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APPLYING GRAPH MODELS TO INTERPRET NICHE OVERLAP IN AN ECOSYSTEM

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Abstract

Ecological networks are abstract representations of nature describing species diversity, trophic (i.e., feeding) and non trophic (e.g., facilitation, mutualism) relationships between species, and flows of energy and nutrients or individuals within an ecosystem. Traditionally, these features have been studied separately, such that each net work describes just one type of interaction. Combining different types of interactions is more challenging, and there is a growing body of literature and methods attempting to tackle this problem [1]. This paper made an attempt in investigating the different network by means of graph models which representing the food web in ecosystem and the space of interaction between the different species. Niche overlap graphs (frequently called competition graphs) have become familiar models of food webs. This work used graph to model the interaction, coexistence and competition between species overlapping each other in an ecosystem.

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1. Introduction

When the diets of different organisms overlap in Natural communities, the possibility arises that the different consumers may compete for food, [2] or may interact mutualistically [3], [4], Competitive or mutualistic interactions over food may influence the evolution of competing or coorperating consumers. Hence, overlaps in the diets of different organisms are of both ecological and evolutionary interest. The diet of organisms, and the relations among the diets of different kinds of organisms, vary greatly from one ecological community to another

A graph model describes the structure of graph data base, and comprises of two core components, nodes and edges. An edge connects two nodes together by describing their relationship to one another. With many nodes connected by many edges, a spider web of interconnected points emerged. We have different kinds of graph that can be used to model different forms of relationship in social network such as Facebook, WhatsApp, Twitter, etc. as by [5], [6] and as well as other field of studies .

A graph is called an overlap graph if each of its vertices can be associated with an interval on the real line in such a way that two vertices are adjacent if and only if the associated intervals overlap, namely they have a nonempty intersection and neither interval contains the other. These intervals are said to form an overlap representation of the graph.

Graphs can be used to describe the situation in which co-occurring species shares parts of their niche space with each other high niche overlap may lead to conflictual interaction (such as competition and excursion) forum some species, in a simplified definition, a niche can be seen as the role organisms play in a certain ecosystem. This includes the biotic and abiotic encompassing it's feeding, living, reproduction, physical and chemical activities. This competition between species in an ecosystem can be moderated using a Niche overlap graph. Each species is represented by a vertex. An undirected edge connects two vertices if the two species represented by those vertices compete.

1.1. Niche overlap

Niche overlap refers to the partial or complete sharing of resources or other ecological factors (predators, foraging space, soil type, and so on) by two or more species. For example, warblers in a woodlot might all feed on insects and thus overlap in their diets, or plants in a meadow might all overlap in their need for light. Niche overlap is an important concept in community ecology because it is expected to determine how many and which species can coexist in a community. Interest in niche overlap began with the competitive exclusion principle, which states that two species using identical resources and/or environments cannot coexist. The niche concept developed as a way to describe the range of environments and resources required for the persistence of individual species. The competitive exclusion principle was then reformulated using niche terminology; two species overlapping completely in their niches cannot coexist. Theory then developed to link limits to niche overlap with limits to the number of coexisting species.

Niche overlap of a lien resource for which they compete, organisms or species that compete such as the owl and the cat feed on mice as well as shrews. Since they are ecological equivalents, although their environment is different, they lived in the same niche.

Ecological communities are comprises of numerous species that interact with each other through various ecological processes. Understanding the extent of niche overlap which occurs when multiple species occupy similar ecological niche, is vital for assessing community structure, competition and biodiversity conservation. Traditional methods of studying niche overlap often involve complex mathematical equations or qualitative assessments, which may lack precision or fail to capture the complexity of real-world ecological system. For this very reason this work decides to explain this concept in simple graph models.

1.2. Innovative contributions of this work

Graph models in ecosystem analysis are essential tools for understanding the complex relationships and interactions between different species and their environments. By studying the structure of the graphs and analyzing the flow of energy, nutrients and other resources through the ecosystem, researchers can gain valuable insights into its functioning and resilience.

Overall graph models provide a powerful framework for studying ecosystems in a holistic and quantitative way, shedding light on their interconnectedness and vulnerability to various stressors. By delving into these models and their applications, researchers and conservationist can make informed decisions to better protect and manage our precious natural resources.

2. Methodology

2.1. Graph models

A graph model describes the structure of graph data base, and comprises of two core components, nodes and edges. An edge connects two nodes together by describing their relationship to one another. With many nodes connected by many edges, a spider web of interconnected points emerged. We have different kinds of graph that can be used to model different forms of relationship in social network such as Facebook, WhatsApp as discussed by [5], [6], Twitter, etc. as well as other field of studies.

Many scientific works have been carried out using different methods to determine niche overlap of species in an ecosystem. However, we desire to bring some of this work by means of Graph model for better understanding of subject matter

Graph models of ecological community representation were constructed. Where species represents nodes and ecological interactions as edges.

