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MATHEMATICAL MODELING ON COVID-19 PANDEMIC WITH AWARENESS AND ELASTICITY ANALYSIS

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Abstract

In this work, I have proposed a six dimensional deterministic epidemiological model of coronavirus disease (COVID-19). I have studied the model and analyzed local stability of both the disease free and the endemic equilibrium point. I have calculated the basic reproduction number with awareness (R_{01}) and without awareness (R_{02}). The real fact is that still now there is no absolute treatment procedure but scientists were doing work on it. So in many countries the vaccination process is ongoing. Further it is not ensured that if an individual is given a vaccine then there is no chance to be infected by the virus. For this perspective my main motive of this work is to control disease transmission. From the elasticity analysis I have mentioned some sensitive parameters which prevent disease transmission. Awareness that is social distancing, wearing masks, quarantine and avoiding much more gathering is one of the sensitive parameters which I have found from our study. Finally, I have verified our analytical results numerically with the help of MATLAB programming.

1. Introduction

Today the most important research area is various kinds of life killing diseases such as HIV, Ebola, Dengue, Cancer, COVID-19 etc. The first coronavirus (COVID-19) was identified in Wuhan city of China, December 2019. Now it is rapidly spread globally across the world and became pandemic (WHO). Mathematical models play a significant role to provides possible strategies to prevent such types of infectious diseases. [1-7] In 6th July, 2021 all over the world total coronavirus confirmed and death cases was 183, 934, 943 and 3, 985, 022 [as per WHO coronavirus (COVID-19)

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dashboard]. All over the world people were facing various trouble from different aspects i.e. economical, educational, health related, travel etc. So many major initiative were taken all the health authority of the world but they were not succeed to prevent to prevent the disease transmission. In a very short period COVID-19 was spread all over the world and changes its nature. Therefore there is very difficult to predict its nature and taken initiative steps and provides adequate facilities to the effected people. In India the first COVID-19 cases was recognised in January 27, 2020 in Kerala. The etiological agent of COVID-19 is Severe Acute Respiratory Syndrome Coronavirus 2 (SARSCoV-2). Actually SARS-CoV-2 name was given by the ICTV (International Committee on Taxonomy of Viruses in February 11, 2020. [8] All kinds of viruses generally alter their structure and its depends on time. Through mutation SARS-CoV-2 virus has also altered their structure. Now SARS-CoV-2 virus is going its six strains. According to Xie et al. [9] first episode was sprouted in Guangdong Province of China in (2002-2003). In (2012-2020) the second one MERS was originated in Saudi Arabia. Third attack of SARS-CoV-2 has been occurred in 2021 and its fatality rate is lower than MERS-CoV.

Every people being treat as susceptible to effected by the virus. However the older people and those people who are already suffering some specific diseases is high risk of getting COVID-19 disease. Many mathematician already published numerous paper on COVID-19. [10-19] Most of the cases near about 81 percent of COVID-19 effected people is asymptomatic and no need to oxygen support treatment. Basic symptoms of coronavirus infected individuals is fever, dry cough, tiredness and breathing problem. Among the infected individual 14 percent acute case need to oxygen support (ICU) treatment when their oxygen saturation level is less than 93 percent (95 to 100 percent is normal range). COVID-19 is the most effective contagious diseases spread human to human through NPIs. To reduce the community spread of coronavirus only lock-down is the reliable effective mechanism. [20, 21] Some researcher also used fractional order differential equation to study the COVID-19 disease [22-25].

In this paper I have considered a non linear six dimensional deterministic epidemic model and suppose that quarantine and hospitalization under isolation population do not consider in the total active population. Main motive of this work is to find out an efficient way to prevent the disease transmission.

2. The Mathematical Model

Some ideas which I gather from already published work. Most of the work they were considered all the classes involved in the total active population. Here I have considered except quarantine and hospitalization class all other classes involved into the active population class. Basis on this assumption I have proposed a six dimensional deterministic model on COVID-19 disease. So many paper considered the nonlinear incidence term in the form $\frac{\beta SI}{N}$ Specially I have considered the nonlinear incidence term in the form $\frac{\beta SI}{N - Q_1}$. I divided the total populations into six subclasses that is; the susceptible class $S(t)$, asymptomatic or exposed class $A(t)$, the active infected class $I(t)$, quarantine class $Q(t)$, hospitalization or isolation class $H(t)$ and $R(t)$ the recovery class. Here the total population is denoted by $N(t) = S + A + I + Q + H + R$. Also the total active population is consider $N - Q_1$, where $Q_1 = Q + H$. Also we imposed a awareness (wearing mask, maintain social distance, etc.) term into the proposed SAIQHR model. My model is taken in the following form.

