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THE USE OF INTUITIONISTIC FUZZY NUMBERS IN RELIABILITY CALCULATIONS

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Abstract

Data insolubility arises as a result of the complicacy of a system with a many number of components in reliability calculations. Because of the tincture of devices with unified Circuits, there is a paucity of failure rate data to examine systems and comparison with uncertainty structure. In this reference, we frequently have to make assumptions about the failure rate of components/systems that have never failed before. Fuzzy set theory offers the ability to deal with problems like this. Present study evaluated the reliability of components/systems is calculated using exponential and Weibull distribution. All parameters in exponential and Weibull distribution considered as Intuitionistic Fuzzy Number (IFN). Lower and upper bounds of all parameters are also emphasizing with the help of α-cut method.

Defuzzification of reliability is also calculated with the help of average method. Some numerical examples are also given in the end of the study.

1. Introduction

It is quite difficult to determine reliability with a high degree of accuracy. In present-time complex systems like electronics, electrical components etc., as the quality of a component or system improves, there is a huge problem

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that component failure data can be very limited.

Even when the systems are the most complicated collection of components, exact values of reliabilities can be performed once a specific quantity of failure rates or component reliabilities is available. The accuracy of the results depends on the accuracy of the data, not the evaluation of reliability. Our main problem is to calculate reliability based on inadequate and inaccurate failure rate data. Conventionally for estimation the reliability of systems is indirect failure data is equipped with standard reliability distribution model and All of the parameters have been approximated using reasonable approximations. These calculated parameters could be based on inadequate and can be inaccurate data. Fuzzy set theory is one way to deal with such inaccurate information. The concept of fuzzy set theory is introduced by Zadeh in [11]. According to Zadeh, fuzzy sets are based on the assumption that the degree of membership is equal to 1 minus the degree of non-membership. The uncertainty in the membership degree can be represented by intuitionistic fuzzy sets. Atanassov [2] proposed the idea of an intuitionistic fuzzy set by introducing an intuitionistic fuzzy set that exists due to the uncertainty of information. Intuitionistic fuzzy sets (IFS) are a generalization of fuzzy set theory. The membership and non-membership degree hold the condition $0 \leq \xi(x) + \eta(x) \leq 1$. Atanassov [3] defined some operations over intuitionistic fuzzy numbers.

Baloui Jamkhaneh ([4] and [5]) calculated reliability under the exponential lifetime distribution and Weibull lifetime distribution using fuzzy numbers. D. Kumar et al. [6] talks over in the fuzzy reliability of unrepairable systems and evaluated formulae of mean time to failure (MTTF) when all parameters his study works as intuitionistic fuzzy numbers. P. Kumar [7] discussed fuzzy reliability of some complex systems using lifetime parameters are in the form of octagonal intuitionistic fuzzy numbers. P. Kumar [7] calculated fuzzy reliability of different systems when reliability of any component of systems using generalized trapezoidal intuitionistic fuzzy numbers. P. Kumar [9] focuses on the fuzzy dependability of some complex systems by utilising the lifetime rate in the intuitionistic fuzzy Weibull distribution, which is referred to as intuitionistic fuzzy numbers. A. K. Verma et al. [10] discussed the uses of fuzzy numbers in different form for reliability evaluation using exponential and Weibull distribution.

2. Exponential Distribution

In general, an Exponential distribution is used to indicate the lifetime of a component *X*. The pdf is given as

$$f(x, \lambda) = \lambda e^{-\lambda x}, \; x > 0 \tag{1}$$

where λ is the failure rate of the component and is a crispset.

The reliability of any component using exponential distribution is given by:

$$R(t) = \int_t^\infty f(x, \lambda) = \int_t^\infty \lambda e^{-\lambda x} dx = e^{-\lambda t} \tag{2}$$

3. Weibull Distribution

The Weibull distribution is one of the most generally utilized distribution functions in reliability analysis. Its probability density function (pdf) is given as

$$f(x) = \frac{\beta}{\theta}\left(\frac{x}{\theta}\right)^{\beta-1} \exp\left(-\left(\frac{x}{\theta}\right)^{\beta}\right), \; x > 0, \; \theta > 0, \; \beta > 0 \tag{3}$$

where θ and β are the scale and shape parameter respectively.

The Weibull distribution is equivalent to the exponential distribution if $\beta = 1$, and it is simplified to the Rayleigh distribution if $\beta = 2$.

