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COMPARISON OF THE PERFORMANCE OF THE ZERO SUFFIX AND MODIFIED ZERO SUFFIX METHODS IN TRANSPORTATION PROBLEMS

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Abstract

Sudhakar, V. J. et al. (2012) developed a new direct method for finding the optimal solutions of Transportation Problems (TPs) using Zero Suffix Method (ZSM), in which the suffix value of a 0-entry cell is obtained by computing the average of the adjacent costs of that 0-entry in its row and column of a reduced cost matrix (RCM). By observing a very few drawbacks in the ZSM, Shambhu Sharma et al. (2013) introduced a Modified Zero Suffix Method (MZSM) for finding the optimal solutions of TPs by computing the suffix value of a 0-entry cell in the RCM as the average of the difference of the lowest and the next lowest cost value in its row and the difference of the lowest and the next lowest cost value in its column. Akilbasha A. et al. (2015) introduced another new method called Modified Zero Suffix Method (MZSM) for finding the optimal solutions for TPs in a single stage, in which the suffix value of a 0-entry cell in the allotment table is obtained by adding all the costs (which is greater than zero) of adjacent rows

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and columns corresponding to the 0-entry cell and then dividing the result by the number of costs added. In this paper, we have studied in detail and compared the performance of each of the three methods on some challenging TPs. Experimental results validate that no one method is the direct method to find the optimal solutions directly to TPs. Each of the methods is just a method to find only the Initial Basic Feasible Solution (IBFS) of TPs.

1. Introduction

The Transportation Problem (TP) is a particular division of Linear Programming Problem (LPP) in which the main objective is to find the shipping program that minimizes the overall cost of shipping commodities from various sources to various destinations. Solving a given TP completely involves two stages. In the first stage, an initial basic feasible solution (IBFS) is obtained for the given TP. The obtained IBFS may or may not be optimal. The optimality of the obtained solution is tested and improved towards optimal, if it is not optimal, in the second stage. For finding the IBFS of a TP, there are several numbers of methods available today in the literature. However, for testing the optimality of an obtained solution and also optimizing it, if it is not optimal, the following three methods only are available:

1. Stepping Stone Method (SSM) due to Charnes A. and Cooper W.W. (1954) [2]

2. Modified Distribution Method (MODI Method) due to Dantzig G.B. (1963) [3]

3. Modified Allocation Method (MODA Method) due to Murugesan R. (2023) [4]

At present day, most of the authors claim that their proposed methods are for finding the optimal solutions of TPs directly just by testing their algorithms only on very simple problems. Very few authors like us only test the algorithms on some challenging and more challenging problems and find that their proposed methods are not direct. In this line, during 2012, Sudhakar et al. [6] developed a direct method for finding an optimal solution of a TP using Zero Suffix Method (ZSM). In this method, first a Reduced Cost Matrix (RCM) is generated by applying the Row Minimum Subtraction (RMS) operation followed by the Column Minimum Subtraction (CMS) operation on the cost matrix of the given TP. Each row and each column of the obtained

RCM will contain at least one 0-entry cell. The suffix value of a 0-entry is obtained by taking the average of the adjacent costs of that 0-entry in its row and column. Then the allocation is made at the 0-entry cell having the largest suffix value and by deleting the exhausted row or the satisfied column, the further reduced table is obtained. This procedure is repeated till all the rim conditions are satisfied. By observing the following two drawbacks of the ZSM due to Sudhakar et al.:

1. It is does not generate optimal solution all the time

2. It does not provide any tie breaking technique to break the tie among certain 0-entry cells having the same largest suffix value.

Shambhu Sharma and et al. [5] suggested a method called Modified Zero Suffix Method (MZSM) for finding an optimal solution of TPs. According to the authors, the optimal solution obtained by this method is better than or equal to that of by the ZSM of Sudhakar et al. [6] and is also free from the problem of degeneracy and requires least numbers of iterations to reach optimality than the ZSM. Without citing the ZSM and the MZSM, in 2015, Akilbasha et al. [1] introduced a new method (with the same name) called the Modified Zero Suffix Method (MZSM) for finding an optimal solution for TPs in single stage. In the article [1], the authors have given a conflicted statement that their method has much easier heuristic approach for finding the optimal solution directly with lesser numbers of iterations and very easy computations. In their algorithm, we have identified one drawback in Step 4. This step says, Draw the minimum number of horizontal lines and vertical lines to cover all the zeros of the reduced transportation table such that some entries of row(s) or/and column(s) which do not satisfy the condition of the Step 3 are not covered. They have not given the way of how to draw the minimum number of horizontal and vertical lines to cover all the zeros of the reduced table. This drawback has been rectified by us by introducing ME Rules to cover all the zeros of the reduced table by drawing minimum number of horizontal and vertical lines.

