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A PRICE SKIMMING STRATEGY FOR DETERIORATING ITEMS

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Abstract

This study developed an economic order quantity model for deteriorating items using price skimming strategy. The model charged a high selling price at the beginning and then lowered it over time. The optimum inventory level and the optimum time to offer discount were determined by maximizing the profit function. Results were illustrated using numerical examples and graphs which showed that the price deductions not only have an effect on the profit function but also the cycle time of the model.

1. Introduction

Price skimming is a pricing strategy in which a seller first sets a fairly high starting price for a product and then reduces the price over time. Usually the seller employs this strategy for new technologies, seasonal fruits and vegetables, newly launched fashion items, etc., because these products deteriorate over time. Seasonal fruits and vegetables decay over time for their biological effect like microorganisms. New technologies and fashion items also deteriorate or become outdated over time as competitors launch similar products, or the brand develops a new product. The price skimming strategy generates a high profit margin for the seller/producer as they set a high introductory price and then gradually reduces the price to attract the next and subsequent layers of the market. Researchers have developed many inventory models for deteriorating items [2], [3], [5], [6], [11], [13], [14], [15], [16], and [18].

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Many researchers have adopted price to develop their inventory models [4], [8], [9], [10] and [12]. Adopting a price skimming strategy for the EOQ model for a deteriorating item will be a fine new addition to this field. This study develops an EOQ model with a price skimming strategy for deteriorating items like seasonal fruits, vegetables, newly launched fashion items, new technologies etc. The research article [1] developed a sustainable recycling process for an imperfect production system where the authors considered a fixed ratio of recyclable defective products and a piecewise constant demand rate of the non-defective items. The research article [7] developed an inventory model considering the optimal replenishment cycle time which addressed various inventory related issues like the partly imperfect quality items and deterioration after the inspection time. The authors also considered the permissible delay in payments which promoted their sales and reduced their on-hand stock level. The article [17] developed a two-level trade-credit model with a finite replenishment rate by considering an alternate due date of payment and limited storage capacity together. The authors developed 4 theorems to find the optimal solutions according to the cost-minimization strategy.

2. Formulation of the Model

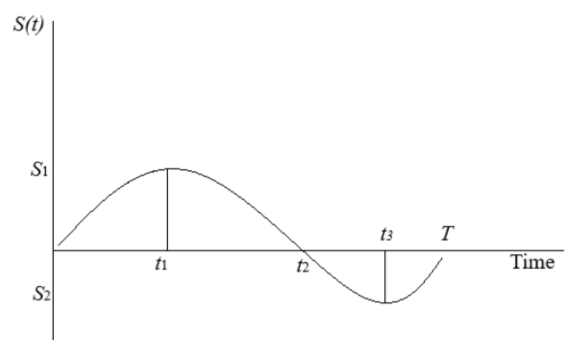

Figure 1. The diagram of the stock over time.

Production starts at the beginning when time $t = 0$. During the time span $[0, t_1]$ production stops when the stock level reaches the level S_1. During time span $[t_1, t_2]$ the stock level decreases due to market demand of the product and deterioration and eventually becomes zero by the end of this interval. This is the time when shortage occurs. To avoid high cost due to deterioration backlogging demand is a highly realistic consideration. When shortage level reaches S_2 at time $t = t_3$ production starts again and stops at time $t = T$ when backlog is cleared. The diagram of variation within the inventory cycle is shown in Figure 1. The following notations and assumptions are used in this model:

a) $S(t) =$ Inventory level at time $t(\geq 0)$.

b) $Q_p =$ Production rate

c) $D_p =$ Demand rate $Q_p > D_p$.

d) $f(p) =$ Selling price.

e) $T =$ Duration of the inventory cycle.

f) Shortages are allowed and fully backlogged.

g) Lead time is zero.

h) $\tau =$ deterioration rate, $0 < \tau << 1$.

i) $C_u =$ Unit cost.

j) $C_s =$ Set up cost per unit per unit time.

k) $C_h =$ Carrying cost per unit per unit time.

l) $C_{sh} =$ Shortage Cost per unit per unit time.

m) $C_d =$ Cost of a deteriorated unit.

