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A NEW APPROACH FOR SOLVING TRANSPORTATION PROBLEM WITH Z-NUMBER PARAMETERS

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Abstract

The transportation problem is a special linear programming problem that arises in many practical applications. In this paper, we present an innovative approach to solving the transportation problem with Z-number parameters (ZTP). First, we convert the ZTP to a FTP with a certain reliability. Then the FTP problem can be changed to TP by using suitable ranking methods, and then the TP can be solved by the usual method. This approach ensures that information regarding reliability is not lost.

Introduction

The transportation problem is an essential type of linear programming problem. The concept of fuzzy sets was introduced by Zadeh [1] in 1965, and it dealt with imprecision and vagueness in real-world situations. Fuzzy transportation problems were investigated by Chiang Kao [2], Chanas et al. [3], Chanas and Kutcha [4], Kaur and Kumar [5], Nagoor Gani and Abdul Rezak [6], Pandian et al. [7], Dinagar and Palanivel [8], Ganesan and others [9].

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In our previous paper, we had proposed a new approach to ZTP [10]. In this paper, we give an innovative approach to solving the TP with Z-number parameters (ZTP). We convert the Z-transportation problem (ZTP) to a fuzzy transportation problem (FTP) with a certain reliability. Then the FTP problem can be tackled by using suitable ranking methods, and then the transportation problem can be solved by the usual method. The proposed method utilizes R-type operations introduced by Stephen [11].

Preliminaries

Definition 1. Fuzzy set. A fuzzy set in X is defined as a set of ordered pairs $A = \{(x, \mu_A(x)) \mid x \in X\}$, where $\mu_A(x)$ is referred to as the membership function for the fuzzy set. If X is a group of objects represented by the generic symbol X, then X is the definition of a fuzzy set A in X. Each element of X is assigned a membership value between 0 and 1 using the membership function.

Definition 2. Triangular fuzzy number. A fuzzy number $\widetilde{A} = (a, b, c)$ is defined as a triangular fuzzy number only if the membership function of this is represented as:

$$\mu_A(x) = \begin{cases} \dfrac{x-a}{b-a} & a \le x \le b \\ \dfrac{c-x}{c-b} & b \le x \le c \\ 0 & c \le x \le d \end{cases}$$

Definition 3. Ranking function. Let $F(R)$ be a set of fuzzy numbers that is defined as a set of real numbers. Function $\Re : F(R) \to R$, is a ranking function that maps every fuzzy number into the real line.

Definition 4. Zadeh's definition of Z-number. A Z-number is an ordered pair of fuzzy numbers $Z = (A, B)$, associated with the uncertain real-valued variable X, with the first component A, a restriction on the possible values for X, as well as the second component B, a measure of the first component's reliability.

Definition 5. Triangular Z-number. In the Z-number $Z = (A, B)$, if both components A and B are triangular fuzzy numbers, then the corresponding Z-number is called a Triangular Z-number.

Definition 6. MIN R Type operation. Let $* \epsilon \{+, -, \times, /\}$, the MIN R operation on the set of all continuous Z-number is defined to be $(A, B)(*, MIN)(C, D) = (A * C, MIN(B, D))$, where the extension principle is used to calculate $A * C$ and $MIN(B, D) = B$ if $R_k(B) < R_k(D)$ and D if $R_k(D) < R_k(B)$.

Definition 7. Sum of two Triangular Z-numbers by MIN R. Let $Z_1 = (A_1, B_1)$ and $Z_2 = (A_2, B_2)$ be any two triangular Z-numbers, then $Z_1(+, MIN)Z_2 = (A_1, B_1)(+, MIN)(A_2, B_2) = (A_1 + A_2, MIN(B_1, B_2))$.

Definition 8. Product *R* Type operation.

Let $* \epsilon \{+, -, \times, /\}$, the product R operation on the set of all continuous Z-number is defined to be $(A, B)(*, \cdot)(C, D) = (A * C, B \cdot D)$, where $A * C$ and $B \cdot D$ are calculated using extension principle.

Definition 9. Sum of two Triangular Z-numbers by Product R. Let $Z_1 = (A_1, B_1)$ and $Z_2 = (A_2, B_2)$ be any two triangular Z-numbers, then $Z_1(+, \cdot)Z_2 = (A_1, B_1)(+, \cdot)(A_2, B_2) = (A_1 + A_2, B_1 \cdot B_2)$.

Converting ZTP to FTP

The ZTP can be mathematically formulated as follows

Minimize $W = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} x_{ij}, \; i = 1, 2, \ldots, m \; j = 1, 2, \ldots, n$

Subject to

$$\sum_{j=1}^{n} x_{ij}, \; a_i, \; i = 1, 2, \ldots, m$$

$$\sum_{i=1}^{m} x_{ij}, \; b_i, \; j = 1, 2, \ldots, n$$

and $x_{ij} \geq 0$, for all i and j,

where $C_{ij} = (C_{ij1}, C_{ij2})$, $a_i = (a_{i1}, a_{i2})$ and $b_j = (b_{j1}, b_{j2})$ are all Z-numbers.

