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SYMMETRIC GENERALIZED REVERSE $(\alpha, 1)^*$-BIDERIVATIONS IN $*$-RINGS

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Abstract

Let R be a ring and α be the endomorphism of R. In this paper, we introduce the notions of generalized reverse $(\alpha, 1)^*$-derivation and symmetric generalized reverse $(\alpha, 1)^*$-biderivation. It is to show that if a semiprime ring admits a generalized reverse $(\alpha, 1)^*$-derivation with an associated reverse $(\alpha, 1)^*$-derivation d, then d maps R into $Z(R)$ and also to show that if a non-commutative prime ring admits a generalized reverse $(\alpha, 1)^*$-derivation F with an associated reverse $(\alpha, 1)^*$-derivation d, then F is right α^*-multiplier on R. Analogous results have been proved for symmetric generalized reverse $(\alpha, 1)^*$-biderivation.

1. Introduction

In [1] Bresar and Vukman proved that if a prime $*$-ring R admits a $*$-derivation (resp. Reverse $*$-derivation) d, then either R is commutative or $d = 0$. Ashraf Ali in [11] extended the above mentioned results for semiprime $*$-rings in the setting of $(\alpha, \beta)^*$-derivations. Shakir Ali [12] proved that if a

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semiprime $*$-ring admits a generalized $*$-derivation (resp. generalized reverse $*$-derivation) F, then F maps R into $Z(R)$. Jaya Subba Reddy in [5] first introduced the concept of $(\alpha, 1)$-reverse derivations in rings and Generalized reverse $(\alpha, 1)$-derivations in rings [4]. He extended some results on Generalized $(\alpha, 1)$-reverse derivations in $*$-prime rings [5], generalized biderivations [6] and Symmetric reverse $(\alpha, 1)$-biderivations in rings [7]. The concept of symmetric biderivations was introduced by Maksa [8, 9]. In [12] the notion of symmetric generalized $*$-biderivation and symmetric generalized reverse $*$-biderivation were defined. Recently, Ashraf Ali in [10] the notion of symmetric generalized reverse $(\alpha, \beta)^*$-derivation on the $*$-ring R, α^*-multiplier and α^*-bimultiplier were defined. The previous work on biderivation motivates us to define symmetric generalized reverse $(\alpha, 1)^*$-biderivation on the $*$-ring R. The aim of this paper is to introduce the concept of generalized reverse $(\alpha, 1)^*$-biderivation and symmetric generalized reverse $(\alpha, 1)^*$-biderivation, then obtain some results concerning commutativity of rings.

2. Preliminaries

Throughout this paper, R will represent an associative ring with center Z. A ring R is said to be prime if $xRy = 0$ implies that either $x = 0$ or $y = 0$ and semiprime if $xRx = 0$ implies that $x = 0$, where $x, y, \in R$. The commutator $[x, y] = xy - yx$. Basic commutator identities $[xy, z] = x[y, z] + [x, z]y$ and $[x, yz] = [x, y]z + y[x, z]$. An additive mapping $x \to x^*$ satisfying $(xy)^* = y^*x^*$ and $(x^*)^* = x$ for all $x, y \in R$, is called an involution on R. A ring R equipped with an involution is called $*$-ring or ring with involution. An additive mapping $F : R \to R$ is called a left α^*-multiplier (resp. right α^*-multiplier) if $F(xy) = F(x)\alpha(y^*)$ (resp. $F(xy) = F(x)\alpha(x^*)$, holds for all $x, y \in R$. An additive mapping $d : R \to R$ is said to be a reverse derivation on R if d $d(xy) = d(y)x + yd(x)$. An additive mapping $F : R \to R$ is called a generalized reverse derivation if there exists a derivation d such that $F(xy) = F(y)x + yd(x)$ holds for all $x, y \in R$. Let R be a semiprime ring and suppose α be the endomorphism of R. Following [7], an additive mapping $F : R \to R$ is called a generalized reverse $(\alpha, 1)$-derivation if there exists a reverse $(\alpha, 1)$-derivation such that $F(xy) = F(y)\alpha + yd(x)$ holds for all $x, y \in R$. Thus, the concept of generalized reverse $(\alpha, 1)$-derivation covers the concepts of $(\alpha, 1)$-reverse derivation. Moreover, generalized reverse $(\alpha, 1)$-derivation with $d = 0$ covers the concept of reverse left α-multiplier. Now we extend the concepts of generalized reverse derivation in the following way: An additive mapping $d : R \to R$ is called reverse $(\alpha, 1)^*$-derivation if $d(xy) = d(x)\alpha(x^*) + yxd(x)$ holds for all $x, y \in R$, where R is a ring with involution. An additive mapping $F : R \to R$ is called a generalized reverse $(\alpha, 1)^*$-derivation if there exists a derivation d such that $F(xy) = F(x)\alpha(x^*) + yd(x)$ holds for all $x, y \in R$. A symmetric biadditive mapping $B : R \times R \to R$ is said to be a symmetric reverse biderivation on R if $B(xy, z) = yB(x, z) + B(y, z)x$ holds for all $x, y, z \in R$. A symmetric biadditive mapping $B : R \times R \to R$ is said to be a symmetric reverse $(\alpha, 1)$-biderivation on R if $B(xy, z) = B(y, z)\alpha(x)$ $+yB(x, z)$ holds for all $x, y, z \in R$. A symmetric biadditive mapping $G : R \times R \to R$ is said to be a symmetric generalized reverse biderivation B on R if $G(xy, z) = G(y, z) + yB(x, z)$, for all $x, y, z \in R$. A symmetric biadditive mapping $G : R \times R \to R$ is said to be a symmetric generalized reverse $(\alpha, 1)$-biderivation on R if there exists a symmetric reverse $(\alpha, 1)$-biderivation B on R such that $G(xy, z) = G(y, z) + yB(x, z)$, for all $x, y, z \in R$. The previous work on reverse biderivations motivates us to define symmetric generalized reverse $(\alpha, 1)^*$-biderivation on the ring R. Reverse $(\alpha, 1)^*$ biderivation on R if there exists a symmetric reverse $(\alpha, 1)^*$-biderivation B on R such that $G(xy, z) = G(y, z)\alpha(x^*) + yB(x, z)$, for all $x, y, z \in R$.