$$\frac{dS}{dt} = R_s - \frac{\beta SI}{N - Q_1} - \frac{\alpha\beta SA}{N - Q_1} - (\mu + \delta)S + \eta Q,$$

$$\frac{dA}{dt} = \frac{\beta SI}{N - Q_1} + \frac{\alpha\beta SA}{N - Q_1} - (\mu + \gamma + r_1)A,$$

$$\frac{dI}{dt} = \gamma A - (\mu + \rho + r_2)I,$$

$$\frac{dQ}{dt} = (1 - \sigma)\rho I - (\eta + \theta + \mu)Q,$$

$$\frac{dH}{dt} = \sigma\rho I + \theta Q + (\mu + r_3)H,$$

$$\frac{dR}{dt} = r_1 A + r_2 I + r_3 H + \delta S - \mu R. \tag{1}$$

Where, susceptible class recruited (birth, immigration) by the rate R_s, β is the disease transmission rate, α is a commutation rate of asymptomatic or exposed class, δ is the awareness rate, η is the quarantine period, μ is the natural death rate of all the classes, asymptomatic individuals became symptomatic at the rate γ, r_2 is the rate at which the infected individuals became recover, r_1 recovery rate of asymptomatic individuals, $\sigma(0 \leq \sigma \leq 1)$ is the hospitalization or isolation rate of infected individuals, ρ is the quarantine rate of infected individuals, θ is the hospitalization rate of quarantine individuals and r_3 is the recovery rate of hospitalization individuals.

3. Some Preliminary Analysis of the System (1)

Mainly my proposed model describes such types of disease which is only spread human to human. So I must need to prove that all the state variable is positive for $\forall\, t$.

Theorem 3.0.1. *All solutions* S, A, I, Q, H *and* R *of the SAIQHR system* (1) *are remain positive for* $\forall\, t > 0$ *in* R_+^6 *according to the initial conditions* $S(0) \geq 0,\ A(0) \geq 0,\ I(0), 0,\ Q(0) \geq 0,\ H(0) \geq 0$ *and* $R(0) \geq 0$ *in the region* Ω.

Proof. Proof is shown in Appendix A.

Theorem 3.0.2. *All non negative solutions of the SAIQHR system* (1) *are bounded and contain in the region*

$$\Omega = \left\{(S, A, I, Q, H, R) \in R_+^6 : 0 \leq S + A + I + Q + H + R \leq \frac{R_s}{\mu}\right\}.$$

Proof. Proof is shown in Appendix B.

4. Basic Reproduction Number R_0

In an infection disease dynamics basic reproductive number play an important role to spread disease. Here I analytically computed the value of the basic reproduction number of the system 1 with the help of next generation matrix technique which has discussed. [26] The calculated value is given below.

$$R_0 = R_{01} + R_{02} = \frac{\mu\beta\gamma}{(\mu+\delta)(\mu+\gamma+r_1)(\mu+\rho+r_2)} + \frac{\alpha\beta\mu}{(\mu+\delta)(\mu+\gamma+r_1)}$$

First term comes from infected class and second term from asymptomatic class. Asymptomatic individuals moves to infected individuals by the proportion $\frac{\gamma}{\mu+\gamma+r_1}$.

5. Equilibria Analysis and its Existence

The SAIQHR model (1) consists two equilibria namely,

(i) The disease free equilibrium point $E_d^0 = \left\{\frac{R_s}{(\mu+\delta)}, 0, 0, 0, 0, \frac{\delta R_s}{\mu(\mu+\delta)}\right\}$

(ii) The endemic equilibrium $E_{en}^* = (S^*, A^*, I^*, Q^*, H^*, R^*)$ with

$$S^* = \frac{R_s}{\mu+\delta} - d_0A^*, \; A^* = \frac{R_s(R_0-1)}{\mu+\mu a_0 + r_1 + r_2a_0 + r_3c_0 + d_0\mu + \delta d_0(R_0-1)},$$

$I^* = a_0A^*$, $Q^* = b_0A^*$, $H^* = c_0A^*$ and

$$R^* = \frac{\delta R_s}{\mu(\mu+\delta)} + \frac{A^*(r_1 + a_0r_2 - c_0r_3 - \delta d_0)}{\mu}, \text{ where } a_0 = \frac{\gamma}{\mu+\rho+r_2},$$

$$b_0 = \frac{\rho\gamma(1-\rho)}{(\mu+\rho+r_2)(\mu+\eta+\theta)}, \; c_0 = \frac{\rho\sigma a_0 + \theta b_0}{\mu+r_3}, \; d_0 = \frac{\mu+\gamma+r_1-\eta b_0}{\mu+\delta}.$$

It is clear that when the values of basic reproduction number is grater than one $(R_0 > 1)$, then the system (1) exists a endemic equilibrium point. If $R_0 < 1$, the model system (1) does not exists any positive endemic equilibrium point.