During $t \in [0, t_1]$, $S(t)$ is governed by

$$\frac{dS(t)}{dt} + \tau S(t) = Q_p - D_p,\ S(0) = 0. \tag{1}$$

and during $t \in [t_1, t_2]$, $S(t)$ is governed by

$$\frac{dS(t)}{dt} + \tau S(t) = -D_p,\ S(t_1) = S_1,\ S(t_2) = 0 \tag{2}$$

The solutions are

$$S(t) = \begin{cases} \frac{1}{\tau}(Q_p - D_p)(1 - e^{-\tau t}), & t \in [0, t_1] \\ \frac{1}{\tau} D_p (e^{\tau(t_2 - t)} - 1), & t \in [t_1, t_2] \end{cases} \tag{3}$$

From (3)

$$S_1 = S(t_1) = \frac{1}{\tau}(Q_p - D_p)(1 - e^{-\tau t_1})$$

$$\Rightarrow t_1 = \frac{1}{\tau} \ln\left(1 + \tau \frac{S_1}{(Q_p - D_p)} + \tau^2 \frac{S_1^2}{(Q_p - D_p)^2}\right) \tag{4}$$

neglecting higher order of τ.

Also from (3)

$$S_1 = \frac{1}{\tau} D_p (e^{\tau(t_2 - t_1)} - 1)$$

$$\Rightarrow t_2 = t_1 + \frac{1}{\tau} \ln\left(1 + \tau \frac{S_1}{D_p}\right). \tag{5}$$

During $t \in [t_2, t_3]$, $S(t)$ is governed by

$$\frac{dS(t)}{dt} = -D_p, \; S(t_2) = 0, \; S(t_3) = -S_2 \tag{6}$$

During $t \in [t_3, T]$, $S(t)$ is governed by

$$\frac{dS(t)}{dt} = Q_p - D_p, \; S(t_3) = -S_2, \; S(T) = 0 \tag{7}$$

The solutions are

$$S(t) = \begin{cases} D_p(t_2 - t), & t \in [t_2, t_3] \\ (Q_p - D_p)(t - t_3) - S_2, & t \in [t_3, T] \end{cases} \tag{8}$$

From (8)

$$D_p(t_2 - t_3) = -S_2 \tag{9}$$

$$\Rightarrow t_3 = \frac{S_2}{D_p} + t_2 \tag{10}$$

Note that the production occurs in continuous time spans $[0, t_1]$ and $[t_3, T]$. Therefore the lot size is

$$S_L = S_1 * t_1 + S_2 * (T - t_3). \tag{11}$$

3. The Pricing Strategy

$$\text{Revenue} = Q_{REV} = f(p) * D_p * t_2 + f(p) * S_2 * (T - t_3) \tag{12}$$

where selling price

$$f(p) = p - \sigma p H(t - \mu),\ \sigma < 1,\ \mu = T^* \in [0, T]$$

Where p is the full price of unit item and σ is the discount that producer offers and $H(t - \mu)$ is a Heaviside function.

$$H(t - \mu) = \begin{cases} 1, & t > \mu \\ 0, & t < \mu \end{cases}$$

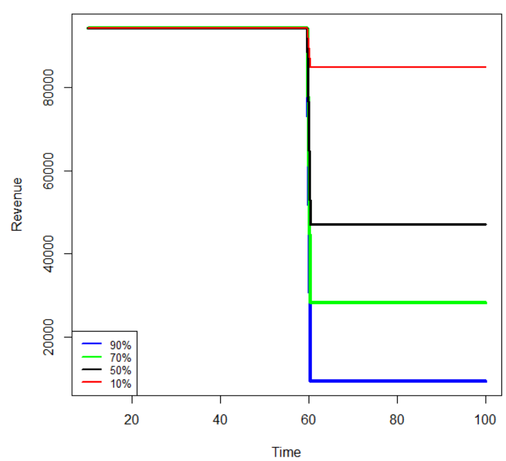

Figure 2. Effect of different price discounts on revenue.

