

IMPACT OF ACTIVATION ENERGY AND VARIABLE VISCOSITY IN MAGNETO PERISTALSIS OF JEFFREY FLUID THROUGH AN ASYMMETRIC CHANNEL

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Abstract

The impact of activation energy with variable viscosity on the magneto peristalsis of Jeffrey fluid through an asymmetric channel is studied in the present work. Also incorporated the consequences of heat, mass transfer effects. In concentration equation activation energy parameter is introduced with variable viscosity. The governing equations are converted to nondimensional form by utilising appropriate dimensionless parameters. By using HPM nonlinear pde are resolved. The impact of various physical factors on velocity, temperature and concentration been analysed by using graphs. The significant outcome of the study is that velocity and temperature rises with viscosity parameter. Further, temperature rises, velocity dropped with magnetic field strength. Also it is seen that chemical reactive parameter and activation energy shows a direct relationship to concentration. Therefore, this work is pertinent to both exothermic and endothermic activities, geothermal, biological and chemical engineering. Additionally, study has applications in blood rheology and cardiovascular physiology.

1. Introduction

The majority of chemical reactions fall into one of the two categories: homogeneous and heterogeneous. There are several applications for chemical processes that involve both kinds. Such as distillation, biological systems, combustion, catalysis, and hydro metallurgical devices. Activation energy is the smallest quantity of energy needed to start a chemical reaction in a system. Svante Arrenius, a Swedish physicist, coined the phrase "activation

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energy" in 1889. Activation energy contributes significantly to binary chemical reactions when combined with heat and mass transfer, especially the development of binary chemically reactive systems with finite activation energy in oil reservoirs, chemical engineering, material degradation, geothermal engineering, oil and water emulsion [25]. Hayat et al. [27] investigated the magneto peristaltic behaviour of Jeffrey material, which exhibits activation energy and non-Darcy number. Salahuddin et al. [51] examined, how mixed convection affects the flow rate and notable impact of high values of activation energy. Rafiq et al. [50], Asjad et al. [9] noticed a reduction in chemical reaction for higher activation energy values after analyzing how different features and activation energy influenced peristaltic flow. Khan et al. [36] investigations reveals that species concentration enhances for higher estimation of activation energy. Bestman [12], Faiz et al. [10] investigated heat and mass transfer application with binary chemical reaction and activation energy. Salamuddin et al. [52] investigated the effect of enthalpy and activation energy on the thermo-physical characteristics of a thirdgrade fluid close to a magnetically induced radioactive surface. Ijaj Khan et al. [32], Hayat et al. [29] investigated impact of activation energy and entropy generation non-Newtonian fluid. Recently Aljaloud et al. [5] investigated cross nanofluid bio-convection flow is caused by a cylinder with activation energy. Anjum et al. [7] studied activation energy aspects on 3D Cross nanofluid flow with gyrotactic microorganisms. Chu et al. [21] worked on activation energy and chemical reactions for the enhancement of thermal energy and solute particles using hybrid nanoparticles. The references listed below will aid to our knowledge of activation energy [3], [33], [37], [38], [34].

In many investigations of fluid motion explored thermo-physical quantities as a constant factors in their mathematical model. But these quantities of fluid is changing with various factors like pressure, temperature and space coordinates etc. Shateyi and Motsa [54], Afsar Khan et al. [2] studied the peristaltic flow of Jeffrey fluid of varied viscosity with space coordinate y. Ajaykumar and Srinivasa [4] due to the major applications in manufacturing process, metal extrusion, and heat transfer across a stretching sheet with temperature dependent viscosity, they explored unsteady laminar boundary flow and transfer of heat. Farooq et al. [24] conducted a comparison of fluids with constant and varied viscosity. Further contrary findings were

found for temperature and concentration in case of variable and constant viscosity. Khan et al. [35] performed a study of nanoparticles submerged in a viscoelastic fluid using numerical methods and given the assumption of zero normal flux. Hayat et al. [28] analysed many aspects of different nanoparticles in peristalsis with entropy generation with variable viscosity. Recently Salamuddin et al. [53] investigated many aspects of the mixed convention, variable viscosity, and activation energy of the three-dimensional shear thinning model. Again Hussein et al. [31], Anjum et al. [7], Rafic et al. [50] investigated activation energy impact with variable properties and its biological applications. The references below will aid in our knowledge of varying viscosity [6], [55], [45]. It is note that the modification of viscosity caused by changes in temperature and y co-ordinate are high significant. Variable viscosity must be considered in order to accurately anticipate the flow behaviour.