6. Local Stability of E_d^0 and E_{en}^*

Theorem 6.0.1. *In absence of disease the equilibrium point* E_d^0 *is locally asymptotically stable while* $0 < R_0 < 1$ *and unstable when* $R_0 > 1$.

Proof. Proof is shown in Appendix C.

Table 1. Sensitivity indicates of R_0 with respect to the parameters

Parameter (y_i)	β	α	δ	μ	γ	r_1	ρ	r_2
Value	0.51	0.0215	0.0014	0.000391	0.08	0.2	0.031	0.01
$\varepsilon_{R_0}^{y_i}$	1	0.011	-0.782	0.771	0.704	- 0.713	- 0.741	- 0.239

Theorem 6.0.2. *The equilibrium point* E_{en}^* *of the SAIQHR model system* (1) *is locally asymptotically stable*, *while* $R_0 > 1$.

Proof. Proof is shown in Appendix D.

7. Direction of Bifurcation at $R_0 = 1$

In this portion I am trying to show the direction of bifurcation at $R_0 = 1$ with the help Castillo-Chavez and Song bifurcation theorem [27].

Theorem 7.0.1. *The system* (1) *attains a transcritical forward bifurcation at* $R_0 = 1$.

Proof. Proof is shown in Appendix E.

8. Sensitivity and Elasticity Analysis

Here I have discussed some sensitive parameter which is more responsible to prevent the disease transmission. Table 1 shows that elasticity of some sensitive parameters on R_0. In epidemiology disease transmission and prevention are totally depends on R_0. Therefore there must be logic to find out some sensitive parameters. First I have calculated the sensitivity and elasticity values of all the sensitive parameters using the following formula [28].

$$S_{R_0}^{t_i} = \frac{\partial R_0}{\partial y_i}$$

$$\varepsilon_{R_0}^{y_i} = \frac{\partial R_0}{\partial y_i} \times \frac{y_i}{R_0}$$

The value of R_0 will increase (decrease) its totally depends on the elasticity value. While the value of $\varepsilon_{R_0}^{y_i}$ is positive for a certain parameter value then, R_0 will increases if increase the parameter value. Also when $\varepsilon_{R_0}^{y_i}$ attains negative value for a certain parameter value then, R_0 will decreases if increase the parameter value. Table 1 says that, β, α, μ and γ has positive elasticity on R_0. If I increase (decrease) any one parameter value among β, α, μ and γ then R_0 will be increase (decrease) Figure 6. Also δ, r_1, ρ and r_2 has negative elasticity on R_0. If I increase (decrease) any one parameter value among δ, r_1, ρ and r_2 then R_0 will be decrease (increase) Figure 7. If the value of β changes 10 percent, then the value of R_0 also changes $10\varepsilon_{R_0}^{\beta}$ percent. My details elasticity analysis shows that the system (1) has two more sensitive parameters that is β and δ. If I control these two parameters value then I should prevent the disease transmission.

9. Exploration of Analytical Result

In this section I am trying to prove validation of our analytical result of the system (1) with support of numerical technique. Numerical Figure 1-7 indicate our analytical findings. While we choose $\beta = 0.51$ and other parameters values from Table 2, then Figure 1 says that the equilibrium point $E_d^0(44668, 0, 0, 0\ 0, 15994)$ is locally asymptotically stable when $R_0 = 0.7760 < 1$. The concern eigenvalues are $-0.000391, -0.0134, -0.0018, -0.0815, -0.3110, -0.0084$. If I choose all the parameters values from Table 2 then the disease related equilibrium point

$E_{en}^*(27399, 135, 262, 99, 76, 127)$ is locally asymptotically stable when $R_0 = 1.6738 > 1$ which is shown in Figure 2. Here the responsible eigenvalues are $-0.000391, -0.0134, -0.3199, -0.0011, +0.00063i, -0.0011 + 0.00063i, -0.0011 - 0.0063i, -0.0813$. Figure 3 shows that the relation among R_0, β and α. Figure 4 suggest that the elasticity analysis of all the sensitive parameters. Forward bifurcation figure of the system (1) at $R_0 = 1$ is given in Figure 5. Figure 6 indicates that if we increase the values of β, α, γ and μ then the value of R_0 also increases. If the values of δ, r_1, r_2 and ρ increases then the value of R_0 decreases which is given in Figure 7.