We present graph models using network analysis techniques to identify niche overlap, this will help in evaluation of their ecological implications. Interpret the findings and discuss the implications for species co-existence, community dynamics, and conservation strategies.

Generally speaking, ecological networks can be divided into three different categories: consumer-resource (food webs, host-parasite/plant-herbivore networks), mutualistic (plant-pollinator/plant-seed dispersal net works) and competitive networks (usually with nodes belonging to a single guild, e.g., plants). These networks can be described at three different levels: topological, quantitative, and dynamical. Topological networks are binary in that they describe only the presence/absence of interactions among species. Quantitative networks add to the topology a weight for each link. This value can describe the frequency or the strength of interaction. Finally, dynamical networks represent not only the species and the topology/strength of interactions but also the underlying population dynamics. From this perspective, the various descriptions can be "coarse grained" hierarchically: from dynamical networks, we can infer the weight for each connection. Making this weight 0 or 1, we obtain topological networks. Dynam ical networks can capture more features compared to just topological networks, such as indirect effects.

2.2. General network properties

The search for unifying principles that give rise to the structure of ecological networks dates back to the 1970s [1]. Networks are characterized by the components-the nodes-and their interactions-the edges (or links or arrows)-connecting them. From this information, the network structure can be derived. Links can either be undirected (a link from node A to node B implies a symmetric link back from B to A) or directed (the connection from A to B can differ from that of B to A). The links can be binary (representing their presence or absence only) or weighted (i.e., have a measure of their strength). In network analysis, the focus is the topological structure and functional relationships in the network. A path in a network is a route between different nodes where each node and link are visited only once. The length of such a 470 path is the sum (weighted or unweighted) of all the links visited. The shortest path between any two nodes is a frequently used measure in network theory. The longest of all the shortest paths is known as the diameter of the net work. The average of all the shortest paths in a network is called the characteristic path length. Because of the large number of nodes and the even larger number of edges that combine to form such an intricate structure, ecological networks are a prominent example of complex systems. Networks are potentially difficult to understand because of their structural complexity, their dynamic nature (the number of nodes and edges frequently change through time), the diversity of types of links and nodes, the presence of nonlinear dynamics in the relationships between nodes, and the fact that various network proper ties can often influence each other. Other characteristics that are commonly described for ecological networks are the proportion of nodes belonging to a specific group or guild, intervality, modularity, and nestedness. Presented below is a brief summary of these different properties.

3. Application of Graphical Models in Ecosystem

3.1. Introduction

In this section, we present the different graph models as related to the relationship existing in a typical ecosystem ranging from co-existence, interaction and competition

The graph model of niche overlap between species is a representation of the interaction, competition and coexistence within an ecosystem. Niche overlap in an ecosystem refers to the situation where two or more species in the same ecosystem have similar ecological niches, meaning they occupy similar roles in their environment. This overlap can lead to competition between species for resources such as food, water, or shelter. However, it can also lead to coexistence and the development of specialized adaptations that allow the species to share resources and thrive in the same environment.

A food web is a directed graph that tells which kinds of organisms nourish which other kinds of organisms in a community of species. Each vertex in such a graph corresponds to a kind of organism. Each arrow or directed edge between vertices corresponds to a flow of energy or biomass from one kind of organism to another.

3.2.0. Co-existence and interaction in an ecosystem

3.2.1. Transfer of energy in the niche

In a niche, energy is transferred from one level to the next through a food chain. When two or more species occupy the same niche, they compete for resources and energy, which results in a transfer of energy from one species to another.

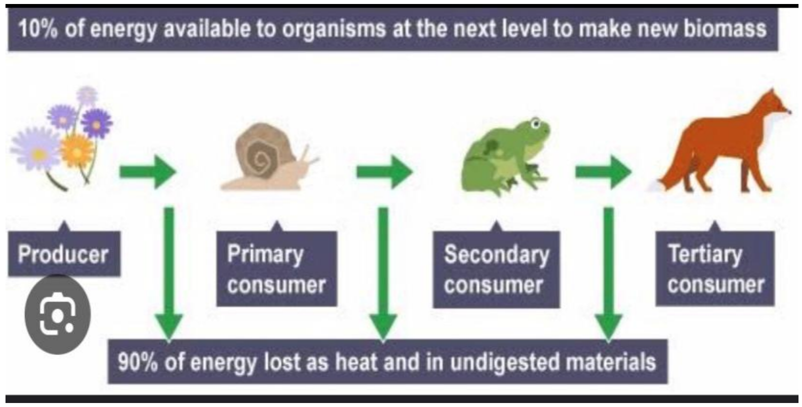
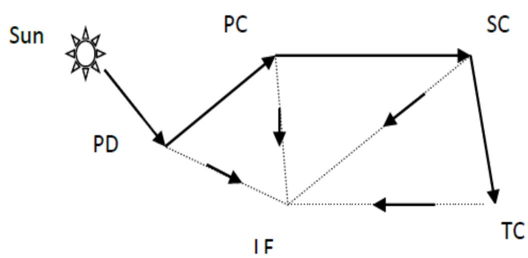


Figure 1a. Energy Transfer in Niche.