In the field of industrial and physical science, the magnetic field has a wide range of uses, including the extrusion of polymer fluids, the petroleum and MHD generators etc. When there is a high magnetic field in MHD flows, Hall and ion slip currents have extraordinary effects. If magnetic field is moderate or low, we can neglect Hall and inertial effect [41]. The effects of Hall and ion slip currents with heat transfer include Hall accelerators, power generators, MHD accelerators, refrigeration coils and electric transformers. Understanding magnetic resonance angiography (MRA), which generates pictures of arteries to assess them for stenosis, and the influence of a magnetic field and the Hall currents on the blood flow through an artery is beneficial [15]. Due to its importance in numerous sectors of medical sciences and bio-engineering, magneto hydrodynamic (MHD) peristaltic flow has attracted substantial interest in recent years because magneto hydrodynamic properties are important in the creation of magnetic devices such as Nuclear Magnetic Resonance (NMR) and Magnetic Resonance Imaging (MRI) [47]. Magneto hydrodynamics (MHD) is a branch of fluid mechanics that studies fluid behavior in the presence of a magnetic field. Krishna et al. [42] worked on Hall and ion slip effects on unsteady MHD convective rotating flow of nanofluids. We can notice the importance of the Hall effect of MHD fluid by the literature [43].

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Heat transfer analysis in two-dimensional flows has attracted the interest of various researchers recently due to its applications. Misra et al. [46] inspired by the peristalsis phenomenon, which is seen in the flow of various bodily fluids that are necessary for life. He examined heat and mass transmission while the fluid flowing. Srinivas and Kothandapani [56] examined how peristaltic movement is influenced by heat and mass transport in a permeable environment with flexible walls. The connection between peristalsis and heat transmission has also piqued some researcher interest since it may be crucial for processes like hemodialysis and oxygenation. Further investigations on heat transfer [58], [17], [39].

Peristaltic pumping is the process of moving fluid by a wave of expansion and contraction from a low-pressure area to a high-pressure area. The peristaltic transport mechanism has been employed in physiological research and industry for things like moving chyme in the gastrointestinal system and transporting blood pumps for heart-lung machines etc. The peristaltic pump idea was initially introduced by Latham [44]. Applications of this technique may be found in a many areas like food intake, urinary bladder etc. With these biological applications many academics are drawn to research towards this area. Nadeem et al. [48] examined the impact of peristaltic flow via an asymmetric channel. Kothandapani and Srinivas [40] studied the peristaltic transport of Jeffrey fluid in an asymmetric channel with the effect of magnetic field. The articles [1], [26], [8], [59] supports to understand the peristaltic motion.

A porous medium contains a lot of tiny holes all over it. There are many examples of fluid flow through porous medium in nature, including water seepage in riverbeds, oil and water movement below the surface, and fluid filtration. Peristaltic flow across porous media is common in many biological processes, such as the passage of urine through the uterus with stones, human lungs, and animal vascular systems. Chamkha [14] have considered Non-Darcy hydro-magnetic free convection flow over a porous medium. We get concluding remarks of porous media effect on fluid velocity by the literature [16], [20].

The wave pattern of peristaltic on the walls is designed to have varied amplitudes and phases to achieve the channel asymmetry. The amplitude ratio has the greatest impact on flow rate when compared to the other factors.

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It is a characteristic that many biological systems have by nature. In living systems food moving via the esophagus, a vertical asymmetric channel is suitable to study peristaltic applications as the bulk of human physiological systems.

From the previous investigations, it has been analysed that the impact of activation energy and variable viscosity combined effect have not discussed through vertical asymmetric channel. This is inspired us to choose vertical asymmetric channel geometry. The authors have therefore begun their efforts to explain the influence of activation energy with varying viscosity of Jeffery fluid through vertical asymmetric channel with porous material. The following articles stand to understand the geometry of a vertical channel [18], [19].