Table 2. Illustration and hypothesis of parameter

Symbol	Explanation	Value and source
R_s	Recruitment rate	80 Estimated
β	Disease transmission rate	1.1 Estimated
α	Commutation rate of asymptomatic individuals	0.0251 Estimated
δ	Awareness rate	0.0014 Estimated
η	Quarantine period	1/14
μ	Natural death rate	0.000391[2]
γ	Asymptomatic individuals to symptomatic	0.08
r_1	Recovery rate of A class	0.2 Estimated
σ	Hospitalization rate of I class	0.002 Estimated
r_2	Recovery rate of I class	0.01 Estimated
ρ	Quarantine rate	0.031 Estimated
θ	Hospitalization rate of Q class	0.0101 [16]
r_3	Recovery rate of H class	0.0013 Estimated

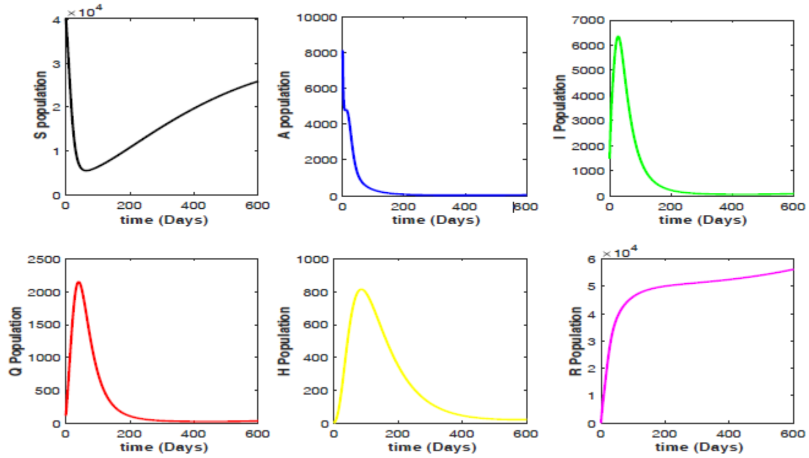

Figure 1. Stability of the disease free equilibrium point E_0.

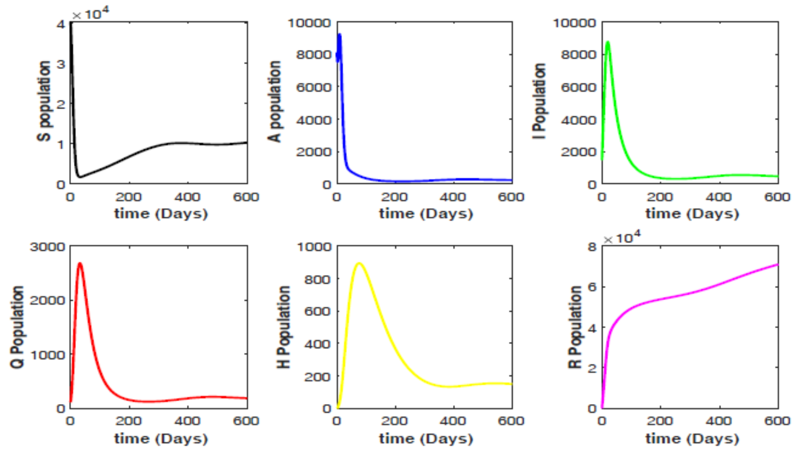

Figure 2. Stability of the endemic equilibrium point E^*.

10. Conclusion

In this work, I have proposed a six dimensional deterministic model on COVID-19 pandemic with awareness. First I have calculated a basic reproductive number R_0 with the help of next generation matrix technique. In disease dynamics R_0 plays an important role to spread of such kinds of diseases. I know that the disease either persists or not it is totally depends on R_0. While $R_0 < 1$ then it dies out and when $R_0 > 1$ then it persists and became endemic. My proposed model consists two non negative equilibria that is disease free and endemic. I have studied the stability of both the equilibrium point. From my analysis it is clear that when $R_0 < 1$ then the disease free equilibrium point is locally asymptotically stable and unstable while $R_0 > 1$. I also prove that the endemic equilibrium point is locally asymptotically stable. I have also analyzed some sensitive parameter on R_0 from their elasticity. My detailed study says that basically the value of R_0 depends on infected class and asymptomatic class. While I take all the parameters value from the Table 2 then the value of R_0 is $1.6738 = 1.6554 + 0.0184 = R_{01} + R_{02}$. Where R_{01} and R_{02} come from infected and asymptomatic class respectively. From elasticity analysis I find out some sensitive parameter and our model says that β and δ is more sensitive. If I decrease the value of β then value of R_0 is also decrease. Although if I increase the value of δ then the value of R_0 is decrease. It is more easier to increase the value of δ rather than to decrease the value of β and it is more realistic. Also if I do not consider any awareness then the value of R_0 is 7.6669. It is much more than the value 1.6738. Therefore considering awareness in my model is more meaningful. Now my study indicates that if I increase the awareness rate from 0.0024 to 0.00261 then the value of R_0 decreases from 1.0741 to 0.9989 while all other parameter value remain same which is given in Table 2. Hence if I increase the awareness rate from my assumption value up to 8.75 percent then the disease dies out from the system. Again if I increase the quarantine rate from 0.055 to 0.060 then the value of R0 decreases from 1.0662 to 0.9918 while all other parameter value remain unchange. Finally I conclude that if I take awareness and increase the awareness and quarantine rate then such types of disease transmission can kept under grip.