The price skimming strategy helps a producer quickly recover their costs of development and generates a high profit margin for the producer. Figure 2 represents the relationship between revenue and discounted price when discounts were offered at $\mu = 60$. The red colored graph represents the relation when $\sigma = 10\%$, the black graph represents when $\sigma = 50\%$, the green colored graph represents when $\sigma = 70\%$, and the purple colored graph represents when $\sigma = 90\%$. The more you increase the percentage of discount, the less you earn. The producer can offer the discounted price at any time $\mu \in [0, T]$ to attract the next and subsequent layers of the market. This study assumes that the time $\mu \in [0, T]$ is the time when the producer is done collecting the max profit to recover its costs.

4. Related Costs

Here are various costs associated with production process.

Production Cost $= P_c = C_u * S_L$ by (11) (13)

Total carrying cost $= H_c = C_h \int_0^{t_2} S(t)dt$

$$= C_h \left[\frac{Q_p S_1^2}{2D_p(Q_p - D_p)} + \frac{\tau S_1^3}{3(Q_p - D_p)^2} \right] \tag{14}$$

Deterioration Cost $= D_c = C_d * [\{(Q_p - D_p)t_1 - S_1\} + \{S_1 - D_p(t_2 - t_1)\}]$

$$= \tau C_d \frac{Q_p S_1^2}{2D_p(Q_p - D_p)} \text{ by (4) and (5).} \tag{15}$$

Shortage Cost $= S_c = C_{sh} * \int_{t_2}^{T} - S(t)dt$

$$= C_{sh} \frac{Q_p S_2^2}{2D_p(Q_p - D_p)} \text{ by (9) and (10).} \tag{16}$$

5. The Maximization of the Profit

Average profit $= Q_{PROFIT}/T = [Q_{REV} - (C_s + P_c + H_c + D_c + S_c)]/T$ by (12), (13), (14), (15), (16)

$$= \Bigg(f(p) \Bigg\{ D_p \Bigg[\frac{S_1}{Q_p - D_p} + \tau \frac{S_1^2}{(Q_p - D_p)^2} \frac{1}{\tau} \ln\left(1 + \tau \frac{S_1}{D_p}\right) \Bigg]$$

$$+ \frac{1}{(Q_p - D_p)} \left[\frac{TD_p(Q_p - D_p)}{Q_p} - S_1 - \tau \frac{S_1^2(2D_p - Q_p)}{2D_p(Q_p - D_p)} \right]^2 \Bigg\} - C_s$$

$$- C_u \Bigg[\frac{S_1^2}{(Q_p - D_p)} + \tau \frac{S_1^3}{(Q_p - D_p)^2} + \frac{1}{(Q_p - D_p)}$$

$$\left\{ TD_p(Q_p - D_p) \frac{1}{Q_p} - S_1 - \tau \frac{S_1^2(2D_p - Q_p)}{2D_p(Q_p - D_p)} \right\}^2 \Bigg]$$

$$- C_h \left[Q_p \frac{S_1^2}{2D_p(Q_p - D_p)} + \tau \frac{S_1^3}{3(Q_p - D_p)^2} \right]$$

$$- C_d \left[\tau \frac{Q_p S_1^2}{2D_p(Q_p - D_p)} \right]$$

$$- C_{sh} \frac{Q_p}{2D_p(Q_p - D_p)} \left[TD_p(Q_p - D_p) \frac{1}{Q_p} - S_1 - \tau \frac{S_1^2(2D_p - Q_p)}{2D_p(Q_p - D_p)} \right]^2 \Bigg) / T$$