The paper is organized as follows: Section 1 briefs the introduction. Section 2 presents the ME rules for covering all the 0-entries by drawing minimum numbers of horizontal and vertical lines. Section 3 presents the ESAN tie breaking technique for selecting an appropriate 0-entry cell for allocation. Section 4 presents the algorithms of the focused three existing methods and the MODA method for optimality testing of a solution. Section 5 lists a set of 7 "More Challenging" TPs. for testing the algorithms. Section 6 discusses and compares the results obtained. Section 7 draws the conclusion.

2. ME Rules for Covering all 0s by Lines

(The word ME is coined from the first letter of the names of the authors Murugesan and Esakkiammal)

In this section, we present the rules of how to draw the minimum number of horizontal and vertical lines to cover all the 0s of a resultant matrix. In Step 4 of the Modified Zero Suffix Method by Akilbasha et al. [1], to cover all the 0-entries by using minimum number of horizontal and vertical lines, the following rules may also be used:

Rule 1. To draw Minimum number of Lines to cover all 0s

(a) Row-wise Marking

(i) Look at the rows successively from first to last until a row with exactly one 0-entry is found.

(ii) Make a mark to this single 0-entry by making a circle around it.

(iii) Draw a vertical line passing through that 0-entry.

(iv) Continue in this way until all the rows have been scrutinized.

(v) After scrutinizing the last row, check whether all the 0s are covered with lines. If yes, go to Rule (2); otherwise, do Column-wise Marking.

(b) Column-wise Marking

(i) Look at the columns successively from first to last until a column with exactly one unmarked 0-entry is found.

(ii) Make an marking to this single 0-entry by placing a circle around it.

(iii) Draw a horizontal line passing through that 0-entry.

(iv) Continue in this way until all the columns have been scrutinized.

(v) After scrutinizing the last column, check whether all the 0s are covered with lines. Go to Rule (2).

Rule 2. To test the optimality condition for Complete Marking

Test whether the Minimum number of lines drawn is equal to k, where $k = Min\{m, n\}$. If yes, compute the suffix value of each of the 0-entry cells or write the Allocation Matrix based on the considered algorithm; otherwise, select the smallest element (say d_{ij}) out of those which do not lie on any of the lines in the above matrix. Then subtract by d_{ij} each element of the uncovered rows or columns, which d_{ij} lies on it. This operation creates some new 0-entries to this row or column. Then, go to Rule 1.

If the optimality condition for complete marking is not achieved through the above two rues, then apply rule 3.

Rule 3.

After performing row-wise marking and column-wise marking completely, if more than one 0-entry is present in certain rows and columns, then

(i) Select any one 0-entry arbitrarily and make a marking to that 0-entry by creating a circle around it.

(ii) Draw a line through the row of the marked 0-entry and put an X mark on all the remaining 0s on the column of that marked 0-entry. (Or) Draw a line through the column of the marked 0-entry and put an X mark on all the remaining 0s on the row of that marked 0-entry.

(iii) Repeat (i) and (ii) until the optimality condition for complete marking is reached.

Also, we have seen that the one drawback in the ZSM due to Sudhakar et al. [6] indicated by Shambhu Sharma et al. [5] is that the ZSM does not provide any tie breaking technique to break the tie among certain 0-entry cells having the same largest suffix value. The same situation arises in both of the MZSMs. In order to break the tie, we have developed a technique called ESAN, which is given below:

3. ESAN Tie Breaking Technique for selecting an exact 0-entry cell

In this section, a detailed sequence of rules to break the tie when tie occurs among certain 0-entry cells, for identifying an opt 0-entry cell for the allocation has been presented. The name ESAN has been coined from the last four letters of the name of the second author Murugesan R.

(a) List each of the 0-entry cells present in the RCM (or in the Allocation Table) row-wise along with their original Unit Transportation Costs (UTCs).

(b) Choose the 0-entry cell having the minimum UTC for the allocation. While choosing the 0-entry cell for allocation, give priority to the non-dummy cell in case of solving an unbalanced TP.

(c) If tie occurs in case of (b), for each such cell, count the total number of 0-entry cells occurred (excluding the considered one) in its row and column. Now, choose the 0-entry cell for allocation for which the number of 0-entries counted is the minimum.