This graph is a directed graph, which shows the flow of Energy from the source (Sun) to PD-PC-SC-TC and their excess Energy to a sink at point LE

Figure 1b. A Graph Model of Energy flow through four trophic level food chain.

From Figure 1 (a and b) PD = Producers (Plants), PC = Primary Consumer, SC = Secondary Consumer, TC = Tertiary Consumer and Lost Energy, LE = Lost Energy (Undigested material).

Producers, such as plants, convert sunlight into energy through photosynthesis. Primary consumers, such as herbivores, eat the producers and obtain energy from them. Secondary consumers, such as carnivores, eat the primary consumers and obtain energy from them. Tertiary consumers, such as apex predators, eat the secondary consumers and obtain energy from them. Each level of the food chain transfers energy from the previous level to the next.

3.2.2. An energy pyramid in niche

An energy pyramid, also known as a trophic or ecological pyramid, is a graphical representation of the energy found within the trophic levels of an ecosystem. The bottom and largest level of the pyramid is the producers and contains the largest amount of energy. As you move up the pyramid, through the trophic levels to primary, secondary and tertiary consumers, the amount of energy decreases and the levels become smaller.

An energy pyramid is a graphical representation of the energy flow through different trophic levels of an ecosystem. At the base of the pyramid are the producers, such as cacti, which convert sunlight into energy through photosynthesis. The primary consumers, such as mice, eat the producers and obtain energy from them. The secondary consumers, such as snakes, eat the primary consumers and obtain energy from them. The tertiary consumers, such as hawks, eat the secondary consumers and obtain energy from them.

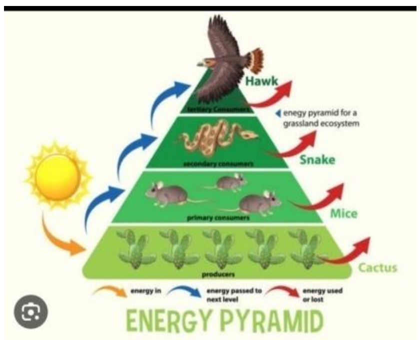


Figure 2a. Energy Pyramid in Niche.

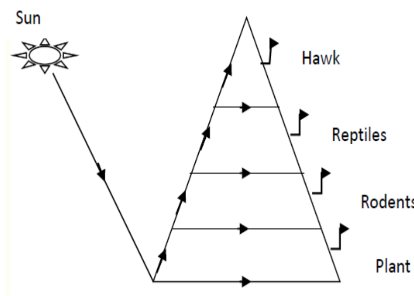


Figure 2b. Graph model showing the flow of Energy Pyramid in a Niche by a directed graph.

Each level of the pyramid represents a decrease in the amount of energy available, with only a portion of the energy from the previous level being transferred to the next level. This is due to energy loss through processes such as respiration, movement, and waste production.

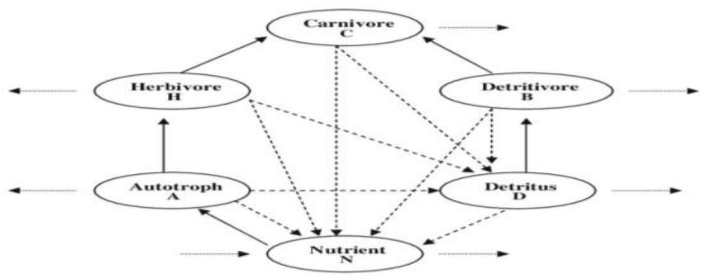


Figure 3. A Graph Model of food web with grazing and detrital food chains linked by nutrient cycling and generalist top predators.

The solid arrows represent consumer-resource interactions, whereas the dashed arrows represent the flow of nutrients from the living components to detritus or the nutrient pool. The dotted arrows represent losses of each food web component from the system and external input of limiting-nutrients.

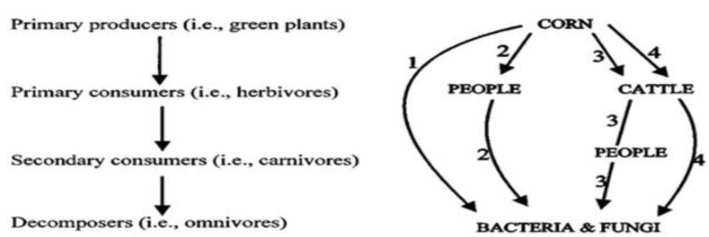


Figure 4. Graph Model showing food Web in ecosystem.

