

BROWNIAN DIFFUSION, RADIATION, CHEMICAL REACTION AND VISCOUS DISSIPATION EFFECTS OVER A MOVING WEDGE ALONG CONVECTIVE BOUNDARY CONDITIONS

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Abstract

This research is mainly concerned about the effects of Brownian diffusion, Chemical Reaction, Radiation, Viscous dissipation, heat generation/absorption as well as the effects of convective boundary conditions on the MHD free convective flow of Casson fluid over a moving wedge. The governing non-linear coupled partial differential equations with auxiliary conditions are transformed into the system of coupled Ordinary differential equations via the similarity transformations and then solved numerically by Runge-Kutta-Ferlbarge along shooting technique. The numerical values of skin friction as well as the heat transfer factor are listed in the tables. Finally, the result shows that the momentum profile speed up with an increase in suction/ injection parameter. But it depreciates with magnetic field parameter. Temperature of the system is enhanced with increase in suction/ injection, heat generation/ absorption together with Prandtl number parameters, the reverse is the case with Eckert number and moving wedge ratio parameter. Concentration, Nusselt number and Sherwood number profiles are considered as decreasing functions.

1. Introduction

Scientists and Engineers are greatly interested in the investigation of the field technology of non-Newtonian fluids and their thermo physical properties, due to their wide range of industrial applications. Non-Newtonian fluid exhibit various viscosity due to some applied forces. The main divergence between Newtonian and Non-Newtonian fluid is Pseudoplasticity. In the study of non-Newtonian fluids researchers usually

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encounter difficulties, among which is highly non-linear boundary layer equations and a greater degree of complexity than Newtonian fluids. Several researchers in the field of fluid dynamics have taken greater interest in the Nano fluid heat transfer for its novelty in improving heat transfer and other uses such as polymer processing, Solar energy, gas cooling, glass production, nuclear reactors, space machinery, furnace design, etc. It was discovered by Choi et al. [1] that when a small amount of nanoparticles are added to the base fluid, it speeds up its thermal conductivity. A lot of researches followed this trend. The characteristics of Newtonian and non-Newtonian fluids flow has been discussed by using the above theory severally by different researchers such as [2-10].

The concept of boundary layer theory is important in a wide variety of engineering disciplines and day to day activities. This theory is mostly applicable in the calculation of Skin friction drag force applied exerted by the fluid such as the drag of an aircraft wing, an entire ship or a turbine blade. Furthermore, based on the Prandtl boundary layer theory, Felkner-Skan created the wedge flow model. For this, great effort has been expended in the previous years. The concept of 2-D laminar forced convectionheat transfer of incompressible Falkner-Skan flow from a wedge has been scrutinized by Lin et al. [11]. Yakob et al. [12] studied the concept of nanofluid flow of Felker-Skan with either a static or moving wedge. The concept of similarity transformations in the study of a power law fluid through a porous stretching wedge has been studied by Postelnicu and Pop [13]. The demonstration mixture of magnetohydrodynamic flow of viscous fluid via a porous stretched wedge was successfully done by Su et al. [14]. Hossaini et al. [15] worked to stabilize the unsteady mixed convection boundary layer fluid flow through a symmetric wedge with fluctuating temperature. The solution of MHD mixed convection flow over a wedge with fluctuating heat as well as chemical reaction was done by Deka and Sharma [16]. They used Falkner-Skan transformation in their research. Chamkha et al. [17] studied the effect of radiation o mixed convection; this was achieved in the presence of a wedge which was implanted in a porous media filled with a nanofluid. The investigation of heat and mass transfer characteristics which are stable in a laminar magnetohydrodynamic flow across a wedge in the presence of changing magnetic fields was studied by Srinivasacharya et al. [18] Khan and

Pop [19] examined the boundary layer flow via a moving wedge in a nanofluid. The investigation of an unsteady magnetohydrodynamic boundary layer fluid flow of a moving stretched porous wedge containing tangent hyperbolic 2 phase nanofluid was established by Mahdy and Chamkha [20]. Numerous researchers worked tremendously on the concept of magnetohydrodynamicnano fluid flow with several impacts [21-30]. Furthermore, various researchers researched several features of flow and heat transfer across a stretching sheet [31-36], as well as diverse fluid flow phenomena studied by other researchers [37-40], in the recent past as alternative strategies for obtaining the approximate solution of differential equations on unbounded domains.