The main objective of this work is to provide a mathematical model for Jeffrey fluid that accounts for both peristaltic flow and activation energy across a porous walled conduit with variable properties. Introduction is found in Section 1 of this article. The Jeffrey model with a transverse magnetic field, variable viscosity and activation energy via porous media is mathematically expressed in Section 2 of the article. Non-dimensional equations are solved in section 3 using the semi analytical technique (HPM). Using graphs, the impacts of physical parameters on temperature, velocity and concentration profiles are explored in section 4. Section 5 includes a summary of the results.

2. Problem Formulation

Travelling waves with differing phase and amplitude propagate along the channel walls, causing asymmetry in the flow. The Jeffrey fluid model may accurately depict the viscoelastic properties of blood. The fluid fluxes are also significantly impacted by several chemical processes occurring inside human physiological systems, notably during blood flow. Vertical asymmetric channel is more appropriate for this mathematical model of diverse physiological systems that are more realistic by taking into account these impacts.



Figure 1. Geometry of physical problem.

 H'_1 and H'_2 are the left and right wall of the channel spacing $d_1 + d_2$. The flow of an in-compressible Jeffrey fluid propagation along X' axis and Y' is perpendicular to the flow. The magnetic field B_0 is applied perpendicular to the direction of the flow. a_1 and b_1 are different amplitudes of the wave and the phase difference Φ shows the asymmetry of the channel. The flow starts when the peristaltic waves phase changed. The wave moves along the channel walls at constant speed c to initiate the peristaltic flow. Let T_0 , T_1 and C_0 , C_1 be temperature and concentration of the left and right side walls respectively.

The geometry of the wall surface is given by the equations

$$Y' = H'_{1} = -d_{2} - b_{1} \cos\left[\frac{2\pi}{\lambda}(X' - ct) + \Phi\right],$$
(1)

$$Y' = H'_{2} = d_{2} - a_{1} \cos\left[\frac{2\pi}{\lambda}(X' - ct)\right],$$
(2)

Here a_1 and b_1 are wave amplitudes, λ is wavelength, velocity of prorogation is c, time t and $0 \le \Phi \le \pi$. Here $\Phi = 0$ and $\Phi = \pi$ means waves are out and are in phase respectively. a_1 , b_1 , d_1 , d_2 and Φ are related by this equation [49]

$$a_1^2 + b_1^2 + 2a_1b_1\cos\Phi \le (d_1 + d_2)^2,$$
(3)

For fluid flow the state equations [27]

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$$\frac{\partial U'}{\partial X'} + \frac{\partial V'}{\partial Y'} = 0, \tag{4}$$