The structural organization of all ecosystems can be described as consisting of four components, one non living and three living. The abiotic (i.e., non-living) component defines the chemical and physical environment of the biotic (i.e., living) component. It includes such things as climate, atmosphere, and soils. It is the water in the fishbowl and the soil, air, and sunlight in the garden, cornfield, and pasture. The three biotic components are producers, consumers, and decomposers. Producers are organisms that capture solar energy. They are the phytoplankton in the fishbowl, the vegetables in the garden, the corn in the cornfield, and the grasses, forbs, and shrubs growing in the pasture. Consumers are organisms that obtain their energy by consuming other organisms. Consumer organisms are animals except in very rare instances (e.g., the Venus fly trap). Consumers that consume plants are called herbivores, those consuming other animals are called carnivores, and those consuming both plants and animals are called omnivores. Cattle are herbivores, coyotes are primarily carnivores, and people are omnivores. Decomposers are the final or last consumers of organic matter, [7]. They are the microorganisms, primarily bacteria and fungi that complete the decomposition process. Food chains are energy processing pathways that determine the pattern of energy flow through an ecosystem (Figure 4). There are two types of food chains, detrital and grazing. In both chains, the first trophic level consists of the primary producers or green plants. The difference between the chains comes at the second trophic level is that if the primary consumers are decomposers, then the food chain is a detrital food chain (e.g., chain #1, Figure 4), otherwise that defined food chain is called a grazing food chain (e.g., chains #2, 3, and 4, Figure 4).

The diet of organisms, and the relations among the diets of different kinds of organisms, vary greatly from one ecological community to another.

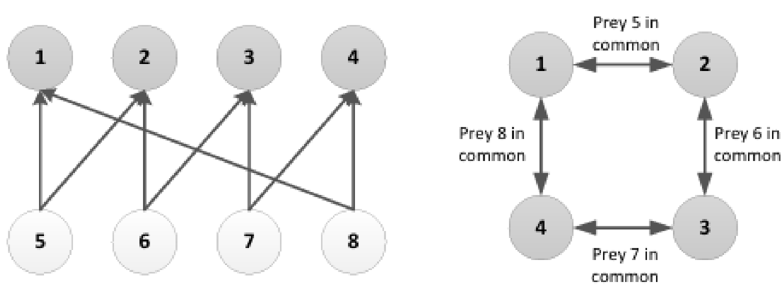


Figure 5. A Graph Model Visualization of a food-web graph of order 8 and its associated niche-overlap graph. The grey nodes are the predators and the white ones are the prey.

This Graph is a bipartite graph of Food-web which shows who eat whom in natural communities, this is presented by drawing a directed graph where each node corresponds to a species and each directed link describes the flow of energy or biomass. From this graph, it is possible to create the niche-overlap graph which represents the competition graphs. Two predators are linked if they share at least one prey. Figure 5 explain a food-web graph and its associated niche-overlap graph.

3.2.3. Transfer of nutrients in niche

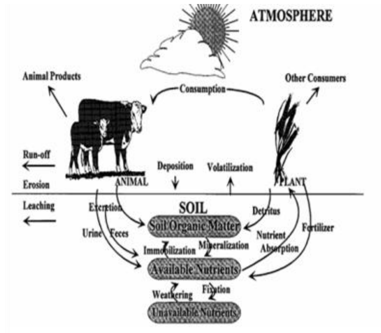


Figure 6a. A Graph Model of ecosystem level nutrient cycle.

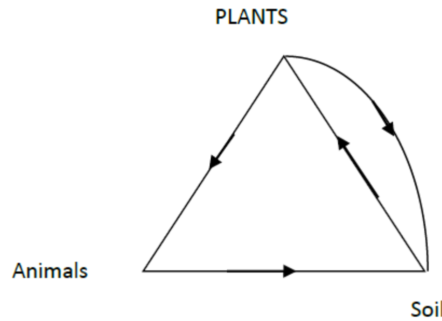


Figure 6b. The Graph Model Representation of Nutrient cycle. We have two cycles Plant-Soil and Plant-Animal-Soil.

Directed graph is use to explain the indispensable function performed by ecosystems which is the cycling of nutrients. Nutrients are the abiotic raw materials required by organisms to capture and process solar energy. Carbon, nitrogen, oxygen, and water are examples of nutrients that are continually cycled by ecosystems (Figure 6). The cycle revolves around the assimilation of nutrients by the primary producers followed by the sequential reduction of complex organic compounds by consumers to simpler, less complex forms [8], [9].

3.2.4. Transfer of heat in niche

According to thermodynamic systems, heat transfer is defined as “The movement of heat across the border of the system due to a difference in temperature between the system and its surroundings.” Interestingly, the difference in temperature is said to be a ‘potential’ that causes the transfer of heat from one point to another.

In an ecological context, heat transfer can occur between organisms in a niche through processes such as conduction, convection, and radiation. For example, a warm-blooded predator might use the heat generated by its metabolism to maintain its body temperature, while a cold-blooded prey animal might bask in the sun to warm up. Often the regulation of temperature, water and nutrients is considered separately. There are strong connections and interactions.