The above researchers considered the wedge which is either static or moving whereas, Ullah et al. [41] studied the impact of radiation, suction/ injection on the heat transfer Casson fluid flow through a stretching wedge when a magnetic field is implemented.

Hatami and Ghasemi [42] in their research they considered Galerkin weighted residual finite element method to find the thermophoresis and Brownian diffusion motion of two phase nanofluid flow around a vertical cone in a porous media. Dharmaiah et al. [43] studied the impact of non-linear thermal radiation, Brownian and thermophoresis on an MHD through a wedge with dissipative impacts for Jeffrey fluid. Mishra and Kumar [44] they examined MHD flow, generative/absorptive heat and mass transfer of nanofluid flow past a wedge in the presence of viscous dissipation through a porous medium.

They used hybrid nanofluids which were made by suspending non identical nanoparticles, to mathematically analyze the MHD flow of Ag-TiO2 hybrid nanofluid over a permeable wedge with heat radiation and viscous dissipation Kho et al. [45]. The study of the influence of heat and mass transfer characteristics of an unsteady, two dimensional stagnation-point flows of Williams nanofluid along a static/moving wedge in the presence of velocity slip and chemical reaction effects was conducted by Hamid et al. [46]. Pandey and Kumar [47] identified the effects of viscous dissipation and Suction/ Injection on MHD flow of a nanofluid past a wedge with convective surface in the appearance of slip flow and porous medium. The examination of the convective heat transfer of nanofluid past a wedge subject to first order

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chemical reaction, heat generation/absorption and suction effects was conducted by Kasmani et al. [48] they incorporated the influence of wedge angle parameter, Thermophoresis, Dufour and Suret type diffusivity. Ali et al. [49] studied chemically reacting flow over a heated porous wedge, they applied Eyring-Powell fluid model in order to comprehend the characteristics of dynamic wedge. The influence of viscous dissipation and thermal radiation on the magnetohydrdynamic heat transfer flow of Casson fluid across a moving wedge with convective boundary condition in the presence of internal heat generation/ absorption.

The literature above demonstrates that to the best of the knowledge of the authors no attempt is made to study the combined effects of Brownian diffusion, chemical reaction, radiation, viscous dissipation as well as heat generation/absorption including the influence of convective boundary conditions on the MHD free convective flow of Casson fluid over a static/moving wedge. Hence the main contribution of this study is to extend the 3 work of Amar et al. [50] by considering the effects of the aforementioned parameters over the published article.

2. Mathematical Formulation

We considered the flow and heat transfer of a viscous radiative Casson fluid flow over a moving wedge in the presence of the magnetic field and radiation parameter. The magnetic field is ignored, this is because the magnetic Reynolds number is low. The wedge's wall temperature is higher than the free stream temperature. Convective heat transfer helps to maintain a constant temperature on the moving wedge surface and then lower surface of the wedge heated by convection fron a hot fluid at temperature T_f which produce heat transfer quantity h_f . A wedge surface which is extended at a constant velocity $U_w(x)$ is subject to the laminar boundary layer that is indicated as $U_w(x) = ax^m$. Where $U_w(x)$ is positive, i.e., the direction of the extending wedge is in the identical way to the fluid flow when is negative. The wedge is in reverse direction when negative shrinking. Also $U(x) = U_{\infty}x^m = cx^m$, in this case c, m and are fixed values as seen in figure 1 below:



Figure 1. Geometry of the problem.