(d) If tie occurs in case of (c), then choose the 0-entry cell for allocation for which the total sum of all the values in the corresponding row and column is the maximum.

(e) Again, if tie occurs in case of (d), then choose the 0-entry cell for allocation for which maximum quantity of allocation can be made.

(f) Yet again, if tie occurs in case of (e), then choose the cell for allocation for which the sum of the demand and supply quantities in the original transportation table is the maximum.

(g) Over again, if tie occurs in case of (f), then choose the cell for allocation for which the i-value (row number) is less in case of occurrence of tie in the same column [or the j-value (column number) is less in case of occurrence of tie in the same row].

4. Algorithms of the Focused Existing Zero Suffix Method and Modified Zero Suffix Methods

In this section, the individual algorithm of the focused ZSM and MZSMs followed by the MODA method for optimality testing and optimizing a solution, if it is not optimal have described one by one.

4.1 Algorithm of the Zero Suffix Method by Sudhakar et al. [6]

According to the authors it is a method for finding the optimal solution of the TPs. The sequence of steps to solve the given TP by using this method is described as given below.

Step 1. Construct the transportation table.

Step 2. Subtract each row entries of the transportation table from the minimum row. Then, subtract each column entries of the transportation table on a certain minimum column.

Step 3. In the matrix cost, there is an entry with zero value in each row and column, then find the suffix value denoted by S. $S =$ (Add the cost of nearest adjacent sides of zero)/(Additional Cost).

Step 4. Choose the maximum of S if it has one of the maximum value. If it has two or more of the same value, then choose just one, the cost becomes the allocation of goods by regarding to the demand and supply.

Step 5. After Step 4, select minimum of $\{a_i, b_j\}$, then allocate it into transportation table. The resulting table should have at least one cost of 0 in each row and column, otherwise repeat Step 2.

Step 6. Repeat step 3 to step 5 until the optimal solution is obtained. The optimal cost is obtained if the column or row is saturated (suffix values = 0).

4.2 Algorithm of the Modified Zero Suffix Method by Shambhu Sharma et al. [5]

The sequence of steps of the modified zero suffix method for finding an optimal solution of TP is described below.

Step 1. Construct the transportation table for the given transportation problem.

Step 2. Subtract the least element of each row from all the elements of the corresponding row.

Step 3. Subtract the least element of each column from all the elements of the corresponding column of the transportation table obtained in step 2.

Step 4. Find the suffix value, S of each zero in the reduced cost transportation table as follows: $S =$ Average of the difference of the smallest and the next smallest elements of its row and the smallest and the next smallest elements of its column.

Step 5. Search the greatest suffix value of a zero and allocate to the corresponding cell and delete the exhausted row/column to get the reduced transportation table. Go to Step 2.

Step 6. Repeat steps 2 to 5 until all the demands and supplies are exhausted.

4.3 Algorithm for the Modified Zero Suffix Method by Akilbasha et al. [1]

The algorithm of the modified zero suffix method for finding an optimal solution to a TP proceeds as follows.

Step 1. Construct the transportation table for the given TP.

Step 2. Subtract each row entries of the transportation table from the row minimum and then subtract each column entries of the resulting transportation table after using the Step 1 from the column minimum.

Step 3. Check if each column demand is less than to the sum of the supplies whose reduced costs in that column are zero. Also, check if each row supply is less than to sum of the column demands whose reduced costs in that row are zero. If so, go to Step 7. (Such reduced table is called the allotment table). If not, go to Step 4.

Step 4. Draw the minimum number of horizontal lines and vertical lines to cover all the zeros of the reduced transportation table such that some entries of row(s) or / and column(s) which do not satisfy the condition of the Step3 are not covered.

Step 5. Develop the new, revised and reduced transportation table as follows: (i) Find the smallest entry of the reduced cost matrix not covered by any lines. (ii) Subtract this entry from all the uncovered entries and add the same to all entries lying at the intersection of any two lines and then, go to Step 3.

Step 6. Repeat the steps 3 to 5 till the step 3 is satisfied.

Step 7. In the reduced cost matrix there will be at least one zero in each row and column, then find the suffix value of all the zeros in the reduced cost matrix by following simplification, the suffix value is denoted by S. Therefore $S =$ {Add all the costs (which is greater than zero) of adjacent rows and columns corresponding to each zeros/No. of costs added} Do allocation according to the following rules:

(a) Choose the maximum of S, if it has one maximum value then first supply to that demand corresponding to the cell. If it has more equal values then select $\{a_i, b_j\}$ and supply to that demand maximum possible.