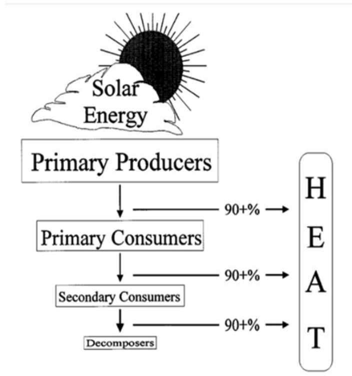


Figure 7a. Graph Model Showing Heat Transfer in Niche represent by a directed Graph.

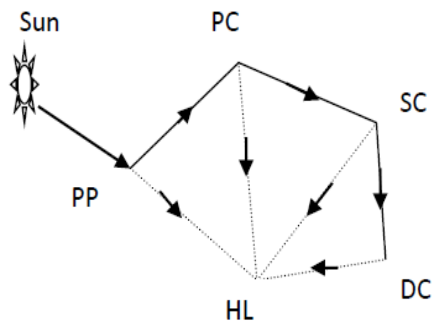


Figure 7b. This graph is a directed graph, which shows the flow of Heat from the source (Sun) to PP-PC-SC-DC and there Heat loses to environment to a sink at point HL.

PP = Primary Producers (Plants), PC = Primary Consumers, SC = Secondary Consumers, DC = Decomposers and HL = Heat Losses (unused heat lost to environment) between these two variables. However, and consequently, an animal's regulatory behaviors must often represent homoeostatic compromise. Moreover, nutrition itself is often considered in a univariate sense (heat only) but in a multinutrition space they can be important homoeostatic compromises both in terms of ingestive behaviors and body composition with ensuring consequences for fitness [10].

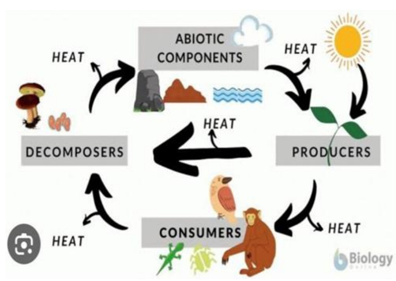


Figure 8a. Graph Model Showing Heat Transfer in Niche represent by a directed Graph.

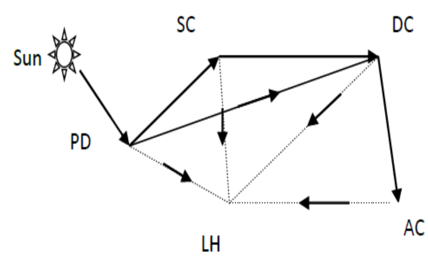


Figure 8b. This graph is a directed graph, which shows the flow of Heat from the source (Sun) to PD-DC, PD-SC-DC-AC and their Heat loses to environment which is a sink at point LH.

PD = Producers (Plants), CS = Consumers, DC = Decomposers, AC = Abiotic Components and LH = Lost Heat (unused heat lost to environment)

Heat is transferred from the abiotic components, such as sunlight or geothermal energy, to the producers, such as plants, through processes such as photosynthesis. Consumers, such as herbivores, obtain heat indirectly from the producers by consuming them. Decomposers, such as bacteria and fungi, obtain heat from the dead organic matter of both producers and consumers. Heat is continuously transferred through these components of the ecosystem, with some of it being lost as waste heat in each step of the process. This process of heat transfer is essential for maintaining the energy balance of the ecosystem and supporting the growth and survival of its inhabitants.

3.3. Competition

Competition in the context of niche overlap in an ecosystem refers to the struggle between two or more species for the same resources, such as food, water, or shelter. When two or more species occupy similar ecological niches, they may compete with one another for these resources. This competition can be intense and may lead to one species outcompete the other, or it can be relatively mild and result in the coexistence of both species. Competition is most typically considered the interaction of individuals that vie for a common resource that is in limited supply, but more generally can be defined as the direct or indirect interaction of organisms that leads to a change in fitness when the organisms share the same resource. The outcome usually has negative effects on the weaker competitors. There are three major forms of competition. Two of them, interference competition and exploitation competition, are categorized as real competition. A third form, apparent competition, Interference competition occurs directly between individuals, while exploitation competition and apparent competition occur indirectly between individuals [11].

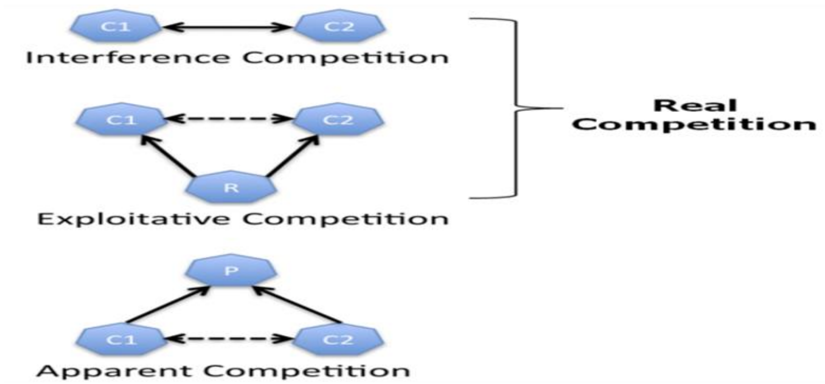


Figure 9. A Graph Model Showing the Three Major Types of Competitive Interaction.