The given Z-transportation problem is said to be balanced if

$$\sum_{i=1}^{m} a_{i_1} = \sum_{j=1}^{n} b_{j_1}$$

i.e., if the total supply equal to the total demand.

In this section, novel approach to solving the Z-transportation problem are suggested.

Let us discuss two distinct cases related to the ZTP.

Case (i). Here the summation is interpreted as a $(+, \min)$ operation.

Consider the ZTP

	D_1	D_2	$\cdots$	D_n	Supply
S_1	(C_{11}, C'_{11})	(C_{12}, C'_{12})	$\cdots$	(C_{1n}, C'_{1n})	(a_1, a'_1)
S_2	(C_{21}, C'_{21})	(C_{22}, C'_{22})	$\cdots$	(C_{2n}, C'_{2n})	(a_2, a'_2)
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
S_m	(C_{m1}, C'_{m1})	(C_{m2}, C'_{m2})	$\cdots$	(C_{mn}, C'_{mn})	(a_m, a'_m)
Demand	(b_1, b'_1)	(b_2, b'_2)	$\cdots$	(b_n, b'_n)	

The above ZTP is converted into the following FTP

	D_1	D_2	$\cdots$	D_n	Supply
S_1	C_{11}	C_{12}	$\cdots$	C_{1n}	a_1
S_2	C_{21}	C_{22}	$\cdots$	C_{2n}	a_2
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
S_m	C_{m1}	C_{m2}	$\cdots$	C_{mn}	a_m
Demand	b_1	b_2	$\cdots$	b_n	

The reliability of this system is

Min $(C'_{11}, C'_{12}, \ldots, C'_{1n}, C'_{21}, C'_{22}, \ldots, C'_{2n}, \ldots, C'_{m1}, C'_{m2}, \ldots, C'_{mn}, a'_1, a'_2, \ldots, a'_m, b'_1, b'_2, \ldots, b'_n)$

Solving the FTP:

Consider the case where $C_{11}, C_{12}, \ldots, C_{1n}, C_{21}, C_{22}, \ldots, C_{2n}, \ldots, C_{m1}, C_{m2}, \ldots, C_{mn}, a_1, a_2, \ldots, a_m, b_1, b_2, \ldots, b_n \in \mathcal{F},$ a set of fuzzy numbers.

The FTP can be converted to TP by using appropriate ranking function $R : F \to (-\infty, \infty)$

	D_1	D_2	…	D_n	Supply
S_1	$R(C_{11})$	$R(C_{12})$	…	$R(C_{1n})$	$R(a_1)$
S_2	$R(C_{21})$	$R(C_{22})$	…	$R(C_{2n})$	$R(a_2)$
⋮	⋮	⋮	⋮	⋮	⋮
S_m	$R(C_{m1})$	$R(C_{m2})$	…	$R(C_{mn})$	$R(a_m)$
Demand	$R(b_1)$	$R(b_2)$	…	$R(b_n)$	

Then the TP can be solved by usual method.

Example 1. Consider the ZTP where $C_{11}, C_{12}, \ldots, C_{1n}, C_{21}, C_{22}, \ldots, C_{2n}, \ldots, C_{m1}, C_{m2}, \ldots, C_{mn}, a_1, a_2, \ldots, a_m, b_1, b_2, \ldots, b_n$ are all triangular fuzzy numbers. Let $A = (a_1, a_2, a_3)$ be a triangular fuzzy number. Consider the ranking function $R(A) = \frac{a_1 + a_2 + a_3}{3}$.

	1	2	3	4	Supply
A	(C_{11}, C'_{11})	(C_{12}, C'_{12})	(C_{13}, C'_{13})	(C_{14}, C'_{14})	(a_1, a'_1)
B	(C_{21}, C'_{21})	(C_{22}, C'_{22})	(C_{23}, C'_{23})	(C_{24}, C'_{24})	(a_2, a'_2)
C	(C_{31}, C'_{31})	(C_{32}, C'_{32})	(C_{33}, C'_{33})	(C_{31}, C'_{31})	(a_3, a'_3)
Demand	(b_1, b'_1)	(b_2, b'_2)	(b_3, b'_3)	(b_4, b'_4)	

Where the C_{ij}, a_i and b_j are given in the following table:

	1	2	3	4	Supply
A	(approx.6, Very sure)	(approx.4, sure)	(approx.1, very sure)	(approx.5, almost sure)	(approx.14, sure)
B	(approx. 8, sure)	(approx.9, very sure)	(approx.2, sure)	(approx.7, sure)	(approx.16, Almost sure)
C	(approx.4, sure)	(approx.3, almost sure)	(approx.6, sure)	(approx.2,very sure)	(approx.5, sure)
Demand	(approx.6, very sure)	(approx.10, sure)	(approx.15, very sure)	(approx. 4, almost sure)	

The linguistic terms can be converted to fuzzy terms in the following table:

	1	2	3	4	Supply
A	((4, 6, 8), (.85, .9, .95))	((3, 4, 5), (.75, .8, .85))	((0, 1, 2), (.85, .9, .95))	((4, 5, 6), (.65, .7, .75))	((12, 14, 16), (.75, .8, .85))
B	((7, 8, 9), (.75, .8, .85))	((8, 9, 10), (.85, .9, .95))	((1, 2, 3), (.75, .8, .85))	((6, 7, 8), (.75, .8, .85))	((15, 16, 17), (.65, .7, .75))
C	((2, 4, 6), (.75, .8, .85))	((1, 3, 5), (.65, .7, .75))	((5, 6, 7), (.75, .8, .85))	((1, 2, 3), (.75, .8, .85))	((4, 5, 6), (.75, .8, .85))
Demand	((4, 6, 8), (.85, .9, .95))	((9, 10, 11), (.75, .8, .85))	((13, 15, 17), (.85, .9, .95))	((3, 4, 5), (.65, .7, .75))	

Solution:

First, we convert the given ZTP into FTP

	1	2	3	4	Supply
A	(4,6,8)	(3,4,5)	(0,1,2)	(4,5,6)	(12,14,16)
B	(7,8,9)	(8,9,10)	(1,2,3)	(6,7,8)	(15,16,17)
C	(2,4,6)	(1,3,5)	(5,6,7)	(1,2,3)	(4,5,6)
Demand	(4,6,8)	(9,10,11)	(13,15,17)	(3,4,5)	

The reliability of this FTP system is Min (very sure, sure, almost sure)

$$= Min\,((.85,\ .9,\ .95),\ (.75,\ .8,\ .85),\ (.65,\ .7,\ .75))$$

$$= (.65,\ .7,\ .75) = \text{almost sure}$$

	1	2	3	4	Supply
A	6	4	1	5	14
B	8	9	2	7	16
C	4	3	6	2	5
Demand	6	10	15	4	35

Since $\sum a_i = \sum b_j = 35,$ the problem is balanced TP. Solving the TP, we get the following optimal solution is

$$x_{11} = 4; x_{12} = 10; x_{21} = 1; x_{23} = 15; x_{31} = 1; x_{34} = 4$$

The total transportation cost

$$= 6 \times 4 + 4 \times 10 + 8 \times 1 + 2 \times 15 + 4 \times 1 + 2 \times 4$$

$$= 24 + 40 + 8 + 30 + 4 + 8$$

$$= Rs.114\,/-$$

The solution for the FTP is

$$x_{11} = 4; x_{12} = 10; x_{21} = 1; x_{23} = 15; x_{31} = 1; x_{34} = 4$$

The fuzzy transportation cost

$$= (4,\ 6,\ 8) \times 4 + (3,\ 4,\ 5) \times 10 + (7,\ 8,\ 9) \times 1 + (1,\ 2,\ 3) \times 15 + (2,\ 4,\ 6) \times 1$$

$$+ (1,\ 2,\ 3) \times 4$$

$$= (16,\ 24,\ 32) + (30,\ 40,\ 50) + (7,\ 8,\ 9) + (15,\ 30,\ 45) + (2,\ 4,\ 6) + (4,\ 8,\ 12)$$

$$= (74,\ 114,\ 154)$$

The solution for the ZTP is

$$x_{11} = 4; x_{12} = 10; x_{21} = 1; x_{23} = 15; x_{31} = 1; x_{34} = 4$$

The Z-transportation cost $= ((74,\ 114,\ 154),\ (.65,\ .7,\ .75))$

Case (ii).

Here the summation is interpreted as a $(+,\ \cdot)$ operation.

The reliability of this system is

Product

$(C'_{11}, C'_{12}, \ldots, C'_{1n}, C'_{21}, C'_{22}, \ldots, C'_{2n}, \ldots, C'_{m1}, C'_{m2}, \ldots, C'_{mn}, a'_1, a'_2, \ldots, a'_m, b'_1, b'_2, \ldots, b'_n)$

Solving the FTP:

FTP can be solved as in Case (i).

Example 2.

