with deadline as $p = 3$, and $OS[3] = 0$, Assign the job c in the optimal schedule:

$j =$ 1	2	3	
b	a	c	OS[j]

$TPOS = ((80, 100, 120), (.8, .85, .9))(+, MIN)((50, 60, 70), (.75, .8, .85))$ $(+, MIN)((10, 20, 30), (.9, .925, .95)) = ((140, 180, 220), (.75, .8, 85))$.

4. Conclusion

This paper gives a certain order of the set of given jobs or tasks with its maximum profits and the Total Profit of Optimal Schedule for the limited timings by using the new ZSPWD algorithm. The advantage of this algorithm is the answer in terms of Z numbers. So, information regarding the reliability is retained.

References

[1] Kenneth R. Baker and Dan Trietsch, Introduction to Sequencing and Scheduling, John Wiley & Sons, Inc. 1974.

[2] Ellis Horowitz, Sartaj Sahni and Sanguthevar Rajasekaran, Computer Algorithms, Computer Science Press, 1997 P.208-215.

[3] Kenneth R. Baker and Dan Trietsch, Principles of Sequencing and Scheduling, John Wiley and Sons, Inc. 2009

[4] John Bruno and Peter Downey, Complexity of task sequencing with deadlines, set-up times and changeover costs, SIAM J. Comput. 7(4) November 1978, Https://Doi.Org/10.1137/0207031.

[5] M. M. M. Fahmy, A fuzzy algorithm for scheduling non-periodic jobs on soft real-time single processor system, Ain Shams Engineering Journal 1 (2010), 31-38.

[6] Rajani Kumari, Dr. Vivek Kumar Sharma and Sandeep Kuma, Fuzzified job shop scheduling algorithm, HCTL Open Int. J. of Technology Innovations and Research HCTL Open IJTIR, Volume 7, January 2014, e-ISSN: 2321-1814 ISBN(Print): 978-1-62951-250-1.

[7] Marin Litoiu and Roberto Tadei, Real-time task scheduling with fuzzy deadlines and processing times, Fuzzy Sets and Systems 117 (2001), 35-45.

[8] Pranab K. Muhuri and K. K. Shukla, Real-time task scheduling with fuzzy uncertainty in processing times and deadlines, Applied Soft Computing 8 (2008), 1-13.

[9] Pranab K. Muhuri and K.K. Shukla, Real-time scheduling of periodic tasks with processing times and deadlines as parametric fuzzy numbers, Applied Soft Computing 9 (2009), 936-946.

[10] Koun-Tem Sun, A Two-dimensional Fuzzy Ranking Approach to Job Scheduling Problems, Institute of Information Education National Tainan Teachers College Tainan, Taiwan. ktsun@ipx.ntntc.edu.tw

[11] Takeshi Itoha, Hiroaki Ishii, Fuzzy due-date scheduling problem with fuzzy processing time, Intl. Trans. in Op. Res. 6 (1999), 639-647.

[12] L. A. Zadeh, A note on Z-numbers, Information Science 181 (2011), 2923-2932.

[13] Akif V. Alizadeh, Rashad R. Aliyev and Oleg H. Huseynov, Algebraic Properties of Z-Numbers Under Additive Arithmetic Operations, AISC 896, pp. 893-900, 2019. https://doi.org/10.1007/978-3-030-04164-9_118.

[14] R. A. Aliev, A. V. Alizadeh and O. H. Huseynov, The arithmetic of discrete Z-numbers, Information Sciences 290 (2015), 134-155.

[15] R. A. Aliev, O. H. Huseynov and L. M. Zeinalova, The arithmetic of continuous Z-numbers, Information Sciences 373 (2016), 441-460.

[16] M. Shahila Bhanu and G. Velammal, Operations on Zadeh's Z-numbers, IOSR Journal of Mathematics, Issue 3, Vol. 11, PP 88-94 (May-June 2015).

[17] P. Rani and G. Velammal, Combining Z-valuations, AIJRSTEM, September-November, 16(1) (2016), 73-79.

[18] S. Stephen, Novel binary operations on Z-numbers and their application in fuzzy critical path method, Advances in Mathematics: Scientific Journal 9(5) (2020), 3111-3120 ISSN: 1857-8365 (printed); 1857-8438.

[19] L. A. Zadeh, Fuzzy logic computing with words, IEEE Trans. on Fuzzy Systems 4 (1996), 103-111.

[20] L. A. Zadeh, Turing, Popper and Occam, Computing with Words, Jerry M. Mendel, University of Southern California, USA, November 2007 | IEEE Computational Intelligence Magazine.

[21] K. Parameswari, Lexicographic order based ranking for z-numbers, Advances in Mathematics: Scientific Journal 9(5) (2020), 3075-3083 ISSN: 1857-8365(printed) 1857-8438(electronic). https://doi.org/10.37418/amsj.9.5.67

[22] K. Parameswari and G. Velammal, Momentum Ranking Function of Z-Numbers and its Application to Game Theory, P-ISSN: 2078-8665, 2023, 20(1 Special Issue) ICAAM: 305-310, E-ISSN: 2411-7986, https://dx.doi.org/10.21123/bsj.2023.8428