$$\begin{split} \rho \bigg(\frac{\partial U'}{\partial t'} + U' \frac{\partial U'}{\partial X'} + V' \frac{\partial U'}{\partial Y'} \bigg) &= -\frac{\partial p'}{\partial X'} + \frac{\partial S'_{XX}}{\partial X'} + \frac{\partial S'_{YX}}{\partial Y'} - \frac{\sigma B_0^2}{1 + m^2} (U' - mV') \\ \rho g \beta_1 (T' - T_0) \sin \alpha_0 + \rho g \beta_2 (C' - C_0) \sin \alpha_0 - \frac{\mu'(y')}{k_1(1 + \lambda_1)} \bigg[U' + \lambda_2 \frac{dU'}{dt'} \bigg], \quad (5) \\ \rho \bigg(\frac{\partial V'}{\partial t'} + U' \frac{\partial V'}{\partial X'} + V' \frac{\partial V'}{\partial Y'} \bigg) &= -\frac{\partial p'}{\partial Y'} + \frac{\partial S'_{YY}}{\partial Y'} + \frac{\partial S'_{XY}}{\partial X'} - \frac{\sigma B_0^2}{1 + m^2} (V' - mU') \\ \rho g \beta_1 (T' - T_0) \cos \alpha_0 + \rho g \beta_2 (C' - C_0) \cos \alpha_0 - \frac{\mu'(y')}{k_1(1 + \lambda_1)} \bigg[V' + \lambda_2 \frac{dV'}{dt'} \bigg], \quad (6) \\ \rho C_p \bigg(\frac{\partial T'}{\partial t'} + U' \frac{\partial T'}{\partial X'} + V' \frac{\partial T'}{\partial Y'} \bigg) &= \kappa \bigg(\frac{\partial^2 T'}{\partial X'^2} + \frac{\partial^2 T'}{\partial Y'^2} \bigg) + 2 \bigg\{ \frac{\partial U'}{\partial Y'} S'_{XX} + \frac{\partial V'}{\partial Y'} S'_{YY} \\ &+ \bigg(\frac{\partial U'}{\partial Y'} + \frac{\partial V'}{\partial X'} \bigg) S'_{XY} \bigg\} + \frac{D_B k_T}{C_s} \bigg(\frac{\partial^2 C'}{\partial X'^2} + \frac{\partial^2 C'}{\partial Y'^2} \bigg) + \frac{\sigma B_0}{1 + m^2} (U'^2 + V'^2), \quad (7) \\ \bigg(\frac{\partial C'}{\partial t'} + U' \frac{\partial C'}{\partial X'} + V' \frac{\partial C'}{\partial Y'} \bigg) &= D_B \bigg(\frac{\partial^2 C'}{\partial X'^2} + \frac{\partial^2 C'}{\partial Y'^2} \bigg) + \frac{D_B k_T}{C_s} \bigg(\frac{\partial^2 T'}{\partial X'^2} + \frac{\partial^2 T'}{\partial Y'^2} \bigg) \\ &- k_T^2 (C' - C_0) \bigg(\frac{T'}{T_0} \bigg)^n \exp\bigg(- \frac{E_a}{k_T} \bigg). \quad (8) \end{split}$$

Fixed and wave frame can be related through the relation [27]

$$X' = x' + ct, \ Y' = y', \ U' = u' + c, \ V' = v',$$
(9)

For Jeffrey fluid the extra stress tensor components are [2]

$$S = \frac{\mu'(y')}{1+\lambda_1} \left(A + \lambda_2 \frac{dA}{dt} \right), \tag{10}$$

$$S_{XX} = \frac{2\mu'(y')}{1+\lambda_1} \left\{ 1 + \lambda_2 \left(\frac{\partial}{\partial t'} + V' \frac{\partial}{\partial Y'} + U' \frac{\partial}{\partial X'} \right) \right\} \frac{\partial U'}{\partial X'}, \tag{11}$$

$$S_{YY} = \frac{2\mu'(y')}{1+\lambda_1} \left\{ 1 + \lambda_2 \left(\frac{\partial}{\partial t'} + U' \frac{\partial}{\partial Y'} + V' \frac{\partial}{\partial X'} \right) \right\} \frac{\partial V'}{\partial Y'}, \tag{12}$$

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$$S_{XY} = \frac{\mu'(y')}{1+\lambda_1} \left\{ 1 + \lambda_2 \left(\frac{\partial}{\partial t'} + U' \frac{\partial}{\partial Y'} + V' \frac{\partial}{\partial X'} \right) \right\} \left(\frac{\partial U'}{\partial X'} + \frac{\partial V'}{\partial Y'} \right), \tag{13}$$

where A denotes first Rivilin-Ericksen tensor, λ_1 is the ratio of relaxation to retardation time, λ_2 is the retardation time, $\mu'(y')$ is variable dynamic viscosity.

The corresponding Boundary conditions are [57]

$$\begin{split} \psi' &= -\frac{q'}{2}, \ u' = \frac{\partial \psi'}{\partial y'} = -c, \ T' = T_0, \ C' = C_0, \ at \ y = h_1, \\ \psi' &= -\frac{q'}{2}, \ u' = \frac{\partial \psi'}{\partial y'} = -c, \ T' = T_1, \ C' = C_1, \ at \ y = h_2, \end{split}$$

and non-dimensional variables are:

$$\begin{split} a &= \frac{a_1}{d_1}, b = \frac{b_1}{d_1}, d = \frac{d_2}{d_1}, x = \frac{x'}{\lambda}, y = \frac{y'}{d_1}, F = \frac{q'}{cd_1}, \\ t &= \frac{ct'}{\lambda}, \psi = \frac{\psi'}{cd_1}, h_1 = \frac{H'_1}{d_1}, h_2 = \frac{H'_2}{d_1}, u = \frac{u'}{c}, \\ v &= \frac{v'}{c}, \delta = \frac{d_1}{\lambda}, \Pr = \frac{\mu_0 C_p}{\kappa}, Ec = \frac{c^2}{C_p (T_1 - T_0)}, \\ Br &= Ec \Pr, \theta = \frac{T' - T_0}{T_1 - T_0}, \phi = \frac{C' - C_0}{C_1 - C_0}, \operatorname{Re} = \frac{cd_1}{v}, \\ Du &= \frac{D_B (C_1 - C_0) k_T}{C_p C_s (T_1 - T_0)}, S_c = \frac{\mu_0}{\rho D_B}, S_r \frac{\rho D_B k_T (T_1 - T_0)}{\mu_0 T_m (C_1 - C_0)}, Da = \frac{k_1}{d_1^2}, \\ M &= \sqrt{\frac{\sigma}{\mu_0}} B_0 d_1, p' = \frac{c\mu_0 \lambda p}{d_1^2}, Gc = \frac{\rho g \beta_2 (C_1 - C_0) d_1^2}{c\mu_0}, Gr = \frac{\rho g \beta_1 (T_1 - T_0) d_1^2}{c\mu_0}, \\ E &= \frac{E_a}{k(T_1 - T_0)}, \lambda_2 = \frac{d_1 \lambda^*}{c\mu_0}, \overline{u} = \frac{\partial \psi}{\partial y}, \overline{v} = -\delta \frac{\partial \psi}{\partial x}, \\ \xi &= \frac{k_r^2 d_1^2}{ScD_B}, ScSr = \frac{k_T (T_1 - T_0)}{T_m (C_1 - C_0)}, \kappa = \frac{c^2 \mu_0}{Ec \Pr(T_1 - T_0)}, \mu(y) = \frac{\mu'(y')}{\mu_0}. \end{split}$$

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The peristaltic motion of various physiological fluids in the small intestine and other ducts involves the flow under the assumption of a long wavelength and a low Reynolds number. Moreover, it is possible to imagine crawling flow in such a channel, in which case the Reynolds number vanishes. Many studies on peristaltic transport considered long wavelengths and low Reynolds values [6, 48]. From the above non-dimensional parameters, the governing equations are reduced to the following stream function, temperature and concentration equations.

$$\frac{\partial p}{\partial x} = \frac{1}{1+\lambda_1} \frac{\partial}{\partial y} \left[\mu(y) \frac{\partial^2 \psi}{\partial y^2} \right] - \frac{M^2}{1+m^2} \left(\frac{\partial \psi}{\partial y} + 1 \right) + (Gr\theta + Gc\phi) \sin \alpha_0 - \frac{\mu(y)}{(1+\lambda_1)Da} \left(\frac{\partial t}{\partial y} + 1 \right),$$
(14)

$$\frac{\partial p}{\partial y} = 0, \tag{15}$$

$$\frac{\partial^2 \theta}{\partial y^2} + \frac{2Br}{1+\lambda_1} \mu(y) \left(\frac{\partial^2 \psi}{\partial y^2} \right) + Du \Pr \frac{\partial^2 \phi}{\partial y^2} + \frac{M^2}{1+m^2} Br \left(\frac{\partial \psi}{\partial y} + 1 \right)^2 = 0, \quad (16)$$

$$\frac{\partial^2 \phi}{\partial y^2} + ScSr \frac{\partial^2 \theta}{\partial y^2} - Sc\xi \phi (1+\theta)^n \exp\left(-\frac{E}{1+\theta}\right) = 0.$$
(17)

Here $\mu(y)$ is the viscosity coefficient depending on co-ordinate space y with peristaltic flow is considered in the form of exponential function as [22]

$$\mu(y) = e^{-\beta y} = 1 - \beta y$$
, where $\beta \ll 1$,

where the Reynolds model viscosity parameter is β and the selection of μ is biologically acceptable. This model demonstrates how fluid concentration is dependent on *y*.