Figure 9 is illustrating the three major types of competitive interaction where the dashed lines indicates the indirect interaction and the solid lines direct interaction, that are part of the ecological communities.

$C1$ = competitor, $C2$ = competitor, P = predator, R = resource.

The outcomes of competition between two species can be predicted using equations, and one of the most well-known is the Lotka-Volterra model [12]. This model relates the population density and carrying capacity of two species to each other and includes their overall effect on each other. The four outcomes of this model are: 1) species A competitively excludes species B; 2) species B competitively excludes species A; 3) either species wins based on population densities; or 4) coexistence occurs. Species can survive together if intra-specific is stronger than inter-specific competition. This means that each species will inhibit their own population growth before they inhibit that of the competitor, leading to coexistence.

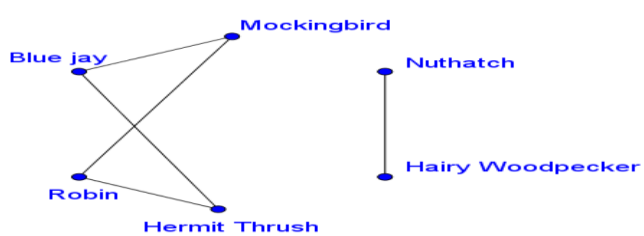


Figure 10. A Graph Model Showing a Construction of a niche overlap graph for six species of birds.

We hereby use an undirected graph to show that the hermit thrush competes with the robin and with the blue jay; the robin also competes with the mockingbird, the mockingbird also competes with the blue jay, and the nuthatch competes with the hairy woodpecker.

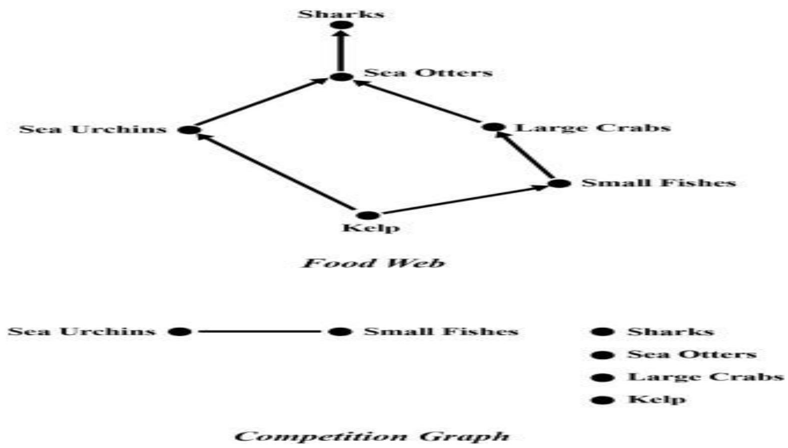


Figure 11. A Graph Model Showing Competition Between Abiotic Factors in Water.

Competition between sharks, sea otters, large crabs, small fishes, kelp, and sea urchins in a shared ecosystem is possible because these species occupy similar ecological niches. For example, sea otters and large crabs are predators that feed on small fishes and sea urchins, while sharks are apex predators that feed on larger fishes. Kelp and sea urchins are herbivores that feed on the same kelp forests, and small fishes may compete for the same food and shelter resources within the kelp forest. This competition can be intense and may lead to one species outcompeting the other, or it can be relatively mild and result in the coexistence of multiple species. Over time, competition can drive the evolution of specialized adaptations that allow species to better exploit their resources and reduce competition.

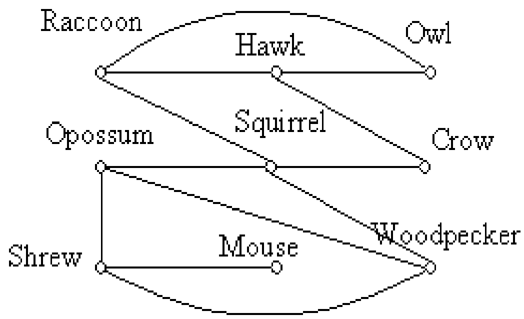


Figure 12. A Graph Model showing Competition between Species represented by an undirected graph.

Niche Overlap Graphs in Ecology graphs are used in many models involving the interaction of different species of animals. For instance, the competition between species in an ecosystem can be modeled using a niche overlap graph. Each species is represented by a vertex. An undirected edge connects two vertices if the two species represented by these vertices compete (that is, some of the food resources they use are the same). A niche overlap graph is a simple graph because no loops or multiple edges are needed in this model. The graph above models the ecosystem of a forest. We see from this graph that squirrels and raccoons, Squirrel and Crow, Squirrel and Opossum, Squirrel and woodpecker competes but that crows and shrews do not, it also shows the species that compete with Hawk are Raccoon, Owl and Crow, only Shrew compete with Mouse.

