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A NOVEL APPROACH FOR FINDING SHORTEST PATH IN Z-GRAPHS

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Abstract

In Graph Theory the problem of finding the shortest path from a specified source node to a destination node is an important combinatorial optimization problem. Fuzzy shortest path problem has also been well studied. Recently interest has arisen in graphs whose edges have Z-number weights. In this paper, we present a novel approach to the solution of shortest path problem (SPP) in Z-graph using Lexicographic ranking procedure of Z-numbers. Usually computations involving Z-numbers are very difficult and time consuming. However, by using the novel binary operations introduced by Stephen, that difficulty has been overcome.

1. Introduction

In Graph Theory the problem of finding the shortest path from a specified source node to a destination node is an important combinatorial optimization problem. A common algorithm used to solve the classical SPP is Dijkstra's algorithm.

While dealing with real life problems it is frequently found that the available data is not precise but vague and uncertain. In such situations, the notion of Fuzzy Numbers [13] is found useful. So Fuzzy versions of various optimisation problems have been considered by various scientists. The Fuzzy shortest path problem (FSPP) was first introduced by Dubois and Prade [1]. Elizabeth and Sujatha [10] developed FSPP by the length of the path in the

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network was trapezoidal fuzzy set where the highest similarity degree was utilized to identify the shortest path. Thus, numerous papers have been published on Fuzzy Shortest Path Problem.

Zadeh has extended the concept of fuzzy numbers and introduced the notion of Z-number in 2011 [14]. Basically, the concept of a Z-number relates to the issue of reliability of information. In 2017 Siddhartha Sankar Biswas [11] investigated Shortest Path Problem in Z-graph. In his article he introduced Z-Dijkstra's Algorithm to solve shortest path problem in Z-graph and to compare two Z-numbers he introduced the notions of "strongly greater than and weekly greater than".

In this paper, we present a Novel approach to the solution of SPP in Z-graph using Lexicographic ranking procedure [8] of Z-numbers.

2. Preliminaries

In many practical situations, we need to employ a binary operation. By Zadeh's procedure, if we do normal addition on Z-number, it is a difficult task. In this paper, we use the operations on Z-numbers, which was introduced by Stephen [9].

For comparing two Z-numbers, we propose to use Lexicographic ordering for Z-number, which was introduced by Parameswari [8].

Definition 1. Graph. A graph is a pair (V, X), where V is a non-empty set, whose elements are called vertices and X is a set of unordered pairs of distinct elements of V. The elements of X are called edges. An acyclic graph is a graph without cycle.

Definition 2. Fuzzy set. A fuzzy set A on a universal set X is a function $A : X \to [0, 1]$.

A is called the membership function, $A(x)$ is called the membership grade of x in A.

We write $A = \{(x, A(x)) : x \in X\}$

Definition 3. Fuzzy Number. A fuzzy number A is a normal and convex fuzzy subset of X.

Definition 4. Triangular Fuzzy Number. A fuzzy number $A = (a, b, c)$ is a triangular fuzzy number if its membership function is

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x - a}{b - a} & \text{if } a \le x \le b \\ \frac{x - c}{b - c} & \text{if } b \le x \le c \\ 0 & \text{otherwise.} \end{cases}$$

Definition 5. Z-number. A Z-number $Z = (A, B)$ is an ordered pair of fuzzy numbers. The first component A is a restriction of real valued uncertain variable X. The second component B is a measure of reliability of the first component. A Z-number not only gives information about the uncertain variable but also the reliability of the information.

Definition 6. Formal definition of Z-number. A Z-number is an ordered pair of fuzzy numbers (A, B) where A is a fuzzy set defined on the real line and B is a fuzzy number whose support is contained in $[0, 1]$.

Definition 7. Shortest Path in Graph. Let $G = (V, E)$ be a weighted graph, directed or undirected. Weight of path $P = \langle v_0, v_1, v_2, \ldots v_k \rangle$ is $w(P) = \sum w(v_{i-1}, v_i)$. Shortest path $\delta(u, v)$ from u to v has weight

$$\delta(u, v) = \begin{cases} \min \{w(P) : P \text{ is path from } u \text{ to } v\} & \text{if path exists} \\ \infty & \text{otherwise.} \end{cases}$$

Definition 8. Lexicographic Order for Z-numbers. Let R_1 and R_2 be any two ranking functions and let $Z_1 = (A_1, B_1), Z_2 = (A_2, B_2)$ be any two Z-numbers. Define $Z_1 \preccurlyeq Z_2$ under the Lexicographic order $L(R_1, R_2)$ if and only if