The component reliability function is determined using the Weibull distribution is as follows:

$$R(t) = \int_t^\infty \frac{\beta}{\theta}\left(\frac{x}{\theta}\right)^{\beta-1} \exp\left\{-\left(\frac{x}{\theta}\right)^{\beta}\right\} dx = \exp\left\{-\left(\frac{t}{\theta}\right)^{\beta}\right\} \tag{4}$$

4. Possibility Distribution of Reliability of any Component

(a) For exponential distributed failure rate model:

The reliability of component when component follow exponential distribution is calculated as

$$R(t) = e^{-\lambda t},\ t \geq 0$$

where λ is the failure rate function which is constant and t is the component's lifetime rate.

According to the preceding sections uncertainty exist in failure rate. Uncertainty behaviour is modelled using a trapezoidal intuitionistic fuzzy number with quadruplets $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_1'$ and λ_4' as $\lambda = \{(\lambda_1, \lambda_2, \lambda_3, \lambda_4), (\lambda_1', \lambda_2, \lambda_3, \lambda_4')\}$.

It is very arduous to investigate the accurate time of lifetime parameters t and also considered as trapezoidal intuitionistic fuzzy number.

In fuzzy set theory $\lambda \times t$ becomes,

$$Z = \lambda \times t \tag{5}$$

where λ and t are considered as intuitionistic fuzzy numbers and x is fuzzy multiplication operator so Z become an IFN.

So the reliability is calculated as

$$R = \exp(-Z),\ Z_L \leq Z \leq Z_R \tag{6}$$

where Z_L are lower limit and Z_R the upper limit of the membership and non membership grade.

(b) For Weibull distributed failure rate model:

The reliability of any component using Weibull distribution is calculated as follows,

$$R(t) = \exp[-((t - t_0)/\theta)^{\beta}],\ t \geq t_0 \tag{7}$$

Taking logarithm of eq. (7) then we get eq. (8)

$$\log_e[\log_e 1/R(t)] = \beta \cdot \log_e((t - t_0)/\theta) \tag{8}$$

where β is shape parameter, θ is scale parameter and t_0 is known as location parameter. Because all three parameters are obtained through experiment, there must be some uncertainty in all of them.

Let

$$z = \beta \otimes \ln[(t - t_0)/\theta] \tag{9}$$

where β, θ, t_0 and t considered as IFN and all the operations in eq. (9) are fuzzy operations.

From eq. (8) and eq. (9), we get

$$\ln[\ln 1/R(t)] = Z,\ Z_L \le Z \le Z_R \tag{10}$$

where Z_L is lower bound and Z_R is the upper bound of the membership grade and non-membership grade.

Example 1.

Given $\lambda = 0.003$ failures per hour and $t = 100$ hours and suppose both failure rate and times are in form of trapezoidal IFN.

i.e. $\lambda = [(0.00270,\ 0.00285,\ 0.00315,\ 0.00330);$

$(0.00260,\ 0.00285,\ 0.00315,\ 0.00340)]$ and

$t = [(90,\ 95,\ 105,\ 110),\ (85,\ 95,\ 105,\ 115)]$

The lower bound and upper bound of the membership and non-membership grade of λ and t are follows in Table 1 and 2.

Table 1. a and β cut of failure rate λ

(a, β)	$(\lambda_L^\alpha, \lambda_R^\alpha)$	$(\lambda_L^\beta, \lambda_R^\beta)$
0.0	(0.0027, 0.0033)	(0.0026, 0.0034)
0.1	(0.002715, 0.003285)	(0.002625, 0.003375)
0.2	(0.00273, 0.00327)	(0.00265, 0.00335)
0.3	(0.002745, 0.003255)	(0.002675, 0.003325)
0.4	(0.00276, 0.00324)	(0.0027, 0.0033)
0.5	(0.002775, 0.003225)	(0.002725, 0.003275)
0.6	(0.00279, 0.00321)	(0.00275, 0.00325)

0.7	(0.002805, 0.003195)	(0.002775, 0.003225)
0.8	(0.00282, 0.00318)	(0.0028, 0.0032)
0.9	(0.002835, 0.003165)	(0.002825, 0.003175)
1.0	(0.00285, 0.00315)	(0.00285, 0.00315)

Table 2. α and β cut of time t

(α, β)	$(t_L^{\alpha}, t_R^{\alpha})$	$(t_L^{\beta}, t_R^{\beta})$
0.0	(90, 110)	(85, 115)
0.1	(90.5,109.5)	(86, 114)
0.2	(91, 109)	(87,113)
0.3	(91.5, 108.5)	(88, 112)
0.4	(92, 108)	(89, 111)
0.5	(92.5, 107.5)	(90, 110)
0.6	(93, 107)	(91, 109)
0.7	(93.5, 106.5)	(92, 108)
0.8	(94, 106)	(93, 107)
0.9	(94.5, 105.5)	(94, 106)
1.0	(95, 105)	(95, 105)

The value of α and β-cut of Z using eq^n. (5) are given in Table 3.