(b) Then allot the next maximum value of S and continuing the process till all the supply and demands are fully supplied and fully received.

4.4 Algorithm for the Existing 'Revised Version of the MODA' Method by Murugesan R. [4]

The term MODA has been coined from the first three letters of the word "Modified" and the first one letter of the word "Allocation". MODA is an iterative method which can be used for testing the optimality of an initial basic feasible solution (IBFS) and also optimize the IBFS, if it is not optimal, for transportation problems. The innovative way of improving a non-optimal solution to an optimal solution by the revised version of the MODA method is based on redistributing the allocation available at a currently allocated cell (basic cell) with largest "unit transportation cost" (UTC) to another un-allocated cell (non-basic cell) and its subsequent induced reallocations. The algorithm of the existing revised version of the MODA method consists of two stages. In Stage #1, an IBFS is obtained to the given TP. In Stage #2, optimality testing of the obtained IBFS and also optimizing it, if it is not optimal, is carried out.

The following notations and abbreviations are used in the development of the algorithm of the revised version of the MODA method:

$m \times n -$ Size of the unit cost matrix of the given TP

TT -Transportation table

BTP-Balanced transportation problem

UTC -Unit transportation cost

C_{ij} -UTC available at the cell (i, j)

$IBFS$ -Initial basic feasible solution

$X = [x_{ij}]$-A solution

X^* -An optimal solution

TTC -Total transportation cost

$Z(X) - TTC$

$Z(X^*)$-Minimum TTC

NBC -Non-Basic Cell

IBC -Identified Basic Cell

NCC -Net Cost Change

SI -Solution Improvement

Stage #1. Obtain an IBFS to the given TP

For the given TP, first obtain an IBFS say $X^{(0)}$ with its associated total transportation cost $Z(X^{(0)})$ using any available method in TPs. We use the I-SOFT method [7] to obtain an IBFS because at present day it has been identified and established as the best method to find the best IBFS to TPs.

Stage #2. Test he optimality of the obtained IBFS

Step 1. Construct the current solution table

Consider the transportation table (TT) en-squared with the obtained allocations (IBFS) $X^{(0)} = [x_{ij}]$ as the current solution table. Also, compute the corresponding TTC $Z(X^{(0)})$.

Step 2. Ensure the non-degeneracy condition

Ensure the numbers of basic cells in the TT exactly equal to $(m + n - 1)$.

Step 3. Perform the Optimality Test on the IBFS $X^{(0)}$

(a) Determine $C(X^{(0)}) = Max\{c_{ij} : x_{ij} > 0\}$ and the corresponding basic cell as the identified basic cell (IBC). Let it be (h, k).

(i) If the IBC is unique, then go to Step (b) directly.

(ii) If there is two or more basic cells having the same largest UTC $C(X^{(0)})$, then select the basic cell having the maximum quantity of allocation as the IBC. Let it be (h, k) and go to Step (b).

(iii) If there is two or more basic cells having the same largest UTC $C(X^{(0)})$ and with the same maximum allocation quantity, then select any one such basic cell as the IBC. Let it be (h, k) and go to Step (b).

(b) Trace a Solution Improvement (SI) loop starting and ending at the Identified Basic Cell (IBC) (h, k) and passing through a non-basic cell. As it is a SI loop, it will have the Net Cost Change (NCC) value as non-positive (≤ 0). If there is a tie between two or more than two SI loops with the same NCC value, then select any one loop. Such a situation may generate alternative solutions to the given TP. If the NCC value of the SI loop is zero, then this will also indicate the existence of an alternative solution to the given TP.

(c) Implement this loop and obtain the better BFS, say $X^{(1)}$ with its associated TTC $Z(X^{(1)})$.

(d) If it is not possible to trace a SI loop starting and ending at the current IBC, then consider the next basic cell having UTC next to $C(X^{(0)})$ as the new IBC (h, k) and go to Step $(a^{(i)})$.

Step 4. Repeat Steps 3(a) to (d) until no SI loop can be traced starting and ending at the new IBC with the current largest UTC. At this level, the solution under optimality test is the optimal one. Write the optimal solution X^* with its minimum TTC as $Z(X^*)$.

Alternative Optimal Solution

At the 'optimal level', if the NCC value of the SI loop is zero, then this indicates that the given TP has an alternative optimal solution. By implementing this loop we can get the alternative optimal solution to the given TP.

Important Note

1. One cannot restrict a SI loop with corner cells having UTCs less than or equal to the UTC of the identified basic cell.