The objective of this study is to maximize the profit function $Q_{PROFIT}(T, S_1)$. The necessary conditions for maximizing the profit are $\frac{\partial Q_{PROFIT}}{\partial T} = 0$ and $\frac{\partial Q_{PROFIT}}{\partial S_1} = 0$. The solutions of $\frac{\partial Q_{PROFIT}}{\partial T} = 0$ and $\frac{\partial Q_{PROFIT}}{\partial S_1} = 0$ will give us T^* and S_1^*. The values of T^* and S_1^* will help us to evaluate the optimal value $Q^*_{PROFIT}(T, S_1)$ of the average profit provided they satisfy the sufficient condition for maximizing $Q_{PROFIT}(T, S_1)$ that is H is negative definite where H is the hessian matrix of $Q_{PROFIT}(T, S_1)$.

$$H = \begin{bmatrix} \dfrac{\partial^2 Q_{PROFIT}}{\partial T^2} & \dfrac{\partial^2 Q_{PROFIT}}{\partial T \partial S_1} \\ \dfrac{\partial^2 Q_{PROFIT}}{\partial S_1 \partial T} & \dfrac{\partial^2 Q_{PROFIT}}{\partial S_1^2} \end{bmatrix}$$

Theorem 5.1. *If* $f(p) - C_u < C_{sh} \dfrac{Q_p}{2D_p}$, $\tau << 1$ *and* $2D_p < Q_p$, *the hessian matrix of* $Q_{PROFIT}(T, S_1)$ *will be negative definite.*

Proof. Since $f(p) - C_u < C_{sh} \dfrac{Q_p}{2D_p}$ implies $f(p) - C_u - C_{sh} \dfrac{Q_p}{2D_p} < 0$ and $2D_p < Q_p$ implies $2D_p - Q_p < 0$.

Therefore

$$\frac{\partial^2 Q_{PROFIT}(T, S_1)}{\partial T^2} = 2(Q_p - D_p) \frac{D_p^2}{Q_p^2} \left[f(p) - C_u - C_{sh} \frac{Q_p}{2D_p} \right] < 0$$

$$\frac{\partial^2 Q_{PROFIT}(T, S_1)}{\partial T \partial S_1} = \frac{\partial^2 Q_{PROFIT}(T, S_1)}{\partial S_1 \partial T}$$

$$= 2 \frac{D_p}{Q_0} \left[1 - \tau \frac{S_1(2D_p - Q_p)}{D_p(Q_p - D_p)} \right] \left[f(p) - C_u - C_{sh} \frac{Q_p}{2D_p} \right]$$

$$\frac{\partial^2 Q_{PROFIT}(T, S_1)}{\partial S_1^2}$$

$$= 2 \frac{1}{(Q_p - D_p)} \left[1 + \tau \frac{S_1(2D_p - Q_p)}{D_p(Q_p - D_p)} \right]^2 \left[f(p) - C_u - C_{sh} \frac{Q_p}{2D_p} \right]$$

$$+ \tau D_p f(p) \left[\frac{2}{(Q_p - D_p)^2} - \frac{1}{D_p^2} \left(1 + \tau \frac{S_1}{D_p} \right)^{-2} \right]$$

$$- 2\tau \frac{(2D_p - Q_p)}{D_p(Q_p - D_p)^2} \left[T D_p (Q_p - D_p) \frac{1}{Q_p} - S_1 - \tau \frac{S_1^2(2D_p - Q_p)}{2D_p(Q_p - D_p)} \right]$$

$$\left[f(p) - C_u - C_{sh}\frac{Q_p}{2D_p}\right]$$

$$- C_h\frac{1}{(Q_p - D_p)}\left[\frac{Q_p}{D_p} + 2\tau\frac{S_1}{(Q_p - D_p)}\right] - \tau C_d\frac{Q_p}{D_p(Q_p - D_p)} < 0.$$