(i) $R_1(A_1) < R_1(A_2)$ or (ii) $R_1(A_1) = R_1(A_2)$ and $R_2(B_1) \ge R_2(B_2)$

Definition 9. Flipped Lexicographic Order for Z-numbers. Let R_1 and R_2 be two ranking functions. Let $Z_1 = (A_1, B_1), Z_2 = (A_2, B_2)$ be any two Z-numbers. Define $Z_1 \preccurlyeq Z_2$ under the Flipped Lexicographic order $FL(R_1, R_2)$ if and only if

(i) $R_1(B_1) > R_1(B_2)$ or (ii) $R_1(B_1) = R_1(B_2)$ and $R_2(A_1) \leq R_2(A_2)$.

Definition 10. Operation on fuzzy numbers. Let $A_1 = (a, b, c)$, $A_2 = (d, e, f)$ be two Triangular fuzzy numbers. Then $A_1 + A_2 = (a + d, b + e, c + f)$.

Definition 11. Binary operation on Z-numbers. Let (A_1, B_1) and (A_2, B_2) be two Z-numbers.

Then $(A_1, B_1)(+, \min)(A_2, B_2) = (A_1 + A_2, \min(B_1, B_2))$.

Definition 12. Z-weight of an edge. A Z-graph is a generalised concept of Fuzzy graph. In Z-graph, at least one of the weights is Z-number.

The Z-weight of an edge is the Z-distance between the corresponding two nodes.

Definition 13. Z-length of the path. Consider a path P from a vertex u to vertex v. Suppose it consists of edges $e_1, e_2, \ldots, e_n$. Suppose the weights of these edges are the Z-numbers $(A_1, B_1), (A_2, B_2) \ldots (A_n, B_n)$. The Z-length of the path P is denoted by $ZL(P)$

$$ZL(P) = (A_1, B_1)(+, \min)(A_2, B_2) \ldots (+, \min)(A_n, B_n).$$

The shortest path problem in Z-graphs is the problem of finding a path between two specified nodes such that the Z-length of the path is minimum.

Here our aim is to find the path between u and v whose Z-length is minimum.

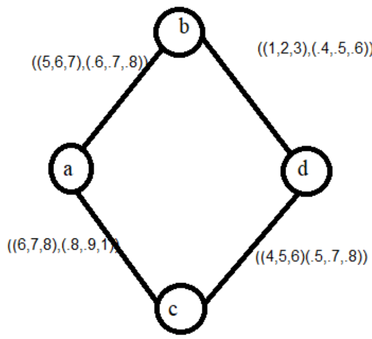
(Z- SP between u and v).

3. Adapting Dijkstra's Algorithm to Shortest path Problem in Z-Graphs

Dijkstra's Algorithm is an algorithm for finding the shortest path from one node to every other node within the same graph whose edges have Z-number weights. For finding the shortest path from a source node to the end node in a Z-graph having Z-weights we can use a modified form of Dijkstra's Algorithm.

To compare the Z-lengths of different paths, we need some procedure to rank or order Z-numbers, Lexicographic or flipped lexicographic method can be used.

Consider the following example.



There are two paths from a to d. Path P consists of edges ab and bd. Path Q consists of edges ac and cd.

Consider the ranking function $r(x, y, z) = \frac{x + y + z}{3}$

$ZL(P) = ((6, 8, 10), (.4, .5, .6))$ and $ZL(Q) = ((10, 12, 14), (.5, .7, .8))$

If we use $L(r_1, r_2)$ with $r_1 = r_2 = r$, we get $ZL(P) < ZL(Q)$.

On the other hand, if we use $FL(r_1, r_2)$ with $r_1 = r_2 = r$ we get $ZL(Q) < ZL(P)$.

The proposed algorithm finds the shortest path from a source node to a destination node of acyclic graph. So, the shortest path depends on what ranking method we use for comparing Z-numbers. Once we fix on the ranking procedure, we can apply Dijkstra's algorithm to solve the problem.

Modified Dijkstra's Algorithm for SPP in Z-Graphs.

We fix suitable ranking functions R_1, R_2. Comparing two Z-numbers is done based on (R_1, R_2). The source node is denoted by s. P is used to store all the nodes of the Z-graph G, which are currently unvisited.