3. Main Results

Theorem 3.1. *Let R be a semiprime $*$-ring and α be the endomorphism of R. If $F : R \to R$ is a generalized reverse $(\alpha, 1)^*$-derivation with an associated reverse $(\alpha, 1)^*$-derivation d, then d maps R into $Z(R)$.*

Proof. Given that $F(xy) = F(x)\alpha(x^*) + yd(x)$, for all $x, y \in R$.

Consider $F(xyz) = F((xy)z)$

$$= F(z)\alpha(y^*x^*) + zd(y)\alpha(x^*) + zyd(x), \text{ for } x, y, z \in R. \tag{3.1}$$

On the other hand, $F(xyz) = F(x(yz))$

$$= F(z)\alpha(y^*x^*) + zd(y)\alpha(x^*) + yzd(x), \text{ for all } x, y, z \in R. \tag{3.2}$$

Comparing the equations (3.1) and (3.2), we obtain

$$[y, z]d(x) = 0, \text{ for all } x, y, z \in R. \tag{3.3}$$

Replacing y by $d(x)y$ in the equation (3.3) and using equation (3.3), we get

$$[d(x), z]yd(x) = 0, \text{ for all } x, y, z \in R. \tag{3.4}$$

Substituting y by yz in (3.4), we have $[d(x), z]yzd(x) = 0$, for all $x, y, z \in R$. (3.5)

Now right multiplying equation (3.3) by z, we obtain that $[d(x), z]yd(x)z = 0$. (3.6)

Comparing (3.5) and (3.6), we get $[d(x), z]y[d(x), z] = 0$ and hence $[d(x), z]R[R(x), z] = 0$, for all $x, y, z \in R$.

By the semiprimeness of R, we have $[d(x), z] = 0$, for all $x, y \in R$.

Hence, we conclude that d maps R into $Z(R)$.

Theorem 3.2. *Let R be a non-commutative prime $*$-ring and α be the endomorphism of R. If $F : R \to R$ is a generalized reverse $(\alpha, 1)^*$-derivation* *with an associated reverse* $(\alpha, 1)^*$- *derivation d, then F is right* α^*-*multiplier on R.*

Proof. From equation (3.3) we can directly have $[y, z]d(x) = 0$, for all $x, y, z \in R$.

Replacing y by ry, we get $[r, z]yd(x) = 0$, for all $x, y, z, r \in R$.

That is, $[r, z]Rd(x) = 0$, for all $x, z, r \in R$.

The primeness of R forces that either $d(x) = 0$ or $[r, z] = 0$, for all $x, z, r \in R$.

Since R is non-commutative ring, we conclude that $d(x) = 0$, for all $x \in R$.

Hence F is left reverse α^*-multiplier on R.

Theorem 3.3. *Let R be a semiprime* $*$*-ring and* α *be the endomorphism of R. If R admits a symmetric generalized reverse* $(\alpha, 1)^*$-*biderivation* $G : R \times R \to R$ *with a nonzero associated symmetric reverse* $(\alpha, 1)^*$-*biderivation B, then G maps* $R \times R$ *into* $Z(R)$.

Proof. Let G be a symmetric generalized reverse $(\alpha, 1)^*$-biderivation on R.

We have $G(xy, z) = G(y, z)\alpha(x^*) + yB(x, z)$, for all $x, y, z \in R$.

Replacing y by y in the above relation, we get

$$G(xyw, z) = G(x(yw), z) = G(yw, z)\alpha(x^*) + ywB(x, z)$$

$$= G(w, z)\alpha(y^*x^*) + wB(y, z)\alpha(x^*) + ywB(x, z). \tag{3.7}$$

On the other hand, $G(xyw, z) = G((xy)w\ z)$

$$= G(w, z)\alpha(xy) + wB(y, z)\alpha(x) + wyB(x, z). \tag{3.8}$$

From equations (3.7) and (3.8), we get $[w, y]B(x, z) = 0$, for all $x, y, z, w \in R$. (3.9)

This equation is similar to equation (3.3), using the same technique as used in the proof of Theorem 3.1, we obtain the required result.

Corollary 3.1. *Let R be a semiprime $*$-ring. If R admits a symmetric generalized reverse biderivation $G : R \times R \to R$ with an associated nonzero symmetric reverse biderivation $B : R \times R \to R$, then B maps $R \times R$ into $Z(R)$.*

Theorem 3.4. *Let R be a non-commutative prime $*$-ring and α be the endomorphism of R. If R admits a symmetric generalized reverse $(\alpha, 1)^*$-biderivation $G : R \times R \to R$ with an associated symmetric reverse $(\alpha, 1)^*$-biderivation B, then $B = 0$.*

Proof. From equation (3.9), we can directly have $[w, y]B(x, z) = 0$, for all $w, x, y, z \in R$.

Replacing y by vy in the last relation and using equation (3.9), we get $[w, v]yB(x, z) = 0$, for all $v, w, x, y, z \in R$. The primeness of R forces that either $[w, v] = 0$ or $B(x, z) = 0$ for all $v, w, x, y, z \in R$. Since R is non-commutative ring, we conclude that $B = 0$.

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