Appendix A: Proof of theorem 3.0.1.

Proof. From system (1), I obtained $\left.\frac{dS}{dt}\right|_{S=0} = R_s + \eta Q$, $\left.\frac{dA}{dt}\right|_{A=0} = \frac{\beta SI}{S+I+R}$, $\left.\frac{dI}{dt}\right|_{I=0} = \gamma A$, $\left.\frac{dQ}{dt}\right|_{Q=0} = (1-\sigma)\rho I$, $\left.\frac{dH}{dt}\right|_{H=0} = \rho\sigma I + \theta Q$ and

$= \left.\dfrac{dR}{dt}\right|_{R=0} = r_1 A + r_2 I + r_3 H + \delta S.$ and With the help of the above expression and initial conditions all solutions of the SAIQHR system (1) are remain positive for $\forall\, t > 0$ in Ω.

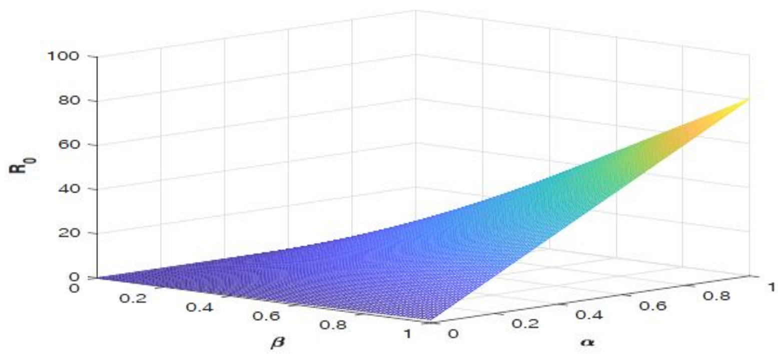

Figure 3. Relation among β, α and R_0

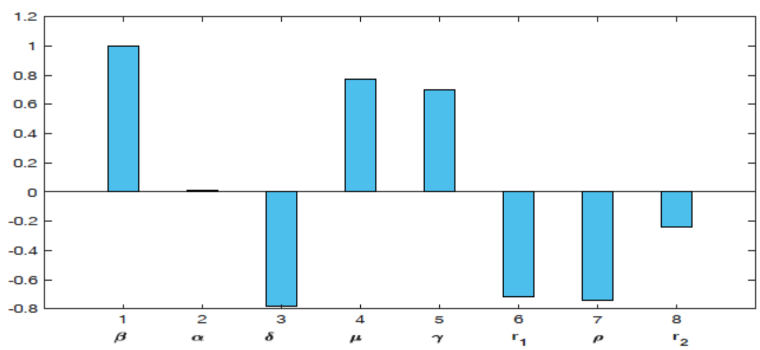

Figure 4. Elasticity on R_0 with respect to the parameters.