The governing equations, in the presence of Brownian diffusion, chemical reaction, radiation, and viscous dissipation and based on the figure 1 above, T_f is the convective surface temperature, T_{∞} denotes the ambient fluid temperature. So, the 2-D steady boundary layer equations of incompressible fluid flow van be presented as Amar et al. 50

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,\tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\rho}\frac{dp}{dx} + v\left(1 + \frac{1}{\gamma}\right)\frac{\partial^2 y}{\partial y^2} + \frac{\sigma B_0^2(x)}{\rho}(U(x) - u),\tag{2}$$

Where u and v are the velocity components along the x and y directions respectively. ρ is the density of the fluid, v denoted the kinematic viscosity, γ shows the Casson fluid parameter, σ demonstrates the electrical conductivity, whereas, B_0 demonstrates the uniform magnetic field along the y-axis. Hereby, described boundary conditions for the considered problem, we have:

$$u = U_w(x), v = v_w(x) = v_0 \sqrt{x^{m-1}} \text{ at } y = 0$$

$$u \to U_\infty(x), \text{ as } y \to \infty, \tag{3}$$

Here, we introduce the dimensionless variables in the form of Amar et al. [50].

$$\eta = \sqrt[y]{\frac{(m+1)u(x)}{2\vartheta x}}, \ \Psi = \sqrt{\frac{2\vartheta x u(x)}{m+1}}f(\eta),$$

$$\theta(\eta) = \frac{T - T_{\infty}}{T_f - T_{\infty}}, \ \phi(\eta) = \frac{C - C_{\infty}}{C_f - C_{\infty}},$$
$$u = U \frac{df}{d\eta}, \ R = \frac{4T_{\infty}^3 \sigma_i}{3kK^*},$$
$$v = -\sqrt{\frac{(m+1)9u(x)}{2x}} \left[f(\eta) + \left(\frac{m-1}{m+1}\right) \eta \frac{df}{d\eta} \right],$$
(4)

Together with a stream function $\psi(x, y)$ as follows

$$u = \frac{\partial \Psi}{\partial y}$$
 and $v = -\frac{\partial \Psi}{\partial x}$, (5)

Employing the similarity transformations in eq. (4) to obtain the velocity factors by using eqs. (4) and (5). Eq. (1) is identically satisfied while Eqs. (2) and (3) can be re-written in this form:

$$\left(1+\frac{1}{\gamma}\right)f''' + ff'' + \beta[1-f'^2] + M(f'-1) = 0$$
(6)

$$f(0) = S, f'(0) = \lambda, f'(\infty) \to 1,$$

$$\tag{7}$$

In Eq. (7) above S stands for suction and λ is the velocity ratio factor and M is the non-dimensional magnetic field.

The wedge angle parameter is given by

$$\beta = \frac{2m\pi}{m+1} \tag{8}$$

We considered the following cases

Case 1. $\beta = 0$:

It implies that the flow is along a horizontal flat plate.

Case 2. $\beta = \frac{1}{2}$:

This implies that the wedge is applied at right angle (900) to the vertical surface.

Case 3. $\beta = 1$:

This implies a plane stagnant flow that revolves (1800) around a wedge.

Case 4. $\beta = 4$: (double stream)

Case 5. $\beta = 5$: (double flow at 900)

It implies a plane at double right angle to the surface.

Case 6. $0 \le \beta \le 2$:

The flow is opposite a half-angle wedge.

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = (\alpha + D_{CT}) \frac{\partial^2 T}{\partial y^2} + \frac{9}{\rho C p} \left(\frac{\partial u}{\partial y}\right)^2 + \frac{Q_0}{\rho C p} \left(T - T_{\infty}\right) - \frac{1}{\rho C p} \left(\frac{\partial q_r}{\partial y}\right)$$
(9)