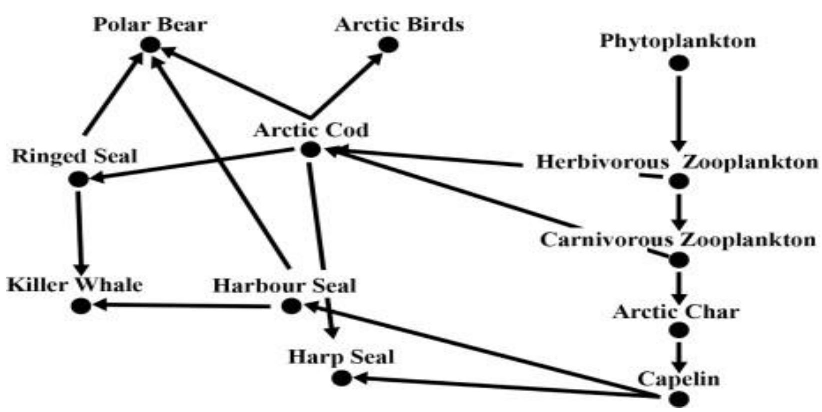


Figure 13. A Graph Model of Food web for the Polar bear.

A food web is a directed graph that tells which kinds of organisms nourish which other kinds of organisms in a community of species. Each vertex in such a graph corresponds to a kind of organism. Each arrow or directed edge between vertices corresponds to a flow of energy or biomass from one kind of organism to another [13]. Another description of communities of species represents each kind of organism by a multidimensional hypervolume in a hypothetical ecological niche space. In niche space, each dimension corresponds to some environmental variable or some variable characterizing the food consumed by the organisms. The multidimensional hypervolume associated with each kind of organism is called its niche. The projection of ecological niche space onto the dimensions characterizing the food consumed is called trophic niche space, and the same projection of a niche is called the trophic niche.

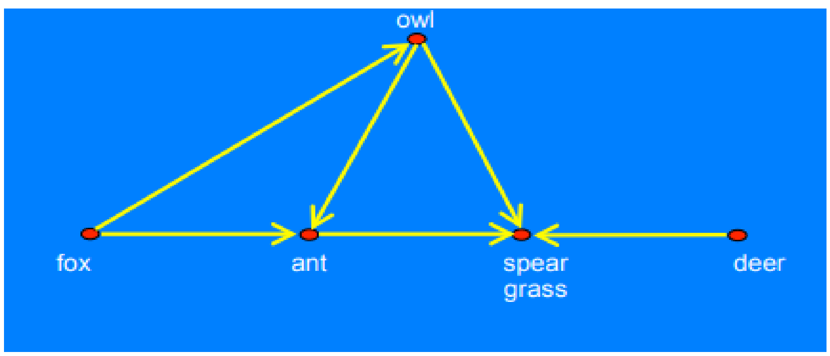


Figure 14. A Graph Model for Competition.

Competition Graphs of Food Webs Food Webs Let the vertices of a directed graph (digraph) be species in an ecosystem. Include an arc from x to y if x preys on y. usual assumption for us: no cycles.

Competition Graphs More generally: Given a digraph $D = (V, A)$ (Usually assumed to be acyclic.) The competition graph $C(D)$ has vertex set V and an edge between a and b if there is an x with arcs $(a, x) \in A$ and $(b, x) \in A$.

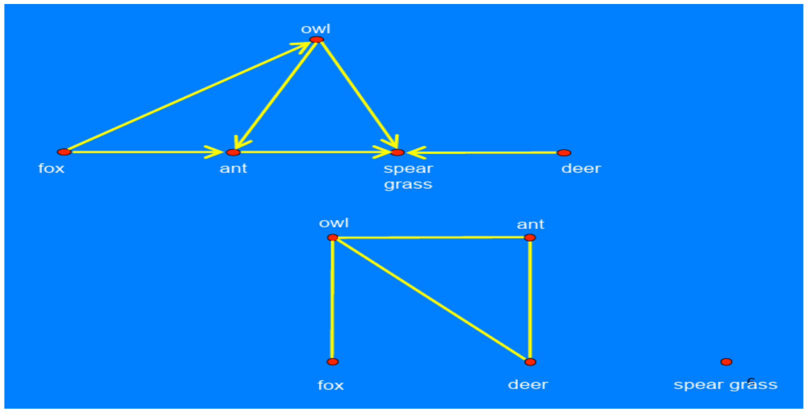


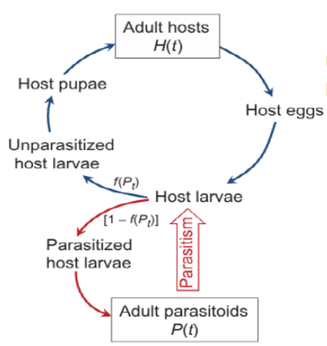
Figure 15. Competition Graphs of Food Webs consider a corresponding undirected graph.