Table 3. α and β-cut of Z.

(α, β)	$(Z_L^{\alpha}, Z_R^{\alpha})$	$(Z_L^{\beta}, Z_R^{\beta})$
0.0	(0.243, 0.363)	(0.27075, 0.33075)
0.1	(0.24571, 0.35971)	(0.26602, 0.33655)
0.2	(0.24843, 0.35643)	(0.26133, 0.34240)
0.3	(0.25117, 0.35317)	(0.25668, 0.34830)

0.4	(0.25392, 0.34992)	(0.25207, 0.35425)
0.5	(0.25669, 0.34668)	(0.24750, 0.36025)
0.6	(0.25947, 0.34347)	(0.24297, 0.36630)
0.7	(0.26227, 0.34027)	(0.23848, 0.37240)
0.8	(0.26508, 0.33708)	(0.23403, 0.37855)
0.9	(0.26791, 0.33391)	(0.22962, 0.38475)
1.0	(0.27075, 0.33075)	(0.22525, 0.39100)

The fuzzy reliability is calculated using eq^n. (6). The α and β-cut of fuzzy reliability are given in Table 4.

Table 4. α and β-cut of reliability

(α, β)	(R_L^α, R_R^α)	(R_L^β, R_R^β)
0.0	(.78427, .695586)	(.76281, .718385)
0.1	(.78215, .69788)	(.76643, .71423)
0.2	(.78003, .70017)	(.77003, .71006)
0.3	(.77789, .70246)	(.77362, .70589)
0.4	(.77575, .70474)	(.77719, .70169)
0.5	(.77361, .70703)	(.78075, .69751)
0.6	(.77146, .70931)	(.78429, .69329)
0.7	(.76931, .71158)	(.78782, .68908)
0.8	(.76715, .71385)	(.79134, .68485)
0.9	(.76498, .71612)	(.79484, .68062)
1.0	(.76281, .71838)	(.79832, .67638)

Example 2.

Given $\beta = 5, t_0 = 110, \theta = 800, t = 550$ and suppose all the parameters are in form of trapezoidal IFN.

$\beta = [(2, 4, 6, 8), (1, 4, 6, 9)]$, $t_0 = [(80, 100, 120, 140), (70, 100, 120, 150)]$,

$\theta = [(500, 700, 900, 1100), (400, 700, 900, 1200)]$ and

$t = [(250, 450, 650, 850), (200, 450, 650, 900)]$

The lower and upper bound of the membership grade and non-membership grade of β, t_0 and θ and t as follows in Tables 5 and 6 respectively.

Table 5. a and β cut of β and t_0

(a, β)	$(\beta_L^\alpha, \beta_R^\alpha)(\beta_L^\beta, \beta_R^\beta)$	$(t_{0L}^\alpha, t_{0R}^\alpha)(t_{0L}^\beta, t_{0R}^\beta)$
0.0	(2.0,8.0) (4.0,6.0)	(80,140) (70,150)
0.1	(2.2,7.8) (3.7,6.3)	(82,138) (73,147)
0.2	(2.4,7.6) (3.4,6.6)	(84,136) (76,144)
0.3	(2.6,7.4) (3.1,6.9)	(86,134) (79,141)
0.4	(2.8,7.2) (2.8,7.2)	(88,132) (82,138)
0.5	(3.0,7.0) (2.5,7.5)	(90,130) (85,135)
0.6	(3.2,6.8) (2.2,7.8)	(92,128) (88,132)
0.7	(3.4,6.6) (1.9,8.1)	(94,126) (91,129)
0.8	(3.6,6.4) (1.6,8.4)	(96,124) (94,126)
0.9	(3.8,6.2) (1.3,8.7)	(98,122) (97,123)
1.0	(4.0,6.0) (1.0,9.0)	(100,120) (100,120)

Table 6. a and β cut of θ and t.