2. One cannot restrict the place (even position or odd position) of the non-basic cell in a SI loop.

5. Numerical Examples

In order to evaluate the efficiency of the focused three methods, we have solved a set of seven numbers of "more challenging" TPs of balanced category in different small sizes, from various literatures and textbooks, which are listed in Table 1.

Table 1. A set of some "More Challenging" balanced TPs

Problem No.
Problem 1 $[C_{ij}]3 \times 3 = [6\,10\,14; 12\,19\,21; 15\,14\,17]$, $[S_i]3 \times 1 = [50,\ 50,\ 50]$, $[D_j]1 \times 3 = [30,\ 40,\ 55]$
Problem 2 $[C_{ij}]3 \times 3 = [4\ 8\ 8; 16\ 24\ 16; 8\ 16\ 24]$, $[S_i]3 \times 1 = [76,\ 82,\ 77]$, $[D_j]1 \times 3 = [72,\ 102,\ 41]$
Problem 3 $[C_{ij}]3 \times 3 = [19\ 30\ 50\ 10; 70\ 30\ 40\ 60; 40\ 8\ 70\ 20]$, $[S_i]3 \times 1$ $= [7,\ 9,\ 18]$, $[D_j]1 \times 4 = [40,\ 8,\ 7,\ 14]$
Problem 4 $[C_{ij}]3 \times 4 = [10\ 15\ 12\ 12; 8\ 10\ 11\ 9; 11\ 12\ 13\ 10]$, $[S_i]3 \times 1$ $= [20,\ 15,\ 12]$, $[D_j]1 \times 4 = [14,\ 12,\ 8,\ 22]$
Problem 5 $[C_{ij}]3 \times 4 = [42\ 48\ 38\ 37; 40\ 49\ 52\ 51; 39\ 38\ 40\ 43]$, $[S_i]3 \times 1$ $= [160,\ 150,\ 190]$, $[D_j]1 \times 4 = [80,\ 90,\ 110,\ 160]$

Problem 6 $[C_{ij}]3 \times 4 = [3\ 48\ 14\ 2; 4\ 230\ 10; 36\ 8\ 12\ 12]$, $[S_i]3 \times 1 = [24, 24, 2]$, $[D_j]1 \times 4 = [6, 12, 3, 44]$
Problem 7 $[C_{ij}]4 \times 3 = [2\ 7\ 14; 3\ 3\ 1; 5\ 4\ 7; 1\ 6\ 2]$, $[S_i]4 \times 1 = [5, 8, 7, 15]$, $[D_j]1 \times 3 = [7, 9, 18]$

6. Results and Discussion

For evaluating the performance of the focused three methods, simulation experiments were carried out on certain challenging balanced TPs shown in Table 1. The main purpose of the experiment was to evaluate the effectiveness of the solutions generated by the three focused methods by comparing them with the optimal solutions. The assessment of the results for the 7 problems is shown in following Table 2.

Table 2. Comparison of results obtained by the focused Zero Suffix Methods on TPs.

Problem No. #	ZSM by Sudhakar et al. [6]	MZSM by Shambhu S. et al. [5]	MZSM by Akilbasha et al. [1]	Optimal Solution by MODA method [4]
1.	1250	1160	1320	410
2.	183	187	204	183
3.	2850	2810	2980	2700
4.	318	316	382	316
5.	450	450	510	430
6.	1113	1102	1126	1102
7.	1775	1620	1625	1475

From Table 2, we discover that the ZSM by Sudhakar et al. [6] has produced optimal solution directly to 1 TP only, the MZSM by Shambhu Sharma et al. [5] has produced optimal solution directly to 3 TPs and the MZSM by Akilbasha et al. [1] has produced optimal solution to nothing. The remaining are the near optimal solutions generated by the focused methods. For each of the solutions obtained, the optimality testing and optimizing it has been done by applying the MODA method.

7. Conclusion

In this paper, we have evaluated and compared the performance of the optimal solution procedures namely Zero Suffix Method developed by Sudhakar, V. J. et al. [6], Modified Zero Suffix Method introduced by Shambhu Sharma et al. [5] and another new Modified Zero Suffix Method proposed by Akilbasha et al. [1] for finding the optimal solutions of transportation problems. To verify the performance of the said procedures, 7 classical benchmark instances from the literature have been tested. Simulation results authenticate that no one of the said methods has produced optimal solutions directly to all the seven problems. Therefore, it is established and recognized that the said methods are just the methods for finding only the IBFS to TPs. The authors of the methods should not claim that their methods are for finding the optimal solution for TPs.

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