Where T denotes the temperature, α , ρ , and 9 are the thermal diffusivity, the density of the fluid and the kinematic viscosity of the fluid, DCT is the Dufour diffusivity parameter, Q_0 denotes the temperature dependent heat generation/ absorption and radiation heat flow parameter (q_r) can be explained by the use of Rosseland approximation

$$q_r = -\frac{4\sigma_i \partial T^4}{3kK^*} \tag{10}$$

where T^4 stands for linear sum of the temperature, by application of Taylor series along with T_{∞} it can be extended. By the Taylor series T^4 can be expressed as the linear sum of temperature together with T_{∞} in the following pattern:

$$T^{4} = T_{\infty}^{4} + 4T_{\infty}^{3}(T - T_{\infty}) + 6T_{\infty}^{2}(T - T_{\infty})^{2} + \dots$$
(11)

it can be observed from Eq. (11) that $(T - T_{\infty})$ are higher order and hence can be excluded and yield

$$T^4 \cong 4TT^3_{\infty} - 3T^3_{\infty} \dots \tag{12}$$

After solving Eqs. (11) and (12), we obtain the following

$$q_r = -\frac{16T_{\infty}^3 \sigma_i}{3K^*} \frac{\partial T}{\partial y}, \qquad (13)$$

We substituted Eq. (13) into Eq. (9), and it will be

$$\begin{split} u \, \frac{\partial T}{\partial x} + v \, \frac{\partial T}{\partial y} &= \left(\alpha + D_{CT}\right) \frac{\partial^2 T}{\partial y^2} + \frac{v}{\rho C p} \left(\frac{\partial^2 u}{\partial y^2}\right) + \frac{Q_0}{\rho C p} \left(T - T_\infty\right) \\ &+ \frac{1}{\rho C p} \left(-\frac{16 T_\infty^3 \sigma_i}{3(\rho C p)_f K^*} \frac{\partial^2 T}{\partial y^2}\right) \end{split}$$

 σ_i -Stefan-Boltzman constant, K^* is the mean absorption coefficient. Employing the similarity transformation in Eq. (4) along with Eq. (5) together with the following boundary conditions:

$$-K\left(\frac{\partial T}{\partial y}\right) = h(T_f - T_w) \text{ at } y = 0$$
$$T - T_{\infty} \text{ as } y \to \infty$$
(15)

Eq. (14) will be transformed into:

$$\left(1 + Ld + \frac{4R}{3}\right)\theta'' + \Pr f\theta' + PEc(f'')^2 + (2 - \beta)\Pr Q\theta = 0,$$
(16)
$$\eta = 0 : \theta'(0) = -Nc[1 - \theta(0)]$$
$$\eta \to \infty : \theta(\infty) = 0,$$
(17)

Ld is the Dufour diffusivity parameter, R is the thermal radiation parameter, Pr is the Prandtl number, Ec is the Eckert number, β is the wedge angle parameter, Q is the heat generation/ absorption parameter, Nc is the convective parameter of moving wedge.

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} - k_1(C - C_\infty)$$
(18)

In this case, C is the nanoparticle concentration, D_{Bis} the Brownian diffusion, and k is the thermal conductivity.

Just as done above, we employ the similarity transformation in Eq. (4) along with Eq. (5) together with the following boundary conditions:

$$C = C_w \text{ at } y = 0$$

$$C \to \infty \text{ as } y \to \infty,$$
(19)

To transform Eq. (18) to the form below:

$$\varphi'' + Lef \varphi' - \tau \varphi,$$
(20)

 $\eta = 0 : \varphi(0) = 0$

$$\eta \to \infty : \phi(\infty) = 1, \tag{21}$$

Where Le is the Lewis number, τ is the chemical reaction parameter (dimensionless).

The skin friction quantity, the Nusselt number and Sherwood number are the measures of interest in engineering. At the surface of the wedge, the shear stress can be determined in the following passion:

$$T_w = \mu \left(\frac{\partial u}{\partial y}\right)_{y=0} \Rightarrow T_w = \psi \left(\frac{\overline{U(x)}}{vx} \left(\frac{\partial^2 t}{\partial \eta^2}\right)_{\eta=0}\right)$$

The term 'Skin friction' refers to

$$C_f = \left(\frac{2\tau_w}{\rho(U(x)^2)}\right) \Longrightarrow \left(\frac{1}{2}\right) C_f \operatorname{Re} x^{\left(\frac{1}{2}\right)} = \sqrt{\frac{1}{2-\beta}} f''(0),$$