Through compatibility conditions, using equation (15) equation (15) can be taken as

$$\frac{1}{1+\lambda_1}\frac{\partial^2}{\partial y^2}\left[\mu(y)\frac{\partial^2\psi}{\partial y^2}\right] - \frac{M^2}{1+m^2}\left(\frac{\partial^2\psi}{\partial y^2}\right) + \left(Gr\frac{\partial\theta}{\partial y} + Gc\frac{\partial\psi}{\partial y}\right)\sin\alpha_0 - \frac{\partial}{\partial y}$$

$$\left[\frac{\mu(y)}{(1+\lambda_1)Da}\left(\frac{\partial\psi}{\partial y}+1\right)\right] = 0.$$
 (18)

The relation between velocity components u, v and ψ is taken as

$$u = \frac{\partial \psi}{\partial y}$$
 and $v = -\delta \frac{\partial \psi}{\partial y}$,

which satisfies equation(4).

Using equation (18) velocity equation can be written as

$$\frac{1}{1+\lambda_1} \frac{\partial^2}{\partial y^2} \left[\mu(y) \frac{\partial u}{\partial y} \right] - \frac{M^2}{1+m^2} \left(\frac{\partial u}{\partial y} \right) + \left(Gr \frac{\partial \theta}{\partial y} + Gc \frac{\partial \phi}{\partial y} \right) \sin \alpha_0 - \frac{\partial}{\partial y} \\ \left[\frac{\mu(y)}{(1+\lambda_1)Da} \left(u + 1 \right) \right] = 0,$$
(19)

The corresponding boundary conditions are

$$\psi = \frac{-F}{2}, \frac{\partial \psi}{\partial y} = -1, \ \theta = 0, \ \phi = 0, \ \text{when} \ y = h_1 = -d - b \cos(2\pi x + \Phi),$$
(20)

$$\psi = \frac{F}{2}, \frac{\partial \psi}{\partial y} = -1, \ \theta = 1, \ \phi = 1, \ \text{when } y = h_2 = 1 - a \cos(2\pi x),$$
 (21)

Partial differential equations (14) to (19) are solved by Homotopy Perturbation technique using boundary conditions (20) and (21). Further HPM methodology discussed in the next section.

3. Methodology

A semi analytical method HPM which is very power full method to find the approximate solutions for the high non-linear coupled partial differential equations [11, 13, 23, 30].

$$H(u, p) = L(u) - pL(u_0) + p \left\{ \beta^2 \frac{\partial u}{\partial y} - \frac{M^2(1+\lambda_1)}{1+m^2} (1+\beta y) \frac{\partial u}{\partial y} \right\}$$
$$(1+\beta y) (1+\lambda_1) \left(Gr \frac{\partial \theta}{\partial y} + G_c \frac{\partial \phi}{\partial y} \right) \sin \alpha_0 - \frac{1}{Da} \left(\frac{\partial u}{\partial y} - \beta(u+1) \right), \qquad (22)$$

$$H(u, p) = L(u) - pL(\theta_0) + p \left\{ \frac{2Br}{1+\lambda_1} \left[(1+\beta y) \left(\frac{\partial u}{\partial y} \right)^2 \right] + Du \operatorname{Pr} \frac{\partial^2 \phi}{\partial y^2} + \frac{M^2}{1+m^2} Br(u+1)^2 \right\},$$
(23)

$$H(\phi, p) = \left[L(\phi) - pL(\phi_0)\right] + p\left\{ScSr\frac{\partial^2\theta}{\partial y^2} - Sc\xi\phi(\theta+1)^n \exp\left(-\frac{E}{\theta+1}\right)\right\}.$$
 (24)

the initial approximations of $u, \theta \cdot \phi$ are

$$u_0 = \frac{-6(F+h_2-h_1)}{(h_2-h_1)^3} [y^2 - (h_1-h_2)y + h_1h_2] - 1,$$
(25)