Vertices = the species in the ecosystem Edge between a and b if they have a common prey, i.e., if there is some x so that there are arcs from a to x and b to x.

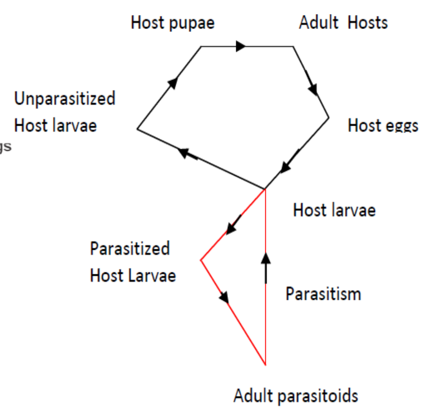
3.3.2. Parasitoids niche overlaps

A large number of species in natural ecosystems are parasites. Despite this fact, parasites have been almost completely disregarded in ecological networks; only recently have ecologists started to include parasites in food webs. Although parasites have been historically neglected in the food web literature, there have been networks centered on parasites. These are commonly referred as parasite-host networks or parasitoid-host network. These networks, even though based on consumer-resource dynamics, share similar features with mutualistic networks. In fact, they are bipartite and display a high degree of nestedness and modularity.

In 1935, A. J. Nicholson and V. A. Bailey [14] developed a model to describe the interaction between an insect host at tacked by a specialist parasitoid. The equilibrium in the Nicholson-Bailey model is unstable, with host and parasitoid population trajectories undergoing oscillations with an amplitude that increases through time. Many modifications of the model have been investigated to attempt to explain the stability of real host-parasitoid systems. PARASITOID LIFE HISTORIES Parasitoids display an amazing range of behaviors and life history strategies, and many of these behaviors and strategies have been incorporated into models. For example, some parasitoid lay a single egg on each host that it attacks; others lay multiple eggs per host. Some parasitoids kill their host immediately upon attack (idiobionts); others allow the host to continue to feed and grow for some time before eventually killing their host (koinobionts). Hymenopteran parasitoids have a haplo-diploid genetic system, with males resulting from unfertilized eggs and females from fertilized eggs, and in many species the female can choose the sex of her offspring on a particular host. According to [15], NICHOLSON-BAILEY HOST PARASITOID MODEL 498 Host pupae Unparasitized host larvae Adult hosts $H(t)f(Pt)$ Host larvae $[1 - f(Pt)]$ Parasitized host larvae Parasitism Host eggs Adult parasitoids $P(t)$ Diagram of the hypothetical host and parasitoid life cycles Figure 16 assumed by the Nicholson-Bailey model. The Nicholson-Bailey model is a discrete-time model that best describes insect host-parasitoid interactions in temperate regions (Figure 16) in which both host and parasitoid have a single, non overlapping generation per year (the term "parasitoid" was not in common usage in 1935, so Nicholson and Bailey instead used the term "entomophagous parasite"). For example, it describes a life cycle in which the adult host emerges in the spring from overwintering pupae, lays her eggs, and then dies.



(a)



(b)

Figure 16 (a and b). A Graph Model of the hypothetical host and parasitoid life cycles, using directed graph.

The host is vulnerable to attack by the parasitoid during only a limited portion of the juvenile host development (e.g., the larval stage), and the adult female parasitoid usually has a relatively short life span (days, weeks, or months) in which to search and attack host prior to death

Looking at figure 16b shows that the graph model form by Nicholson-Bailey Host Parasotoid Model 498 is directed and a two cycle graph.

4. Conclusion

In recent years, graph theory has established itself as an important mathematical tool in variety of subjects [16]. [17] Graph models can be used to study many fields in real life. There are many Scientists, engineers, researchers and scholars are interested to understand and optimize these networks.

Investigating niche overlap through graph modeling offers a valuable approach to studying species interaction and community dynamics in ecosystems. By uncovering the complex network of relationships among species researchers can deepen their understanding of ecological processes and develop strategies for sustainable ecosystem management and conservation.

4.1. Limitation of this work

This work cover the area of graph model using directed and undirected graph showing relationship of Niche overlap in an ecosystem. We present how species co-exist, interact and compete for heat transfer, Energy transfer/nutrients and finally a Parasitoids - Host niche overlap.

4.2. Future research

Future research should aim;

i) At developing an early warning systems that can detect changes in niche overlap and help predict potential ecological consequence. This will allow for the timely implementation of conservation measures.

ii) Developing Graph models of spatial and temporal dimensions of niche overlaps in ecosystems. This would help in understanding how niche overlap changes over time and across different spatial scales, and how it affects ecosystem structure and function.

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