The term 'local Nusselt number' refers to as

$$Nu = \frac{q_w^x}{k_f A T_w} \Rightarrow \frac{Nu}{\sqrt{R_{ex}}} = -\sqrt{\frac{1}{2-\beta}} \theta'(0) \ R_{ex}^{-\frac{1}{2}} Nux = -\frac{k_{hnf}}{k_f} \theta'(0), \qquad (22)$$

Also the term 'local Sherwood number' is refers to as:

$$Sh = \frac{q_m^x}{D_B A C_w} \Rightarrow \frac{Sh}{\sqrt{R_{ex}}} = -\sqrt{\frac{1}{2-\beta}} \varphi'(0)$$
(23)

 q_w is the surface (wall) heat flux, while q_m is the surface (wall) mass flux.

$$Nur = \frac{Nu}{\sqrt{R_{ex}}} = -\theta'(0), \tag{24}$$

$$Shr = \frac{Sh}{\sqrt{R_{ex}}} = -\varphi'(0), \tag{25}$$

3. Results and Discussion

It is clearly visible on figure 2, that magnetic field parameter has an effect on the momentum profile: any slight increase in the parameter it implies a significant decrease in the momentum profile. Figures 3-7 are basically designed to show how far suction/ injection parameter is powerfully impacting the system. One conclusion we can get from these results is that the wall mass suction/ injection is very significant in maintaining the steady boundary layer near the surface by delaying the separation. On figure 3 we can observe that as the values of suction parameter (S > 0) increases it decreases the momentum of the fluid. On the other hand, as the values for injection parameter (S < 0) is increased it causes an increase in the momentum profile. Effect of suction/ injection on profile of temperature is illustrated on Figure 4. It is clearly observable that for an increase in the parameter it causes an instant decrease in the temperature profile as well as its boundary layers thicknesses, this is irrespective of suction parameter (S > 0) or injection parameter (S < 0).



Figure 2. Influence of magnetic field parameter on momentum profile.

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Correspondingly, the influence of suction/injection parameter on the Nusselt number profile is depicted on figure 5, it is clearly observable that in both cases of suction (S > 0) and injection (S < 0) parameters any increase in the parameters, this leads to lowering of the Nusselt number inside the thermal profile area. It is a clear indication that increasing these parameters causes the profile of heat and mass transmission to decrease. The effect of the Nusselt number on concentration and concentration gradient has also been studied through Fig. 6, in this case, it has been observed that the nanoparticle concentration decreases with an increase in the Nusselt number and shear stress profiles decrease drastically with an increase in the Nusselt number and then increases asymptotically.



Figure 3. Influence of suction/injection parameter on momentum profile.



Figure 4. Influence of Suction/injection on temperature profile.



Figure 5. Influence of suction/inject parameter on Nusselt number profile.

Figure 7 is plotted to explain the effects of the Suction/injection parameter over the local Sherwood number profile. Here we have studied that any increment in the values of the parameters either suction (S > 0) or injection (S < 0) brings about a decrease in the local Sherwood number. Physically speaking, this seems to be reasonable. Additionally, it is remarkable to notice that for both parameters the local Sherwood number for the Suction parameter the fluid in the presence of nanoparticles is larger than the fluid in its absence.



Figure 6. Influence of Suction/injection parameter on concentration profile.

Furthermore, Figure 8 is on the impacts of Prandtl number (Pr) on temperature profile. The profiles of temperature behave erratically as Pr increases. The temperature exhibits extemporaneous comportment. The occurrence of heated air near the powdered may perhaps be to culpability. When the external temperature is nullified, Pr intensifies the temperature profile.



Figure 7. Influence of suction/injection on Sherwood number profile.

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As seen on figure 9, it shows that as the Eckert number increases, the thickness of the thermal boundary layer decreases as the curves become increasingly steeper. As a consequence, the reduced Nusselt number, being proportional to the initial slope, increases. This pattern is reminiscent of the free convective boundary layer flow in a regular fluid. Consequently, looking at Figure 10, it explains the enhancement in the values of heat generation/ absorption parameter (Q) which is accompanied by an increase in the temperature distribution. Physically, the increase in the temperature distribution field transport from heat absorption (Q < 0) into heat generation (Q > 0) by the average kinetic energy of the fluid particles, which leads to an increase in the movement and speed of the fluid molecules within the channel because the temperature determines the kinetic energy associated with the movement of the fluid molecules and nanoparticles, which also causes small distances between the fluid molecules.