$$\theta_0 = \frac{y - h_1}{h_2 - h_1},\tag{26}$$

$$\phi_0 = \frac{y - h_1}{h_2 - h_1},\tag{27}$$

The series solutions are obtained by the following equations

$$u = u_0 + pu_1 + p^2 u_2 + \dots, (28)$$

$$\theta = \theta_0 + p\theta_1 + p^2\theta_2 + \dots, \tag{29}$$

$$\phi = \phi_0 + p\phi_1 + p^2\phi_2 + \dots, \tag{30}$$

4. Results and Analysis

To solve equations (16), (17) and (19) we have used HPM. It is a highly efficient approach for solving non-dimensional equations. By using series solutions of velocity, temperature and concentration, from the equations (28), (29), (30) we have plotted the graphs and discussed the influence of different physical parameters namely the mass Grashof Gc, the thermal Grashof Gr, Hartmann number M, Hall current m, Darcy number Da, Schmidt Sc, Soret Sr, Dufour effect Du, Brinkmann number Br, Prandtl number Pr, chemical reaction parameter ξ , viscosity parameter β , activation energy parameter E.

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According to Hayat et al. [27], we have taken the range of physical parameter values phase difference $\phi = 0.1$, $\alpha_0 = \frac{\pi}{2}$, d = 1 by using the fixed values a = 0.85, b = 0.75, x = 0.2, c = 0.2, n = 0.1. While the range of the different physical characteristics varied over are mentioned in the figure caption. Figures (2-25) are plotted to analyse the influence of different parameters on velocity, temperature and on concentration. We done result analysis under the assumption of low Reynolds number (Re $\ll 1$), it means viscous force predominates inertial force. Moreover, it differentiates laminar and turbulent flow. Reynolds number Re < 1000 represents laminar flow.

4.1 Velocity profile

Due to the magnetic field being provided perpendicular to the fluid motion and being high at the channel's centre. In figure 2 as the Hartmann number M grows. For M = 1 it is observed that axial velocity is maximum at y = 0. As M value increases we can see that the velocity decreases at the channel's centre and increases towards the walls. Because the transverse magnetic field produces the Lorentz force and the transverse magnetic field creates a drag-like force which operates against the flow and hence decreases the velocity profile. In the sphere of medicine, this can be extremely useful for eliminating cancer cells. When the Hall parameter m grows in Figure 3, increase in the velocity is observed and it enhance the momentum boundary layer thickness [42].

From figures 4 and 5 Mass and thermal Grashof numbers Gc and Gr have the same nature on velocity, it deceases from the range -1.22462 to 0 and increases from 0 to 1.26266. Gr is the ratio of buoyancy force to viscous force, very less Gr number characterises laminar boundary layer. From figure 6, we noted that at the channel centre velocity increased with viscosity parameter $-1.22462 \le \beta \le 0.5$ and velocity starts to decrease towards right side of the wall in range $0.5 \le \beta \le 1.26266$. If $\beta = 0$ then the viscosity become constant for the study. Hence this investigation can be used to study many human physiological systems especially, the blood flow. The core of the channel experiences an increase in velocity when we transition from a porous to a non-porous material. Due to that reason from figure 7 we observed

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increasing in the permeability parameter Da, velocity increases at the centre of the channel. At Da = 2 we can observe raise in velocity due to high permeability. Fluid flows more easily in this situation. If $Da = \infty$ then porous medium effect becomes zero.



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4.2 Temperature profile

Figures 8 to 17 represent variation of temperature θ . Temperature has more variation by Joule heating. By this fact temperature increases with the magnetic field. From figure 9 we observed that temperature decreases with increasing Hall current m and increasing in temperature with Brinkmann number Br observed in figure 10 due to less conduction of heat. From figure 11 we observed that decreasing in temperature with increasing in Gashrof number Gc. Increasing in permeability parameter Da temperature decreases figure 12.Increasing in temperature with increasing dufour is observed from figure 13.

Temperature decreases with increasing in activation energy E from figure 14. Also we observed from figure (15) that temperature raises with viscosity parameter β . It is observed that from figures 16 and 17 temperature increases with Prandtl number Pr and Schmidt number Sc. Pr is totally independent of the environment in which a fluid flows. If Pr < 1 then conductive heat transfer will dominates convective heat transfer. Momentum boundary layer thinner than thermal boundary layer. In this case temperature flows more readily. Pr clarifies the relative significance of viscous and thermal dissipation. For Sc > 1 momentum transfer is more significant than mass transfer.