Therefore

$$\frac{\partial^2 Q_{PROFIT}(T, S_1)}{\partial T^2}\frac{\partial^2 Q_{PROFIT}(T, S_1)}{\partial S_1^2}$$

$$-\left(\frac{\partial^2 Q_{PROFIT}(T, S_1)}{\partial S_1 \partial T}\right)\left(\frac{\partial^2 Q_{PROFIT}(T, S_1)}{\partial T \partial S_1}\right)$$

$$= 2\left(\tau\frac{S_1(2D_p - Q_p)}{D_p(Q_p - D_p)}\left[f(p) - C_u - C_{sh}\frac{Q_p}{2D_p}\right]\frac{D_p}{Q_p}\right)^2$$

$$+ 4\tau\left[f(p) - C_u - C_{sh}\frac{Q_p}{2D_p}\right]\frac{f(p)D_p^2}{Q_p^2(Q_p - D_p)}$$

$$- 2\tau D_p f(p)\left[f(p) - C_u - C_{sh}\frac{Q_p}{2D_p}\right](Q_p - D_p)\frac{1}{Q_p^2}\left(1 + \tau\frac{S_1}{D_p}\right)^{-2}$$

$$+ 4\tau\frac{S_1(2D_p - Q_p)}{D_p(Q_p - D_p)}\left[f(p) - C_u - C_{sh}\frac{Q_p}{D_p}\right]^2$$

$$T(Q_p - D_p)\frac{D_p^2}{S_1 Q_p^2} - 2\left[f(p) - C_u - C_{sh}\frac{Q_p}{2D_p}\right]$$

$$D_p\left(1 - \frac{D_p}{Q_p}\right)(\tau C_d + C_h) - 4\tau\left[f(p) - C_u - C_{st}\frac{Q_p}{2D_p}\right]S_1 C_h\frac{D_p^2}{Q_p^2(Q_p - D_p)} > 0$$

Hence the hessian matrix of $Q_{PROFIT}(T, S_1)$ is negative definite.

Theorem 5.2. *There exist optimal solutions* $T = T^*$ *and* $S_1 = S_1^*$ *which maximize the profit function* $Q_{PROFIT}(T, S_1)$ *if* $f(p) - C_u < C_{sh}\frac{Q_p}{2D_p}$ *and* $\tau << 1$.

Proof. Since the hessian matrix of $Q_{PROFIT}(T, S_1)$ is negative definite, there exists a unique solution $(T = T^*, S_1 = S_1^*)$ that maximizes the profit function. The optimal values, T^* and S_1^*, are the solutions of the equations

$$\frac{\partial Q_{PROFIT}(T, S_1)}{\partial T} = 0, \frac{\partial Q_{PROFIT}(T, S_1)}{\partial S_1} = 0$$

which implies

$$\frac{D_p}{Q_p}\left[(f(p) - C_u) - C_{sh}\frac{Q_p}{2D_p}\right]\left[TD_p(Q_p - D_p)\frac{1}{Q_p} - S_1 - \tau\frac{S_1^2(2D_p - Q_p)}{2D_p(Q_p - D_p)}\right] = 0$$

and

$$f(p)D_p\left[\frac{1}{(Q_p - D_p)} + 2\tau\frac{S_1}{(Q_p - D_p)^2} + \frac{1}{D_p}\left[1 + \tau\frac{S_1}{D_p}\right)^{-1}\right]$$

$$-2\frac{1}{(Q_p - D_p)}\left[TD_p(Q_p - D_p)\frac{1}{Q_p} - S_1 - \tau\frac{S_1^2(2D_p - Q_p)}{2D_p(Q_p - D_p)}\right]$$

$$\left[1 + \tau\frac{S_1(2D_p - Q_p)}{D_p(Q_p - D_p)}\right]\left[f(p) - C_u - C_{sh}\frac{Q_p}{2D_p}\right]$$

$$-C_h\frac{S_1}{(Q_p - D_p)}\left[\frac{Q_p}{D_p} + \tau\frac{S_1}{(Q_p - D_p)}\right]$$

$$-\tau C_d\frac{S_1 Q_p}{D_p(Q_p - D_p)} = 0$$

Hence solving above two equations we get the optimal values $T = T^*, S_1 = S_1^*$.

6. Numerical Examples

Here are two examples. In example 1, the price of the product is less than that in example 2.