Algorithm

Inputs:

V // $= \{1, 2, 3, \ldots, n\}$ the set of vertices in the given connected Z-graph

s // the source vertex to be taken as 1

Weight matrix $w(i, j) = (A(i, j), B(i, j))$ // $W(i, j)$ is the Z-no weight associated with the edge joining vertex i and vertex j

Initialisation:

- ❖ for every vertex i in V
- ❖ $\{Zd[i] = ((\infty, \infty, \infty), (1, 1, 1))$
- ❖ Predecessor $[i] = \varphi\}$
- ❖ $Zd[s] = ((0, 0, 0), (1, 1, 1))$
- ❖ $Zd \min = ((\infty, \infty, \infty), (1, 1, 1))$
- ❖ {

Main part:

- ❖ $P = V \setminus \{s\}$
- ❖ $N = s$
- ❖ While $P \neq \Phi$
- ❖ {
- ❖ for every vertex i in P
- ❖ {
- ❖ if $Zd[i] > Zd[N](+, \min) w(i, j)$
- ❖ {
- ❖ Predecessor $[i] = N$, $Zd[i] = Zd[N](+, \min) w(i, j)$
- ❖ }

❖ if $Zd[i] < Zd \min$

❖ next $= i$, $Zd \min = Zd[i]$

❖ }

❖ N = next

❖ Delete N from P

❖ $Zd \min = ((\infty, \infty, \infty), (1, 1, 1))$

❖ }

Output:

❖ $Zd[i]$ // Z length of SP from source to i

❖ Predecessor $[i]$ // SP from source to i is {SP from source to predecessor $[i]$} U {edge joining predecessor $[i]$ and $[j]$}

Example.

Here we have to find the shortest path from the source node a to destination node e.

Consider the graph G in figure 1 with Z-weighted edges. The weight of the edges are given in Table 1.

Table 1

Edge	Z-weight
ab	((2, 3, 4), (.7, .8, .9))
ad	((6, 7, 8), (.7, .8, .9))
bd	((1, 2, 3), (.6, .7, .8))
bc	((6, 7, 8), (.7, .8, .9))
dc	((4, 5, 6), (.5, .6, .9))
de	((1, 2, 3), (.6, .7, .8))
ce	((7, 8, 9), (.6, .7, .9))

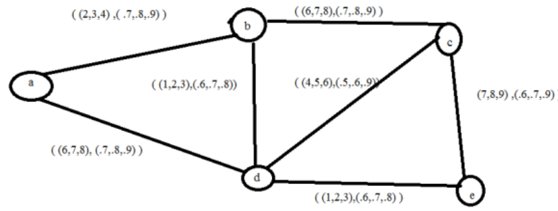


Figure 1.

- ❖ $V = \{a, b, c, d, e\}$
- ❖ Take $S = a$, $Zd[a] = ((0, 0, 0), (1, 1, 1))$

  $Zd \min = ((\infty, \infty, \infty), (1, 1, 1))$
- ❖ Then $P = \{b, c, d, e\}$ and $N = S = a$
- ❖ By applying algorithm, we get the output as

Output:

The shortest path is $a \to b \to d \to e$

- ❖ $Zd[a] = ((0, 0, 0), (1, 1, 1))$
- ❖ Predecessor $[b] = a$ and $Zd[b] = ((2, 3, 4), (.7, .8, .9))$
- ❖ SP from source to $b = \{\ \} U \{ab\} = \{ab\}$
- ❖ Predecessor $[d] = b$ and $Zd[d] = ((3, 5, 7), (.6, .7, .8))$
- ❖ SP from source to $d = SP$ from source to b $U\{bd\} = \{ab, bd\}$
- ❖ Predecessor $[c] = d$ and $Zd[c] = ((7, 10, 13), (.5, .6, .9))$
- ❖ SP from source to $c = SP$ from source to $d\, U\, \{dc\}$

  $$= \{ab, bd, dc\}$$
- ❖ Predecessor $[e] = d$ and $Zd[e] = ((4, 7, 10), (.6, .7, .8))$
- ❖ SP from source to $e = SP$ from source to $d\, U\, \{de\}$

  $$= \{ab, bd, de\}$$

Hence the shortest path from source "a" to destination "e" is $a \to b \to d \to e$.

The length of this path is $((4, 7, 10), (.6, .7, .8))$.

So, it is "sure" that the shortest path length is "around 7".

4. Conclusion

In this paper, we have developed a modified form of Dijkstra's algorithm to find the shortest path in a Z-graph. Usually computations involving Z-numbers are very difficult and time consuming. However, by using the novel binary operations introduced by Stephen, that difficulty has been overcome. Hence the resulting algorithm can be easily applied to real life problems in an efficient manner.

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