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$Gc = 0.2, Gr = 0.1, Br = 0.1, E = 0.1, \xi = 0.1$

4.3 Concentration profile

Gc = 0.2, Gr = 0.1, Br = 0.1, E = 0.1,

 $\xi = 0.1$

Figures 18 to 25 states the impact of various factors on concentration. We observed concentration enhances with increasing in magnetic field M from figure 18. It is clears that concentration is directly proportional to magnetic field due to Lorentz force. In electrochemical processes, it improves mass transfer. Concentration decreases with increasing in Hall parameter m observed in figure 19 and from figure 20 we observed that viscosity parameter β has constant effect on concentration at channel centre and varies at near the walls of the channel.

Concentration increases with Brinkmann number in figure 21. From figure 22 we observed that concentration increases with activation energy because molecule counts increases which need minimum energy, it leads to increasing in reaction rate therefore Figure 23 shows enhancement of concentration with increasing in chemical reactive material ξ . If Sc > 1, momentum boundary layers are thicker than concentration boundary layers at all points and momentum diffusivity outweighs mass diffusivity. From figures 24, 25 we observed that concentration increases with Schmidt number $1 \leq Sc \leq 2$ and concentration increases with Soret number $2 \leq Sr \leq 4$ because greater Soret number leads to increased convective flux.



Figure 18. Variation of M on Φ m = Sc = Sr = Du = Da = 1, Gc = 0.2, $\Pr = Br = n = Gr = 0.1$,

 $\beta = 0.01, \xi = 0.1, E = 0.1$



Figure 20. Variation of β on Φ M = 2m = Sr = Sc = Du = Da = 1, Pr = n = Gr = E = 0.1, Gc = 0.2, $\xi = 0.1, Br = 1.0$



Figure 19. Variation of m on Φ M = 2, Sc = Sr = Du = Da = 1,

Gc = 0.2, \Pr = Br = n = Gr = E = 0.1, β = 0.01, ξ = 0.1



Figure 21. Variation of Br on Φ M = 2, m = Sr = Sc = Du = Da = 1, Pr = n = Gr = E = 0.1, Gc = 0.2, $\xi = 0.1, Br = 1.0$



Figure 22. Variation of E on Φ m = Sr = Sc = Du = Da = 1, Pr = 0.1, $\beta = 0.01$, Gc = 0.2, Br = Gr = 0.1, $\xi = 0.1$, M = 2



Figure 23. Variation of ξ on Φ m = Sr = Sc = Du = 1, Pr = Br = 0.1, $\beta = 0.01$, Gc = 0.2, M = 2, Da = 0.1, Sc = 0.5, Gr = E = 0.1



Figure 24. Variation of Sc on Φ m = Sr = Sc = Du = 1, Br = n = Gr $= E = 0.1, \beta = 0.01, Gc = 0.2, \xi = 0.1,$ Pr = 0.5, M = 2



5. Conclusions

Our present article aims to investigate impact of activation energy and variable viscosity of Jeffrey fluid through an asymmetric channel. HPM method is good agreement to solve high non-linear coupling pde. Instead of taking constant viscosity, Variable viscosity is more prominent to anticipate the flow characteristics. The major outcomes of this investigation are as follows:

- Hartmann number *M* and Darcy number *Da* have opposite behaviour on velocity.
- *Gr* and *Gc* effects on velocity are similar and changes its behaviour at the centre of the channel.
- Velocity increases with variable viscosity parameter β and Hall parameter *m* at centre of the channel.
- Temperature decreases with increasing in activation energy parameter E and increases with viscosity parameter β and with Br, Du, Pr.
- Concentration increases with increasing in activation energy parameter *E*, *Sc* and *Sr*.
- If $Da \to \infty$, $m = Gr = Gc = \beta = 0$, then our findings are in good accord with Kothandapani and Srinivas [40].

Therefore, this work is pertinent to geothermal, biological and chemical engineering. Additionally, the study of variable viscosity is useful in blood rheology and cardiovascular physiology.

Conflict of interest

The author declares that there is no personal or organizational conflict of interest with this work.

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