(a, β)	$(\theta_L^\alpha, \theta_R^\alpha)(\theta_L^\beta, \theta_R^\beta)$	$(t_L^\alpha, t_R^\alpha)(t_L^\beta, t_R^\beta)$
0.0	(500,1100)(400,1200)	(250,850)(200,900)
0.1	(520,1080)(430,1170)	(270,830)(225, 875)
0.2	(540,1060)(460,1140)	(290,810)(250,850)

0.3	(560,1040)(490,1110)	(310,790)(275,825)
0.4	(580,1020)(520,1080)	(330,770)(300,800)
0.5	(600,1000)(550,1050)	(350,750)(325,775)
0.6	(620,980)(580,1020)	(370,730)(350,750)
0.7	(640,960)(610,990)	(390,710)(375,725)
0.8	(660,940)(640,960)	(410,690)(400,700)
0.9	(680,920)(670,930)	(430,670)(425,675)
1.0	(700,900)(700,900)	(450,650)(450,650)

The value of Z using eq. (9) is as:

$$Z = (-2.157619,\ -2.7725887,\ -3.1771065,\ -3.5024039,$$

$$-1.12393, -2.7725887, -3.1771065, -4.230032663)$$

The lower and upper bound of the membership grade and non membership grade of Z are given in Table 7.

Table 7. α and β cut of Z.

(α, β)	$(Z_L^{\alpha}, Z_R^{\alpha})$	$(z_L^{\alpha}, z_R^{\alpha})$
0.0	(-2.1576, -3.5024)	(-1.1239, -4.23003)
0.1	(-2.2219, -3.4699)	(-1.28879, -4.12474)
0.2	(-2.2806, -3.4373)	(-1.45366, -4.01945)
0.3	(-2.3423, -3.4048)	(-1.61853, -3.91416)
0.4	(-2.4036, -3.3723)	(-1.78339, -3.80886)
0.5	(-2.4652, -3.3398)	(-1.94826, -3.70357)
0.6	(-2.5266, -3.3073)	(-2.11315, -3.59828)
0.7	(-2.5881, -3.2747)	(-2.27799, -3.49299)
0.8	(-2.6496, -3.2422)	(-2.44286, -3.38769)

0.9	(-2.7111, -3.2096)	(-2.60773, -3.28239)
1.0	(-2.7726, -3.1772)	(-2.77258, -3.17711)

The reliability is calculated using eq. (10) is given by

$R = (0.800831451, 0.939413061, 0.959151464, 0.970324352,$

$0.72252733, 0.939413061, 0.95915464, 0.985553452)$

The α-cut and β-cut of fuzzy reliability are shown in Table 8.

Table 8. α and β-cut of fuzzy reliability.

(α, β)	$(R_L^{\alpha}, R_R^{\alpha})$	$(R_L^{\beta}, R_R^{\beta})$
0.0	(0.800832, 0.9703244)	(0.722527, 0.985554)
0.1	(0.8146896, 0.969207)	(0.744216, 0.982912)
0.2	(0.8285478, 0.968089)	(0.765905, 0.980277)
0.3	(0.842406, 0.9669725)	(0.787594, 0.977634)
0.4	(0.856265, 0.965856)	(0.809283, 0.974994)
0.5	(0.870123, 0.9647379)	(0.830971, 0.972354)
0.6	(0.883981, 0.9633621)	(0.852659, 0.969714)
0.7	(0.897839, 0.962503)	(0.874347, 0.967074)
0.8	(0.911697, 0.9613861)	(0.896036, 0.964434)
0.9	(0.92556, 0.9602687)	(0.917725, 0.961795)
1.0	(0.939413, 0.9591515)	(0.939413, 0.959155)

5. Conclusion

In this article, we evaluate the formula of the fuzzy reliability for any system when every component of this system uses exponential and Weibull distributions. All the parameters in both distributions are in the form of trapezoidal intuitionistic fuzzy numbers. An important characteristic of the suggested method is the capacity of fuzzy sets to emphasize between different values of variables within a certain range using the membership function. In the result, we also get the fuzzy reliability as a trapezoidal intuitionistic fuzzy numbers (TrIFN) for different reliability distributions (Exponential and Weibull). The α-cut and β-cut of different parameters are also calculated. Numerical examples are used to demonstrate the methodology. In examples, we evaluate α-cut and β-cut of the fuzzy reliability.

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