Example 1. In this example, all parameters are assumed to be in appropriate units. $\tau = 0.01$, $C_h = 10$, $C_{sh} = 165$, $C_u = 15$, $C_s = 200$, $C_d = 10$,

$Q_p = 80$, $d_p = 35$, $p = 200$. The optimum inventory level and the optimum time were determined by maximizing the profit function. Solving $\frac{\partial Q_{PROFIT}}{\partial T} = 0$ and $\frac{\partial Q_{PROFIT}}{\partial S_1} = 0$ we get T^* and S_1^* where $T^* = 36.70$, $S_1^* = 740.06$, and $Q_{PROFIT}^* = 134{,}706.40$.

Example 2. Here σ varies from 10% to 90%. To cover all the costs after offering a 90% discount we selected very high price, otherwise the producer will not have enough earning. Here, all parameters are assumed to be in appropriate units. $\tau = 0.01$, $C_h = 400$, $C_{sh} = 2{,}000$, $C_u = 4{,}000$, $C_s = 500$, $C_d = 20$, $Q_p = 800$, $d_p = 350$, $p = 5{,}000$. Results have been illustrated by a graph in Figure 3.

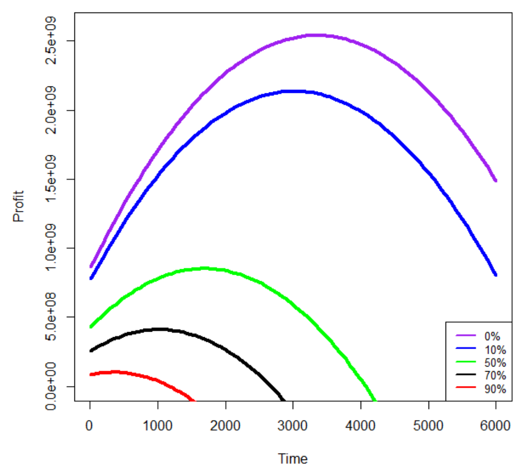

Figure 3. Effect of different price discounts on profit.

7. Conclusion

This study developed an Economic Order Quantity model with Price skimming strategy for deteriorating items. Since the price skimming pricing strategy sets a high introductory price, it helps a producer quickly recover its costs of development. It generates a high profit margin for the producer. Then reduces the price over time. Our model used a Heaviside function to represent the price skimming strategy and showed a revenue vs time relationship graph (Figure 2) for various price discounts ranging from 0% to 90%. It is clear from the graph that the more discount you offer the less you earn. The producer can decide what discount percentage they should offer and when they should offer it so that they can run the business smoothly. This study has a specific time $\mu \in [0, T]$ when discount was offered. In reality, the producer can set up μ. The time μ could be set up before t_1, or sometime between t_1 and t_2 or sometime between t_2 and t_3 or after t_3. If we offer a discount too early there is a chance that the earnings will not be enough to cover the costs associated with the production. Figure 3 showed a very interesting result. It showed that discounts not only affect the profit but also the cycle time. If we were to offer higher discounts the inventory cycle will end up too soon without giving us high profits. If we were to offer no discount at all or too little discount, that would give us maximum profit with a longer cycle time. In Figure 3, the red colored graph represents the relation between the profit function and time when $\sigma = 90\%$, the black graph represents relation between the profit function and time when $\sigma = 70\%$, the green colored graph represents the relation between the profit function and time when $\sigma = 50\%$, and the blue colored graph represents between the profit function and time when $\sigma = 10\%$. The purple colored graph represents the relation between the profit function and time when no discounts were offered at all. The more you increase the amount of discount the less you earn. In Figure 3, all the graphs show profit function when discounts were offered at $T = T^*$ and $S_1 = S_1^*$. That means the optimum inventory level and the optimum time to offer discount were determined by maximizing the profit function using example 2 parameters. Also this example finds that a high selling price earns maximum profit later than discounted selling price. Also high selling price takes longer optimal time to earn maximum profit compare to discounted selling price whereas discounted selling price completes the cycle faster than a selling price without any discount as the discounted selling price attracts more customers. This study could be extended in the future by incorporating price dependent deterioration as the products that deteriorates over time, sold at a very high price, are more likely to be wasted or become outdated.

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