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A NEW GENERALIZED WEIGHTED CROSS ENTROPIC PARAMETRIC MODEL FOR DISCRETE PROBABILITY DISTRIBUTION

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Abstract

The present study focuses on the establishment of a new generalized parametric model for weighted cross entropy based upon discrete probability distribution along with their graphical presentation.

1. Introduction

Shannon [21], who was a communication engineer and had been working across noiseless or noisy channels, introduced information entropy. After this development, a lot of entropic measures with properties and appliances were elaborated and developed by researchers like Kapur ([11], [12]), Chakrabarti and Chakrabarty [5], Herremoes in [8], Nanda and Paul [17], Sharma and Taneja [22], Parkash, Thukral and Gandhi [18], Cincotta and Giordano [4], Majumdar and Jayachandran [16], Goel, Taneja and Kumar [5], Jamaati and Mehri [10], Khozani and Bonakdari [14], Sheng, Shi and Ralescu [23], etc.

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Moreover, it is natural that the concept of distance which already prevalent in the field of mathematics should be extended for its applications to problems in other disciplines also. One of the most important directed divergence measures was given by Kullback and Leibler [15]. Kapur [13] also added the many directed divergence measures. Some other developments regarding the divergence measures have been made by Ararat, Hamel and Rudloff [1], Watson, Nieto-Barajas and Holmes [24], Pinelis [19], Sankaran, Sunoj and Nair [20], Huang, Yong and Hong [9], Handa, Narula and Gandhi [6]-[7], Avlogiaris, Micheas and Zografos [2] etc.

2. The New Generalized Parametric Model for Weighted Cross Entropy

A new weighted measure of cross entropy proposed through this mathematical model is recommended as:

$$D_\alpha(P, Q; W) = \frac{\ln\left(\prod_{i=1}^{n}\left(\frac{p_i}{q_i}\right)^{(\alpha-1)w_i p_i}\right)}{(\alpha-1)}; \ \alpha > 0, \alpha \neq 1 \tag{2.1}$$

Ignoring weights and taking limit as $\alpha \to 1$, we get:

$$\underset{\alpha\to 1}{lt}[D_\alpha(P, Q)] = \underset{\alpha\to 1}{lt}\left(\frac{\ln\left(\prod_{i=1}^{n}\left(\frac{p_i}{q_i}\right)^{(\alpha-1)p_i}\right)}{\alpha-1}\right) = \sum_{i=1}^{n} p_i\left(\ln\frac{p_i}{q_i}\right)$$

Therefore, $D_\alpha(P, Q; W)$ is an extension to Kullback-Leibler's [15] cross entropy measure after attaching weights with the probability distribution.

3. Properties of Cross Entropy Measure to be Convex

(i) $D_\alpha(P, Q; W)$ is non-negative,

i.e.

$$D_\alpha(P, Q; W) = \sum_{i=1}^{n} w_i p_i \ln\frac{p_i}{q_i} \geq 0.$$

(ii) $D_\alpha(P, Q; W) = 0$, if and only if $P = Q$.

(iii) For proving (2.1) as convex, we have

$$\frac{\partial^2(D_\alpha(P, Q; W))}{\partial p_i^2} = \frac{w_i}{p_i} > 0.$$

and $\dfrac{\partial^2 D_\alpha(P, Q; W)}{\partial p_i \partial p_j} = 0, \quad i \neq j.$

Since

$$\begin{bmatrix} \frac{w_1}{p_1} & 0 & \dots & 0 \\ 0 & \frac{w_2}{p_2} & \dots & 0 \\ \vdots & & & \\ 0 & 0 & \dots & \frac{w_n}{p_n} \end{bmatrix}$$

is definitely positive.

So, $D_\alpha(P, Q; W)$ proved to be convex.

Similarly, $\dfrac{\partial(D_\alpha(P, Q; W))}{\partial q_i} = -\dfrac{w_i p_i}{q_i}$

$$\frac{\partial^2(D_\alpha(P, Q; W))}{\partial q_i^2} = \frac{w_i p_i}{q_i^2} > 0.$$

Also,

$$\frac{\partial(D_\alpha(P, Q; W))}{\partial q_i \partial q_j} = 0.$$

Thereupon,

$$\begin{bmatrix} \frac{w_1 p_1}{q_1^2} & 0 & \dots & 0 \\ 0 & \frac{w_2 p_2}{q_2^2} & \dots & 0 \\ \vdots & & & \\ 0 & 0 & \dots & \frac{w_n p_n}{q_n^2} \end{bmatrix}$$

is too positive definite. Therefore, $D_\alpha(P, Q; W)$ comes as convex.

With the above given properties, the mathematical model introduced in (1.2.1) proves its validity to be a true parametric weighted cross entropy measure. Next, constructed Table 3.1 for the presentation of $D_\alpha(P, Q; W)$ and obtained Figure 3.1 proving the convexity of the investigated measure.

Table 3.1. Convexity of $D_\alpha(P, Q; W)$

p_i	q_i	w_i	$D_2(P, Q; W)$	$D_{2.2}(P, Q; W)$	$D_{2.3}(P, Q; W)$	$D_{2.4}(P, Q; W)$	$D_{2.5}(P, Q; W)$
0.1	0.5	1.0	0.159848254	.368064207	.36806421	0.368064	0.368064207
0.2	0.5	1.025	0.085800684	0.197563376	0.19756338	0.197563	0.197563376
0.3	0.5	1.075	0.038415125	0.088454094	0.08845409	0.088454	0.088454094
0.4	0.5	1.1	0.009619217	0.022149065	0.02214906	0.022149	0.022149065
0.5	0.5	1.125	0	0	0	0	0
0.6	0.5	1.15	0.010056454	0.023155841	0.02315584	0.023156	0.023155841
0.7	0.5	1.175	0.041988625	0.096682382	0.09668238	0.096682	0.096682382
0.8	0.5	1.2	0.100449581	0.231293708	0.23129371	0.231294	0.231293708
0.9	0.5	1.225	0.195814111	0.450878654	0.45087865	0.450879	0.450878654

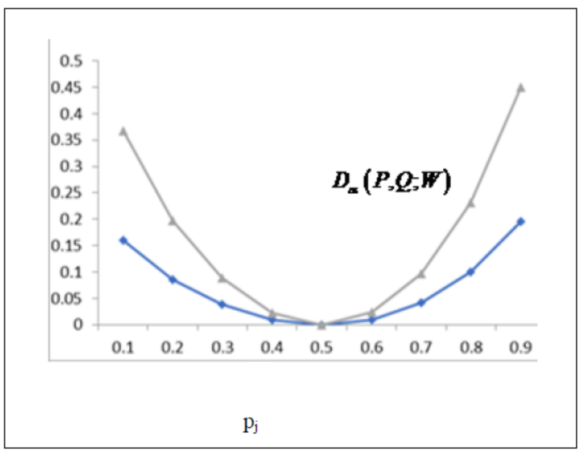

Figure 3.1. Convexity property of $D_\alpha(P, Q; W)$.

Present figure clearly shows $D_\alpha(P, Q; W)$ to be convex.

4. Conclusion

As it is required that the investigation of new generalized probabilistic and weighted measures of information is mandatory. So, we conclude that, by considering information measures that already exist, a new weighted measure including the measure of divergence is established. Along the parallel lines, more theoretic measures may be expanded.

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