Consider the ZTP where $C_{11}, C_{12}, \ldots, C_{1n}, C_{21}, C_{22}, \ldots, C_{2n}, \ldots, C_{m1}, C_{m2}, \ldots, C_{mn}, a_1, a_2, \ldots, a_m, b_1, b_2, \ldots, b_n$ are all triangular fuzzy numbers. Let $A = (a_1, a_2, a_3)$ be a triangular fuzzy number. Consider the ranking function $R(A) = \dfrac{a_1 + a_2 + a_3}{3}$.

	1	2	3	4	Supply
A	(C_{11}, C'_{11})	(C_{12}, C'_{12})	(C_{13}, C'_{13})	(C_{14}, C'_{14})	(a_1, a'_1)
B	(C_{21}, C'_{21})	(C_{22}, C'_{22})	(C_{23}, C'_{23})	(C_{24}, C'_{24})	(a_2, a'_2)
C	(C_{31}, C'_{31})	(C_{32}, C'_{32})	(C_{33}, C'_{33})	(C_{31}, C'_{31})	(a_3, a'_3)
Demand	(b_1, b'_1)	(b_2, b'_2)	(b_3, b'_3)	(b_4, b'_4)	

Where the C_{ij}, a_i and b_j are given in the following table:

	1	2	3	Supply
A	(approx.8, Very sure)	(approx.7, sure)	(approx.3, very sure)	(approx.50, sure)
B	(approx. 3, sure)	(approx.8, very sure)	(approx.9, sure)	(approx.70, Almost sure)
C	(approx.11, sure)	(approx.3, almost sure)	(approx.5, sure)	(approx.90, sure)
Demand	(approx.60, very sure)	(approx.70, sure)	(approx.80, very sure)	

The linguistic terms can be converted to fuzzy terms in the following table:

	1	2	3	Supply
A	((6, 8, 10), (.75,.8,.85))	((6, 7, 8), (.75,.8,.85))	((1, 3, 5), (.85,.9,.95))	((40, 50, 60), (.75,.8,.85))
B	((2, 3, 4), (.75,.8,.85))	((7,8,9), (.85,.9,.95))	((7, 9, 11), (.75,.8,.85))	((50, 70, 90), (.85,.9,.95))
C	((10,11,12), (.85,.9,.95))	((1, 3, 5), (1, 1, 1))	((4, 5, 6), (.75,.8,.85))	((80,90,100), (.75,.8,.85))
Demand	((50,60,70), (.85,.9,.95))	((50,70,90), (.75,.8,.85))	((65,80,95), (.85,.9,.95))	

Solution:

First, we convert the given ZTP into FTP

	1	2	3	Supply
A	(6,8,10)	(6,7,8)	(1,3,5)	(40,50,60)
B	(2,3,4)	(7,8,9)	(7,9,11)	(50,70,90)
C	(10,11,12)	(1,3,5)	(4,5,6)	(80,90,100)
Demand	(50,60,70)	(50,70,90)	(65,80,95)	

The reliability of this FTP system is Product (very sure, sure, completely sure)

$= ((.85, .9, .95) \cdot (.75, .8, .85) \cdot (1, 1, 1))$

$= (.6375, .72, .8075)$

We can rewrite it as the following TP using the ranking function:

	1	2	3	4	Supply
A	8	7	3	50	8
B	3	8	9	70	3
C	11	3	5	90	11
Demand	60	70	80	210	60

Since $\sum a_i = \sum b_j = 210$, the problem is balanced TP. Solving the TP, we get the following optimal solution is

$$x_{13} = 50; x_{21} = 60; x_{23} = 10; x_{32} = 70; x_{33} = 50$$

The total transportation cost

$$= 3 \times 50 + 3 \times 60 + 9 \times 10 + 3 \times 70 + 5 \times 20$$

$$= 150 + 180 + 90 + 210 + 100$$

$$= \text{Rs.}730/-$$

The solution for the FTP is

$$x_{13} = 50; x_{21} = 60; x_{23} = 10; x_{32} = 70; x_{33} = 50$$

The fuzzy transportation cost

$$= (1,\ 3,\ 5) \times 50 + (2,\ 3,\ 4) \times 60 + (7,\ 9,\ 11) \times 10 + (1,\ 3,\ 5) \times 70 + (4,\ 5\ \ 6) \times 20$$

$$= (50,\ 150,\ 250) + (120,\ 180,\ 240) + (70,\ 90,\ 110) + (70,\ 210,\ 350)$$

$$+ (80,\ 100,\ 120)$$

$$= (390,\ 730,\ 1070)$$

The solution for the ZTP is

$$x_{13} = 50; x_{21} = 60; x_{23} = 10; x_{32} = 70; x_{33} = 50$$

The Z-transportation cost $= ((390,\ 730,\ 1070),\ (.6375,\ .72,\ .8075))$

Conclusion

An innovative approach to solving ZTP has been provided in this paper. Here we have demonstrated the approach using the numerical examples.

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