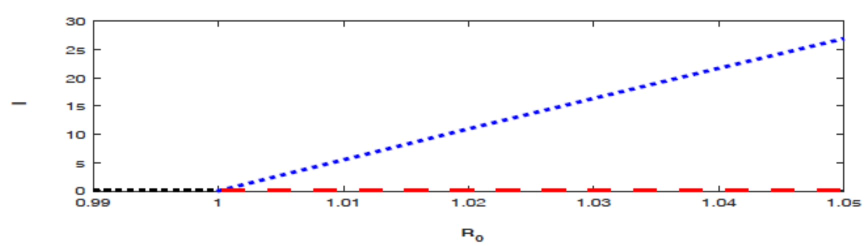

Figure 5. Transcritical forward bifurcation appear when $R_0 = 1$

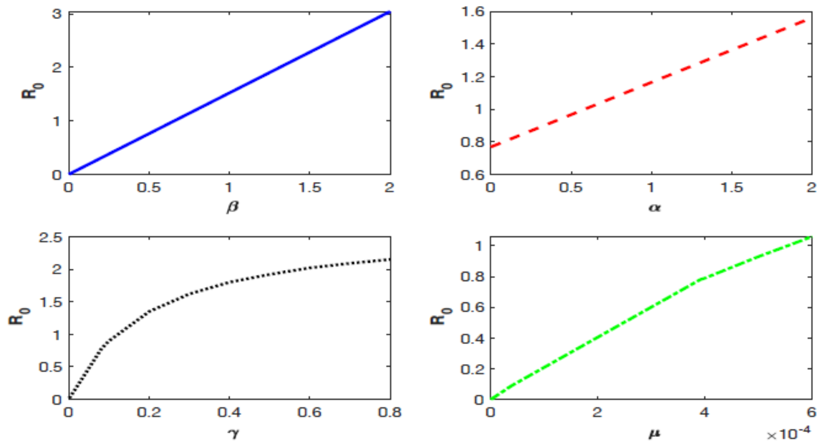

Figure 6. Relation among β, α, γ and μ with R_0

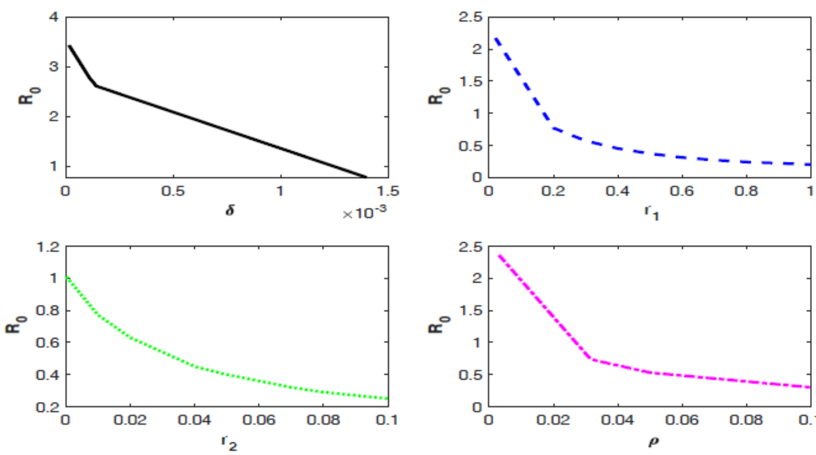

Figure 7. Relation among δ, r_1, r_2 and ρ with R_0.

Appendix B: Proof of theorem 3.0.2.

Proof. My proposed model I suppose that $N = S + A + I + Q + H + R$. Therefore, $\frac{dN}{dt} = \frac{dS}{dt} + \frac{dA}{dt} + \frac{dQ}{dt} + \frac{dH}{dt} = R_s - \mu\ (S + A + I + Q + H + R)$.

Hence $\lim_{t\to\infty} \sup(S + A + I + Q + H + R) \leq \frac{R_s}{\mu}$. Therefore, all non negative solutions of the SAIQHR system (1) are ultimately bounded and contain in the region

$$\Omega = \left\{(S,\ A,\ I,\ Q,\ H,\ R) \in R_{+}^{6} : 0 \leq S + A + I + Q + H + R \leq \frac{R_s}{\mu}\right\}.$$

Appendix C: Proof of theorem 6.0.1.

Proof. First, I construct a Jacobian matrix around the disease free equilibrium point E_d^0 of the system (1)

$$J_0^d\Big|_{E_d^0} = \begin{pmatrix} -l_1 & -\frac{\alpha\beta\mu}{l_1} & -\frac{\beta\mu}{l_1} & \eta & 0 & 0 \\ 0 & \frac{\alpha\beta\mu}{l_1} - n_1 & \frac{\beta\mu}{l_1} & 0 & 0 & 0 \\ 0 & \gamma & -t_1 & 0 & 0 & 0 \\ 0 & 0 & (1-\sigma)\rho & -m_1 & 0 & 0 \\ 0 & 0 & \sigma\rho & \theta & -(\mu + r_3) & 0 \\ \delta & r_1 & r_2 & 0 & r_3 & -\mu \end{pmatrix}.$$

Where $l_1 = (\mu + \delta)$, $m_1 = (\eta + \theta + \mu)$, $n_1 = (\mu + \gamma + r_1)$ and $t_1 = (\mu + \rho + r_2)$. The characteristics equation $|\, J_0^d\big|_{E_d^0} - \lambda_0 I_d' \,| = 0$ has four purely negative eigenvalues (I_d' is the identity matrix) $\lambda_1^0 = -\mu < 0$, $\lambda_2^0 = -(\mu + r_3) < 0$, $\lambda_3^0 = -l_1 < 0$, $\lambda_4^0 = -m_1 < 0$ and the another two eigenvalues of the equation

$$f(\lambda_0) = \lambda_0^2 + p_0\lambda_0 + q_0 = 0$$

where, $p_0 = \left\{\frac{R_0 l_1 t_1^2 + \mu\beta\gamma - \alpha\beta\mu t_1(1 - R_0)}{R_0 l_1 t_1}\right\}$ and $q_0 = t_1 n_1 (1 - R_0)$.