Figure 8. Influence of Prandtl number on temperature profile.

The impact of the moving wedge ratio parameter (λ) over the temperature profile is presented in Figure 11. However, a minute progression in the fluid temperature was seen as the value of the parameter increased. However, figure 12, is plotted to depict the influence of Lewis number over the concentration profile. We can make the conclusion from these results that

the Lewis number (Le) is very significant in maintaining the steady boundary layer near the plate by delaying the separation. On this figure we can also observe that as the values of Le increases the nanoparticle concentration of fluid is decreased drastically. The effect of chemical reaction parameter on concentration and concentration gradient has also been studied through Figure 13, on this graph it has been observed that the nanoparticle concentration decreases with an increase in the chemical reaction parameter and shear stress profiles decreases drastically with its increase and then increases asymptotically.



Figure 9. Influence of Eckert number on temperature profile.



Figure 10. Influence of heat generation/absorption on temperature profile.



Figure 11. Influence of moving wedge ratio parameter on temperature profile.



Figure 12. Influence of Lewis Number on concentration profile.



Figure 13. Influence of chemical reaction parameter on concentration profile.



Figure 14. Influence of chemical reaction parameter on Sherwood number profile.

Figure 14 is plotted to explain the effect of chemical reaction parameter over the local Sherwood number profile. At this juncture we have deliberated that for any increment in the values of the parameter it brings about an increases in skin friction coefficient and a reverse for the case of local Sherwood number. We find this reasonable and expected from a physical point of view. It is exciting to note that skin friction as well as local Sherwood

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number for chemical reaction parameter, the fluid in the presence of nanoparticles are greater than that in the absence of nanofluid.

4. Conclusions

In this study, we explored boundary layer flow past a moving wedge. Appropriate transformations are applied on the non-linear PDE to be converted into non-linear ODE, which are then numerically solved using the Runge-Kutta-Ferlbarge along shooting technique. In this study the momentum rises with an increase in suction/ injection parameter, reverse is the case with magnetic field parameter. However, temperature drops with an increase in Prandtl number, suction/ injection as well as chemical reaction parameter, while it rises with Eckert number and moving wedge ratio parameter. Nanoparticle concentration profile, Nusselt number profile as well as the Sherwood number profiles are found to be decreasing functions.

Nomenclature

C nanoparticle Concentration of the fluid

- C_f Concentration at the surface
- C_w Concentration at the wall
- C_∞ Ambient fluid Concentration
- T Temperature of the fluid
- T_f Convective fluid temperature at the surface
- T_w Wall Temperature
- T_{∞} Ambient fluid Temperature

U velocity component in x-direction

 $U_w(x)$ Stretching/ Shrinking wedge surface velocity

- U(x) free stream velocity
- S suction/injection parameter
- R thermal radiation

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- q_r Radiation heat flux
- Q heat generation/ absorption
- Pr Prandtl number
- *Nc* Convective parameter
- k Thermal conductivity
- Q_0 Temperature- dependent heat generation/ absorption
- ${\cal C}_p\,$ Specific heat of viscous fluid at constant
- k^* Mean absorption coefficient
- u, v Velocity component in (x, y) direction
- Ld Dufour diffusivity
- DB Brownian diffusion
- Le Lewis number

Greek Letters

- σ_i Stefan-Boltzman constant
- γ Casson fluid parameter
- $\sigma\,$ Electric conductivity of the nanofluid
- 9 Kinematic viscosity of the fluid
- ψ Stream function
- ρ Density of the fluid
- η Similarity variable
- β Wedge angle parameter
- λ Moving Wedge ratio parameter
- τ Chemical reaction parameter

Subscripts

- w Condition at the wall
- ∞ Condition far away from the surface

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