It is obvious that while $R_0 < 1$ then both the value of $p_0 > 0$, $q_0 > 0$ are positive. Therefore the above quadratic equation must has two negative roots λ_5^0 and λ_6^0. Hence E_d^0 is locally asymptotically stable while $0 < R_0 < 1$ and unstable when $R_0 > 1$.

Appendix D: Proof of theorem 6.0.2.

Proof. The variational matrix $J^* \,|_{E_{en}^*}$ of the system (1) is given below,

$$J^*\Big|_{E_{en}^*} = \begin{pmatrix} -l_1 - l & -m & -n & \eta & 0 & 0 \\ l & m - n_1 & n & 0 & 0 & 0 \\ 0 & \gamma & -t_1 & 0 & 0 & 0 \\ 0 & 0 & (1-\sigma)\rho & -m_1 & 0 & 0 \\ 0 & 0 & \sigma\rho & \theta & -(\mu + r_3) & 0 \\ \delta & r_1 & r_2 & 0 & r_3 & -\mu \end{pmatrix}.$$

where, $l = \dfrac{(A^* + I^* + R^*)(\alpha\beta A^* + \beta I^*)}{(S^* + A^* + I^* + R^*)^2}$, $m = \dfrac{(S^* + I^* + R^*)(\alpha\beta S^* + \beta S^* I^*)}{(S^* + A^* + I^* + R^*)^2}$

and $n = \dfrac{(S^* + A^* + R^*)(\beta S^* + \alpha\beta S^* I^*)}{(S^* + A^* + I^* + R^*)^2}$. The characteristics equation $| J^*\big|_{E_d^*} - \lambda' I_d' | = 0$ has two negative eigenvalues $\lambda_1' = -\mu < 0$, $\lambda_2' = -(\mu + r_3) < 0$ band another four eigenvalues of the equation

$$\lambda'^4 + a_1\lambda'^3 + a_2\lambda'^2 + a_3\lambda' + a_4 = 0,$$

Where, $a_1 = l_1 + m_1 + n_1 + t_1 - m$, $a_2 = l_1 m_1 + l_1 n_1 + l_1 t_1 + m_1 n_1 + m_1 t_1 + n_1 t_1 + \ln - ml_1 - mm_1 - mt_1 - n\gamma$, $a_3 = l_1 m_1 n_1 + l_1 m_1 t_1 + l_1 n_1 t_1 + m_1 n_1 t_1 + \ln_1 t_1 + lm_1 n_1 - mm_1 l_1 - ml_1 t_1 - nl_1\gamma - mm_1 t_1 - m_1 n\gamma$ and $a_4 = l_1 m_1 n_1 t_1 + lm_1 n_1 t_1 - mm_1 l_1 t_1 - l_1 m_1 n\gamma - l\eta\gamma\rho(1 - \sigma)$. As per Routh-Hurwitz Criterion if all $a_i > 0$, $i = 1, 2, \ldots, 4$ and $a_1 a_2 a_3 > a_3^2 + a_1^2 a_4$ then all roots $(\lambda_3', \lambda_4', \lambda_5'$ and $\lambda_6')$ of the above equation has negative real parts. Hence the theorem is proved.

Appendix E: Proof of theorem 7.0.1.

Proof. Suppose $S = y_1$, $A = y_2$, $I = y_3$, $Q = y_3$, $Q = y_4$, $H = y_5$, $R = y_6$ and the bifurcation parameter is $\psi = \beta$ for $R_0 = 1$. Now at $\psi = \psi^c = \beta^c$, for $R_0 = 1$ I have

$$\beta^c = \frac{(\mu + \delta)(\mu + \gamma + r_1)(\mu + \rho + r_2)}{\mu\gamma + \alpha\mu(\mu + \rho + r_2)}$$

According to this assumption my proposed SAIQHR system (1) will take the form:

$$\frac{dS}{dt} = R_s - \frac{\beta^c y_1 y_3}{y_1 + y_2 + y_3 + y_6} - \frac{\alpha\beta^c y_1 y_2}{y_1 + y_2 + y_3 + y_6} - (\mu + \delta) y_1 + \eta y_4 = f_{1,1}$$

$$\frac{dA}{dt} = \frac{\beta^c y_1 y_3}{y_1 + y_2 + y_3 + y_6} + \frac{\alpha\beta^c y_1 y_2}{y_1 + y_2 + y_3 + y_6} - (\mu + \gamma + r_1) y_2 = f_2,$$

$$\frac{dI}{dt} = \gamma y_2 - (\mu + \rho + r_2) y_3 = f_3,$$

$$\frac{dQ}{dt} = (1 - \sigma)\rho y_3 - (\eta + \theta + \mu) y_4 = f_4,$$

$$\frac{dH}{dt} = \sigma\rho y_3 + \theta y_4 - (\mu + r_3) y_5 = f_5,$$

$$\frac{dR}{dt} = r_1 y_2 + r_2 y_3 + r_3 y_5 + \delta y_1 - \mu y_6 = f_6. \tag{2}$$

At the bifurcation parameter $\beta = \beta^c$ the variational matrix around $E^0_{d'}$ of system (2) is as follows:

$$J_0^{d'}\Big|_{E^0_{d'}} = \begin{pmatrix} -l_1 & -\frac{\alpha\beta^c\mu}{l_1} & -\frac{\beta^c\mu}{l_1} & \eta & 0 & 0 \\ 0 & \frac{\alpha\beta^c\mu}{l_1} - n_1 & \frac{\beta^c\mu}{l_1} & 0 & 0 & 0 \\ 0 & \gamma & -t_1 & 0 & 0 & 0 \\ 0 & 0 & (1-\sigma)\rho & -m_1 & 0 & 0 \\ 0 & 0 & \sigma\rho & \theta & -(\mu + r_3) & 0 \\ \delta & r_1 & r_2 & 0 & r_3 & -\mu \end{pmatrix}.$$

Where, $E^0_{d'} = \left\{\frac{R_s}{(\mu + \delta)}, 0, 0, 0, 0, \frac{\delta R_s}{\mu(\mu + \delta)}\right\}$. The all eigenvalues of the above variational matrix is $\lambda_1^c = -\mu < 0, \lambda_2^c = -(\mu + r_3) < 0, \lambda_3^c = -l_1 < 0,$ $\lambda_4^c = -m_1 < 0$ $\lambda_6^c = -\frac{l_1 t_1^2 + \mu\beta\gamma}{l_1 t_1}$. It is obvious that at $\beta = \beta^c$ the variational matrix has one zero eigenvalues as $R_0 = 1$ and other eigenvalues are negative. Now I have considered the right eigenvector $v = (v_1, v_2, v_3, v_4, v_5, v_6)$ and left eigenvector $w = (w_1, w_2, w_3, w_4, w_5, w_6)^T$ consequent to the zero eigenvalues of $J_0^{d'}\,|_{E^0_{d'}}$ at $\beta = \beta^c$.

Where, $w_1 = -\dfrac{w_2[\alpha\beta^c\mu m_1 t_1 + \mu\beta^c\gamma m_1 - \rho\eta\gamma(1-\sigma)l_1]}{l_1^2 m_1}$, $w_2 = w_2$,

$w_3 = \dfrac{\gamma w_2}{t_1}$, $w_4 = \dfrac{\rho\gamma(1-\sigma)w_2}{m_1 t_1}$, $w_5 = \dfrac{[\rho\sigma\gamma m_1 + \rho\gamma\theta(1-\sigma)]w_2}{(\mu + r_3)m_1 t_1}$,

$w_6 = \dfrac{[r_1(\mu + r_3)m_1 t_1 + r_2\gamma(\mu + r_3)m_1 + r_3(\rho\sigma\gamma m_1 + \rho\gamma\theta - \gamma\sigma\gamma\theta)]w_2}{\mu(\mu + r_3)m_1 t_1}$, $v_1 = 0$,

$v_2 = v_2$, $v_3 = \dfrac{\beta^c v_2}{l_1 t_1}$, $v_4 = 0$, $v_5 = 0$ and $v_6 = 0$.

Hence $a = \sum_{k,i,j=1}^{6} v_k w_i w_j \dfrac{\partial^2 f_k}{\partial y_i \partial y_i}(0, 0)$

$$= -\frac{\mu^2(2v_2w_2^2\alpha\beta^c + v_2w_2w_3\alpha\beta^c + v_2w_2w_3\beta^c + v_2w_2w_6\alpha\beta^c + 2v_2w_3^2\beta^c + v_2w_3w_6\beta^c)}{l_1 R_s}$$

and $b = \sum_{k,i=1}^{6} v_k w_i \dfrac{\partial^2 f_k}{\partial y_i \partial\beta}(0, 0) = \dfrac{v_2\mu(w_2 + w_3)}{l_1}$.

From the above two desired results I say that $a < 0$ and $b > 0$. Hence according to the condition (iv) of Castillo-Chavez and Song bifurcation theorem [27] I conclude that a transcritical bifurcation appear at $R_0 = 1$ and its direction is forward. Hence the theorem is proved.

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